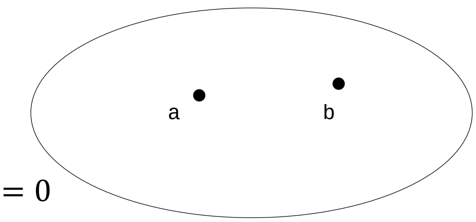
Random Walk in 3-Dimensions

CS 4850 Notes – Lecture 22, Monday, March 9, 2009

Scribes: Cameron Allen (CPA9) and Eric First (ELF24)

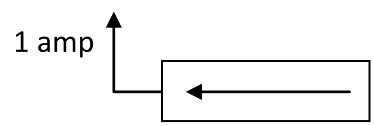
Review

1. v_x is probability that a walk starting at x reaches a before b.



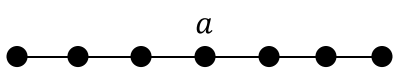
$v_a = 1 \quad v_b = 0$

2. i_{xy} is net traversals of edge xy in walk from a to b.

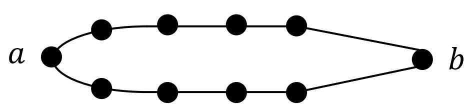


3. $P_{escape} = \frac{c_{eff}}{c_a}$

1D



Bend line:



$\lim \rightarrow \infty$: will return to a with probability 1.

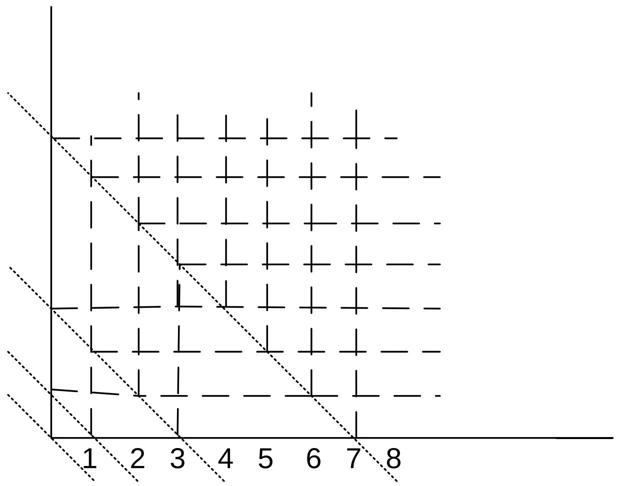
2D

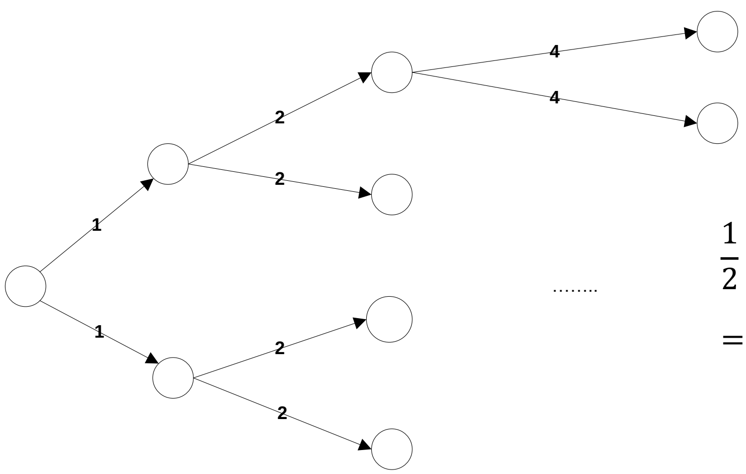
Many parallel paths but resistance is still infinite.

3D

Start with 2D, then will generalize to 3D.

Upper bound on R_{eff}

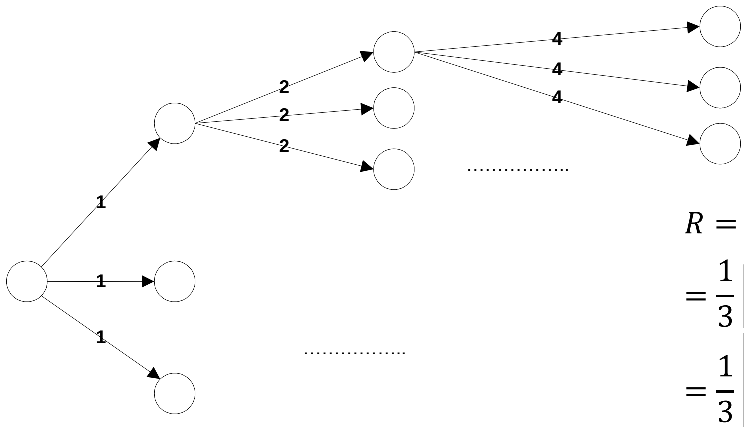




$$\frac{1}{2}\left[1+\frac{1}{2}\left[2+\frac{1}{2}(4+R)\right]\right]$$
$$=\frac{1}{2}+\frac{1}{4}(2)+\frac{1}{8}(4)+\frac{1}{16}(R)$$

