

Random walks on directed Graphs

select a vertex at random or select an edge and continue walk.
This means adding a "restart" node.

Thm: strongly connected and a-periodic implies

- 1) unique stationary probability
- 2) $\lim_{t \rightarrow \infty} \frac{N(i,t)}{t} = \pi_i =$ average # of times at vertex i
- 3) expected time between visits to vertex i is $\frac{1}{\pi_i} =$ return time

Adjacency Matrix: $\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \sim & \sim & \sim & & \\ \sim & \sim & \sim & & \end{pmatrix}$ ← normalize rows so they sum to 1

$$A\pi = \pi$$

similar to: $Ax = \lambda x \Rightarrow (A - \lambda I)x = 0$

to get a non-trivial solution, $\det(A - \lambda I) = 0$

$\Rightarrow n^{\text{th}}$ degree polynomial $\Rightarrow n$ solutions

Def. A Markov process is a random process whose future behavior depends on current state, not how you got there.

persistent state - strongly connected component
with no out edges

periodic

irreducible - single strongly connected component

ergodic state - persistent aperiodic

A Markov chain is ergodic if every state is ergodic \equiv SCC + aperiodic

Discovery time (v) — time to first reach of a vertex v from a uniform random start

to increase page rank:

- 1) capture the random walk — short cycles
- 2) capture the restart: have a lot of pages

Why not rank pages by discovery time? \rightarrow inefficient

- Paper by Altman and Tennenholtz

"Ranking Systems, The Pagerank Axioms"

5 Axioms \Rightarrow ranking is the same as pagerank

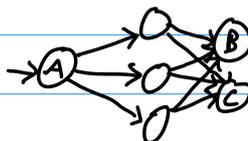
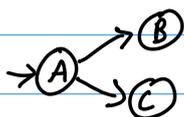
however: the problem discussed is about an unweighted SCC, and is thus not the same as our model

Axiom 1: isomorphic vertices have equal rank

Axiom 2: Adding a self-loop to a vertex v does not change the rank of any pair of other vertices
Rank of v can only increase

Axiom 3: vote by committee

\Rightarrow the following two graphs have the same page-rank for vertices B and C



Axiom 4: If several vertices have the same set of successors, they can be collapsed into a single vertex

Axiom 5: vertices A, B, C have the same rank in the following two graphs:

