CS 4850 LECTURE 17 - FEBRUARY 25, 2009

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Growing Graph with Preferential Attachment

At time $t = 0$ no vertices
At each unit of time, add one vertex
With probability δ add edge connecting new vertex to an existing vertex, selecting the existing vertex with probability proportional to degree of the existing vertex.
Let $d_i(t)$ be degree of i^{th} vertex at time t.

$$\frac{d}{dt} d_i(t) = \delta \frac{d_i(t)}{\sum_{j=1}^{t} d_j(t)} = \delta \frac{d_i(t)}{2\delta t} = \frac{d_i(t)}{2t}$$

Solution to differential equation is $d_i(t) = at^{\frac{1}{2}}$

$$at_i^{\frac{1}{2}} = \delta \Rightarrow a = \frac{\delta}{t_i^{\frac{1}{2}}} \Rightarrow d_i(t) = \delta \sqrt{\frac{t}{t_i}}$$

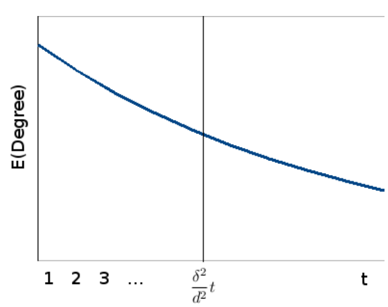

Figure 1

$$d = \delta \sqrt{\frac{t}{t_i}} \qquad d^2 = \frac{\delta^2}{t_i} t \Rightarrow t_i = \frac{\delta^2}{d^2} t$$

$$Pr(\text{vertex degree } > d) = 1 - \frac{\delta^2}{d^2} \qquad \text{pdf} = \frac{\partial}{\partial d}\left(1 - \frac{\delta^2}{d^2}\right) = \frac{2\delta^2}{d^2}$$

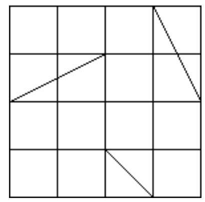

FIGURE 2. n x n graph with 3 long edges added

SMALL WORLD GRAPHS

Find shortest path using path using only local information. Assume an n x n graph, with each vertex connected to 4 neighbors (less on edges), and add one edge to a further vertex. Selection of the further vertex is proportional to distance.

$$Pr[(u,v)] \propto \frac{d^{-r}(u,v)}{\sum\limits_{w \neq v} d^{-r}(u,w)}$$

$r = 0$ edge ends uniformly distributed

$r < 2$ always short paths, but no efficient algorithm to find them
probability of encountering edge that gets you closer too low

$r = 2$ $\exists$ alogrithm $(\ln n)^2$ take edge that gets you closest to destination each time

$r > 2$ there may or may not be a short path, but no algorithm exists to find them
probability of encountering long edge too low

$r = \infty$ no long edges

Proof of algorithm:
s start point, t end point

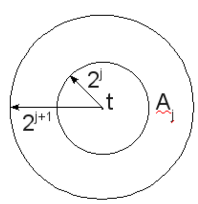

FIGURE 3

$$A_j \in (2^j, 2^{j+1}]$$

at most $\log_2 n$ phases, $E(\text{time}) = \ln n$
Time of algorithm is $(\ln n)^2$
Lemma: For $r = 2$ $\exists$ constant c such that the probability that a long distance edge from u goes to v is at least $c\frac{d^{-2}(u,v)}{\ln n}$.
Proof: $Pr[(u,v)] \propto \frac{d^{-r}(u,v)}{\sum\limits_{w \neq v} d^{-r}(u,w)}$
To Be Continued ...