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Random Walks on Undirected Graphs (3/16/07)

Theorem: Commute time (u,v) = $2mk_{uv}^{eff}$.

Proof: Insert current d_i into each vertex i where d_i is the degree of node i and withdraw 2m amps current from vertex j.

$$d_i = \sum_{kadjtoi}(v_{ij} - v_{kj}) = d_i\, v_{ij} - \sum_k v_{kj}$$

$$v_{ij} = 1 + \frac{1}{d_i} \sum_k v_{kj}$$

$$= \sum_k \frac{1}{d_i}(1 + v_{kj})$$

$$h_{ij} = \sum_k \frac{1}{d_i}(1 + h_{kj})$$

$$v_{ij} = h_{ij}$$

Instead of removing current from j remove it from i.

$$v_{ji} = h_{ji}$$

Reverse current in above step, we get:

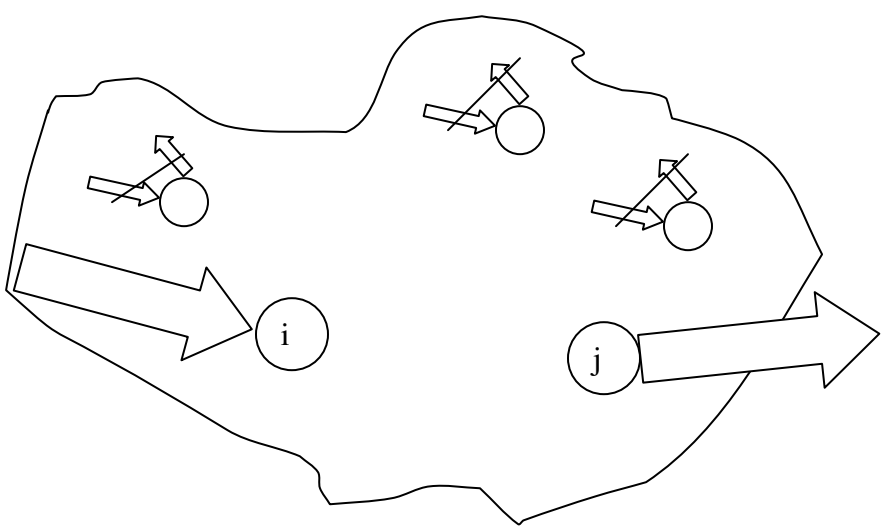
$$-v_{ji} = h_{ji}$$

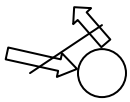
$$h_{ji} = v_{ij}$$

<u>Two Scenarios</u>

1. Put current in proportional to degree at every node and take out at j.
2. Put current in at i and take it out at all individual ones.

Overall we can say we applied 2m current at i and took it out at j.





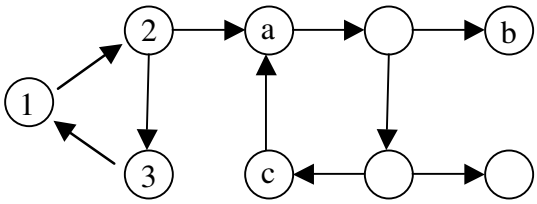
Superimpose both situations:

$$v_{ij} = \underbrace{h_{ij} + h_{ji}} = 2mk_{ij}^{eff}$$

commute time (i, j) $= 2mk_{ij}^{eff}$

k_{ij}^{eff} à number of edges

Directed Walks (PageRank)

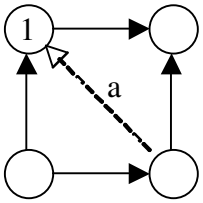


Some problems:

- If you start at a, you never reach 1, 2, 3 (not strongly connected)
- Where do you go once you are at b? (no out edges)

Solutions:

- Put self-loop at b? Not a very good solution.
- With probability *p*, have each node take a random jump to a random vertex. E.g. *p*=0.15; Prob(c goes to a) = 0.85, Prob(c goes to a vertex taken at random) = 0.15
- Essentially, create a new vertex labeled "Restart" and give it an edge from every other vertex, such that the edge to "Restart" is taken with probability *p*.
- PageRank of "Restart" vertex is guaranteed to be *p*.
- This is not typically done in practice, however.



Start at 1. What is the stationary probability of each vertex? The cycles don't converge!

Adding edge *a* solves the problem.