Homework 11 due April 17

Select five problems from below or make up some of your own.

1. (Open problem) Can one shatter any set of six points by a square in general position?

2. What is the VC-dimension of triangles? Right triangles?

3. What is VC-dimension for a corner? I.e. all points (x,y) such that either
   (1) $(x - x_0, y - y_0) \geq (0,0)$,
   (2) $(x_0 - x, y - y_0) \geq (0,0)$,
   (3) $(x_0 - x, y_0 - y) \geq (0,0)$, or
   (4) $(x - x_0, y_0 - y) \geq (0,0)$

for some (x_0, y_0).

4. Show that seven points of a regular septagon are separatable by rotated rectangles.

5. Prove that no set of eight points can be separated by rotated rectangles.

6. Show that the VC dimension of arbitrary right triangles is seven.

7. Show that the VC dimension of arbitrary triangles is seven.

8. Show that the VC dimension of axes-aligned right triangles with the right angle in the lower left corner is four.

9. Prove that the VC dimension of 45°, $45^\circ, 90^\circ$ triangles with right angle in the lower left is four.

10. Prove that no set of six points can be shattered by squares in arbitrary position.

11. Prove that the VC dimension of convex polygons is infinite.

12. Create list of simple shapes for which we can calculate the VC-dimension.

13. Consider a circle containing three points. Is it possible to shrink the circle and move its center so that two of the points are on the resulting circle and the resulting circle is completely contained within the original circle? Show how to do this. Is it impossible to continue shrinking the circle and moving the center in such a way that all three points are on the circumference and the resulting circle is completely contained in the original? Prove your answer.