

Select five problems from below or make up some of your own.

1. Suppose you have a thousand random numbers in the range -1 to 1 and the probability distribution from which they were drawn is semi circular. If you sort the number and then plot their values in increasing order what shape curve will you get? Try to do this mathematically.
2. Generate a thousand random numbers in the range -1 to 1 according to a semicircular distribution. Sort them and then plot their values in increasing order.
3. Create a 1200 by 1200 probability matrix consisting of nine 400 by 400 blocks as shown below where the probabilities within each block have the same value as given.

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$
$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

Convert the matrix to a 0/1 matrix by randomly flipping a coin with the given probabilities of heads. Compute the singular values, sort them, and then plot them. What do you conclude from the plot?

Using the first three singular vectors find an approximation to your 0/1 matrix.

4. Maximize $\sum_{i=1}^n \sigma_i a_i$ subject to the restraint $\sum_{i=1}^n a_i^2 = 1$.

4. Problem should have been Maximize $\sum_{i=1}^n (\sigma_i a_i)^2$ subject to the restraint $\sum_{i=1}^n a_i^2 = 1$.

Do either or some other problem.

5. Prove that $\text{trace}(A) = \lambda_1 + \lambda_2 + \dots + \lambda_n$. Hint: Consider the coefficient of λ^{n-1} in $\det(A - \lambda I)$. The roots of $\det(A - \lambda I) = 0$ are $\lambda_1, \lambda_2, \dots, \lambda_n$.

6. Assuming that $\text{trace}(A) = \lambda_1 + \lambda_2 + \dots + \lambda_n$ prove that $\text{trace}(A^2) = \lambda_1^2 + \lambda_2^2 + \dots + \lambda_n^2$.

7. Prove that

$$\int \sin^n \theta d\theta = -\frac{\sin^{n-1} \theta \cos \theta}{n} + \frac{n-1}{n} \int \sin^{n-2} \theta d\theta$$

8. On the course web site just below the homework there is a file of distances between all pairs of 28 cities. One could convert this distance matrix into a similarity matrix which would be of the form AA^T where the matrix A is a 28 by 2 matrix of city coordinates. From the similarity matrix one could calculate the first two singular vectors and use those for coordinates and draw a map of the cities. Try it.

9. Create some heavy tailed probability distributions. A heavy tailed distribution is where the distribution decays as some power of x . Calculate several of the moments for several of these.