$$R=\frac{1}{2}+\frac{1}{4}(2)+\frac{1}{8}(4)+\cdots=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\cdots=\infty$$

$$x+y+z=2^k-1$$

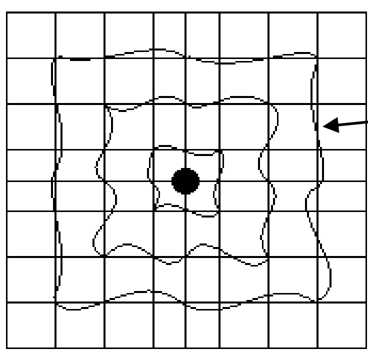


$$R = \frac{1}{3} + \frac{1}{9}(2) + \frac{1}{27}(4) + \cdots$$
$$= \frac{1}{3}\left[1 + \frac{2}{3} + \frac{4}{9} + \cdots\right]$$
$$= \frac{1}{3}\left[\frac{1}{1 - \frac{2}{3}}\right] = 1 = c_{eff}$$

$$P_{escape} \geq \frac{1}{6}$$

($c_a = 6$ because there are 6 edges coming out of a if you consider more than just 1 quadrant)

Lower bound on R_{eff}



short – sets $R = 0$ in between

2D:

$$\frac{1}{4}\left(1 + \frac{1}{3} + \frac{1}{5} + \cdots\right)$$

3D:

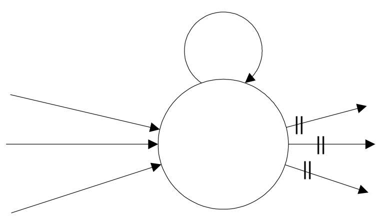
$$R_{lower\ \ bound} = \frac{1}{6}\left[1 + \frac{1}{9} + \frac{1}{25} + \cdots\right] \geq \frac{1.23}{6} \geq 0.2$$

$$P_{escape} = \frac{1}{6\, R_{eff}} \leq \frac{5}{6}$$

Page Rank and Hitting Time

Directed graphs

Stationary probability = page rank

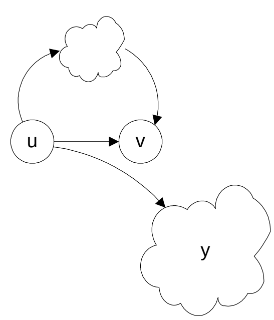


Hitting time h_{uv} = expected time to reach v from u.

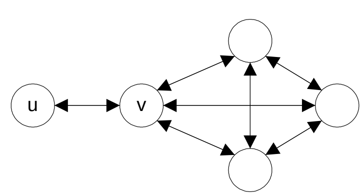
Generalize – personalized page rank and personalized hitting time using probability distribution

Undirected graphs

If I add an edge to graph, do I raise or lower hitting time?

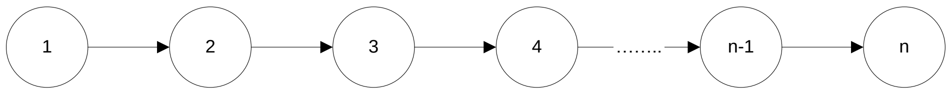


Hitting time is not symmetric:



$$h_{uv} = 1$$
$$h_{vu} > 1$$

Lemma: Expected time of random walk starting at one end of a chain of n vertices to reach the other end is $\Theta(n^2)$.



$$h_{ij} \quad i < j$$

$$h_{12} = 1$$

$$h_{i,i+1} = \frac{1}{2}(1) + \frac{1}{2}\left[1 + h_{i-1,i} + h_{i,i+1}\right]$$

$$\frac{1}{2}h_{i,i+1} = 1 + \frac{1}{2}h_{i-1,i}$$

$$h_{i,i+1} = h_{i-1,i} + 2$$

$$h_{i,i+1} = 2(i-1) + h_{12} = 2i - 1$$

$$h_{1n} = \sum_{i=1}^{n-1} h_{i,i+1} = \sum_{i=1}^{n-1}(2i-1) = (n-1)^2$$