Random walks on directed graphs

select a vertex at random or select an edge and traverse walk. This means adding a "restart" node.

Then, strongly connected and a-periodic implies

1) unique stationary probability
2) \( \lim_{t \to \infty} \frac{M(i,t)}{t} = \pi_i = \text{average } \# M \text{ times at vertex } i \)
3) expected time between visits to vertex i is \( \frac{1}{\pi_i} = \text{return time} \)

Adjacency Matrix: \( \begin{pmatrix} 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \) they sum to 1

\( A \pi = \pi \)

similar to: \( Ax = \lambda x \Rightarrow (A - \lambda I)x = 0 \)

to get a non-trivial solution, \( \det (A - \lambda I) = 0 \)
\( \Rightarrow n^{th} \text{ degree polynomial } \Rightarrow n \text{ solutions} \)

Def. A Markov process is a random process whose future behavior depends on current state, not how you got here.

persistent state - strongly connected component with no cut edges

periodic

irreducible - single strongly connected component

ergodic state - persistent aperiodic
A Markov chain is ergodic if every state is ergodic $\equiv$ scc + aperiodic

Discovery time ($v$) — time to first reach of a vertex $v$
from a uniform random start

to increase page rank:
1) capture the random walk — short cycles
2) capture the restart — have a lot of pages

Why not rank pages by discovery time? ⇒ inefficient

- Paper by Altman and Tenenhaus

"Ranking Systems, The PageRank Axioms"
5 Axioms ⇒ ranking is the same as pagerank
however: the problem discussed is about an
unweighted SCC, and is thus not the
same as our model

Axiom 1: isomorphic vertices have equal rank
Axiom 2: Adding a self-loop to a vertex $v$ does not
change the rank of any pair of other vertices
Rank of $v$ can only increase

Axiom 3: vote by committee
$\Rightarrow$ the following two graphs have the same page-rank
for vertices B and C
Axiom 4: If several vertices have the same set of successors, they can be collapsed into a single vertex.

Axiom 5: Vertices A, B, C have the same rank in the following two graphs:

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Graph 1: O -> A  O -> B  O -> C
Graph 2: O -> A  O -> B  O -> C
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