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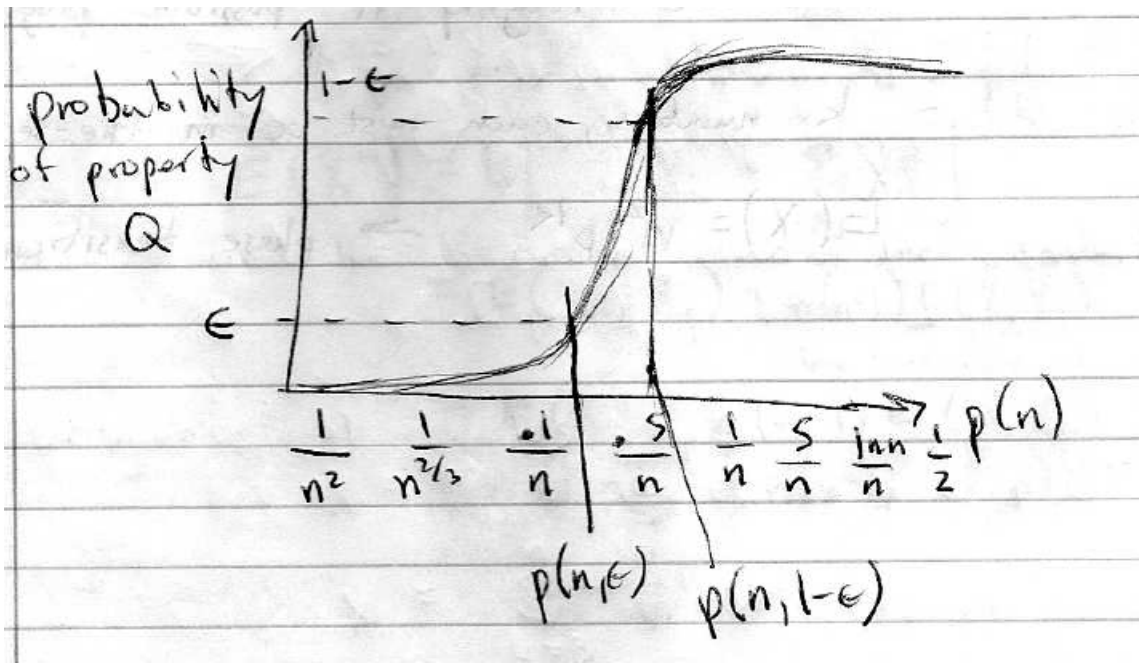
Increasing Properties

Q is an increasing property of a graph G if when a graph has property Q, any graph obtained from G by adding edges has the property Q.

Notation

Let p(n,ε) be the function p(n) such that the probability $N_{p(n)}$ has property Q equal ε.
N_p=N(n,p)



For any ε>0, there exists a constant m such that:
P(n,1- ε)≤mP(n, ε)

Theorem

Every increasing property of N_p has a threshold.

Proof

Let 0< ε< $\frac{1}{2}$ and let m be an integer such that $(1-\varepsilon)^m \le \varepsilon$.

Consider union of m independent copies of $N_{p(n,\ \varepsilon)}$. The union is equivalent to a single N_q where we select an integer with a higher probability.

$$q = 1-[1-P(n,\varepsilon)]^m$$

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For example:

P(n, ε)= $\frac{3}{n}$; N_p={5,17,81}; N_p={10,11}; $N_q \subset$ {5,10,11,17,81,…}

q=1-[1- P(n, ε)]^m≤m P(n, ε)

By the previous inequality and the fact that Q is an increasing property, we have:
A. Prob(N_q εQ) ≤Prob($N_{mp(\varepsilon)}$εQ)

Since Q is an increasing property if one or more N_p(ε) has property then N_q has Property.

If N_q does not have property then for all N_p(ε), N_p(ε) does not have property.

$$\forall N_{p(\varepsilon)} : N_{p(\varepsilon)} \notin Q$$

Prob(N_q ∉ Q) ≤Prob($\forall N_{p(\varepsilon)} : N_{p(\varepsilon)} \notin Q$)

It is possible none of $N_{p(\varepsilon)}$ have Q, but N_qεQ.

Example if the property is that the set has 2 elements, then all N_p(ε) have one element, but N_q might have more.

$$\Pr ob(N_q \notin Q) \le [1 - \Pr ob(N_{p(\varepsilon)} \in Q)]^m \le (1-\varepsilon)^m \le \varepsilon$$

B. Prob(N_q εQ)≥1-ε

Consequently we can sandwich the $\Pr ob(N_q \in Q)$ and conclude:

$$1-\varepsilon \le \Pr ob(N_q \in Q) \le \Pr ob(N_{mp(\varepsilon)} \in Q)$$

C. Therefore, m(p(ε))≥p(1-ε)

For 0<ε<.5: P(ε)≤P(0.5)≤P(1- ε)≤mp(ε)

Since P(1/2) is between P(ε) and some constant m times P(ε), P(1/2) is asymptotically equivalent of P (ε). P(1/2) is a threshold.

[P(ε), P(1- ε)] is bounded by a constant.

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<u>Emergence of Cycles</u>

Let x be the number of cycles in a graph, G(n,p). Define Cycles to length 3 or larger, removing self-loops and two nodes pointing to each other.

$$x = \sum_{k=3}^{n} \binom{n}{k} \frac{(k-1)!}{2}$$

$$E(x) = \sum_{k=3}^{n} \binom{n}{k} \frac{(k-1)!}{2} p^k$$

$$E(X) \leq \sum_{k=3}^{n} \frac{(np)^k}{2k}$$

If p<<1/n E(x) $\rightarrow$ 0, we can surly a graph selected at random, almost surly has no cycle.

p≈c/n	c>1	E(x) goes to ∞
	c<1	E(x) converges

Second moment method:

p<<1/n	no cycles	
p=c/n	c<1	finite number of cycles
p=c/n	c>1	infinite number of cycles