\[
\lim_{d \to -\infty} V(d) = 0
\]

\[
\sqrt{d}
\]

\[
[x_1, x_2, \ldots, x_d]
\]

\[
e^{-\frac{(x_1^2 + x_2^2 + \ldots + x_d^2)}{2}}
\]

\[
E(x_1^2 + x_2^2 + \ldots + x_d^2) = dE(x_i^2) = d\sigma^2
\]

\[
\sigma^2 = E((x_i - \bar{x})^2)
\]

**Alternate Computation of Radius:**

\[
e^{-\frac{1}{2} r^{d-1}}, \text{ where the first term is probability density, and the second term is volume.}
\]

\[
mass = e^{-\frac{1}{2} r^{d-1}}
\]

\[
\frac{d mass}{dr} = (d - 1)r^{d-2} e^{-\frac{1}{2} r^{d-2}} - r^{d-1} e^{-\frac{1}{2} r^{d-2}} e^{-\frac{1}{2} r^2}
\]

\[
\frac{d mass}{dr} = (d - 1)r^{d-2} e^{-\frac{1}{2} r^{d-2}} - r^d e^{-\frac{1}{2} r^2} = 0
\]

\[
d - 1 = r^2
\]

\[
r = \sqrt{d}
\]

**Place** \(r = \sqrt{d}\) into \(e^{-\frac{1}{2} \sqrt{d} \cdot r^{d-1}}\): \(e^{-\frac{1}{2} \sqrt{d} \cdot d^{d-1}}\)

**Let** \(r' = \sqrt{d} + k\), \(e^{-\frac{(\sqrt{d} + k)^{d-1}}{2}}\)
\[
\frac{e^{-\frac{d+2\sqrt{dk+k^2}}{2}\sqrt{d}^{d-1}}}{e^{-\frac{d}{2}\sqrt{d}^{d-1}}} = e^{-\frac{k^2}{2}} e^{\frac{k^2}{2}} = 1
\]
Generating Points:

\[ \sqrt{\delta} + 2d \geq \sqrt{2d} + c \]

\[ d^{1/2} \geq \text{constant} \]

Dimension Reduction
\[ [X_1, X_2, \ldots, X_k, X_{k+1}, \ldots, X_d] \text{ d dimension down to k-dimension} \]

\[ [X_1, X_2, \ldots, X_k, 0, 0, \ldots, 0] \text{ Project on first k-dimensions, zero out the rest} \]

\[ \sqrt{d} \quad \text{d dimension} \]

\[ \sqrt{k} \quad \text{k dimension} \]

Distances get shrunk by \( \sqrt{\frac{k}{d}} \)

\[ \sqrt{\delta^2 + 2k} \geq \sqrt{2k + c} \]

\[ \sqrt{\delta^2 + 2k} = \sqrt{2k \left(1 + \frac{\delta^2}{2k}\right)^{1/2}} = \sqrt{2k \left(1 + \frac{\delta^2}{4k} + \cdots\right)} \geq \sqrt{2k + c} \]

\[ \frac{\delta^2 \sqrt{2k}}{4k} \geq c \quad \delta^2 \geq \text{constant} \]

\[ \begin{bmatrix}
o_1 & 0 & 0 & 0 & 0 \\
0 & o_2 & 0 & 0 & 0 \\
. & . & . & . & V \\
. & . & . & . & .
\end{bmatrix} \]

Data Streams:

\( a_1, a_2, a_3, \ldots \)

Assume the a's are credit card #’s

1. Number of distinct elements

2. Number of occurrences of a given element

3. Number of occurrences of frequently occurring elements
4. Sketch (Signature of the data Stream)

1) What is a wanted: a low space algorithm:
   If m types of elements you could get exact answers if you have m spaces. If you have space
   less than m, then you cannot get an exact answer. If two streams have different number of
   distinct elements then they must lead to different internal states.

2) If same number but different subsets, then different stages. 2^n states, and need m memory.
   Log(m) space algorithm.