February 26, 2007

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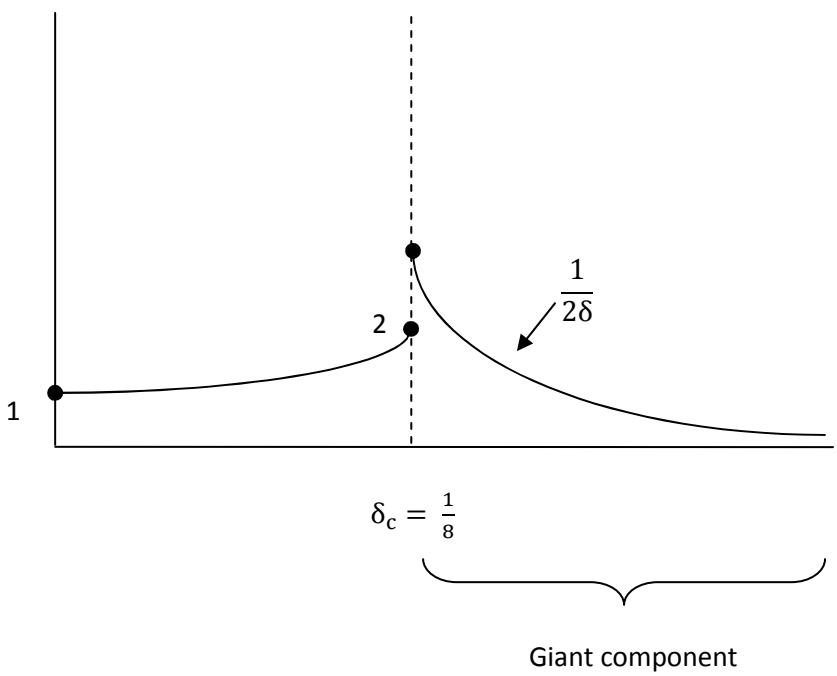
<u>Growth Model with Uniform Attachment</u>

Phase transition for giant component

g(1) = 1	all vertices are in finite component
g(1) < 1	there exist giant component

1 - g(1) = fraction of vertices which are in giant component



$g'(1) = \sum_{k=0}^{\infty} k^2 a_k$

If giant component does not exist, then g = 1 and the numerator and denominator of g' are both 0.

Use l'Hopital's rule:

$$g'(x) = \frac{1}{2\delta} \frac{-\frac{xg'(x) - g(x)}{x^2}}{-g'(x)}$$

$$g'(1) = \frac{1}{2\delta} \frac{g' - g}{-g'}$$

$$(g')^2 - \frac{1}{2\delta}g' + \frac{1}{2\delta}g = 0$$

$$g' = \frac{\frac{1}{2\delta} \pm \sqrt{\frac{1}{4\delta^2} - 4\frac{1}{2\delta}g}}{2}$$

$$g' = \frac{1 \pm \sqrt{1 - 8\delta g(1)}}{4\delta} = \frac{1 \pm \sqrt{1 - 8\delta}}{4\delta}$$

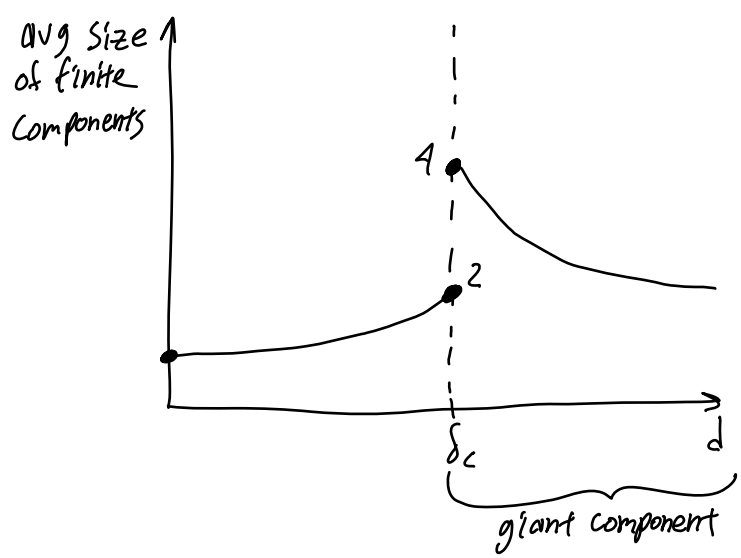
$\delta \leq \frac{1}{8}$ otherwise solutions become complex

let $\delta \to 0$ all components should have size -> 1

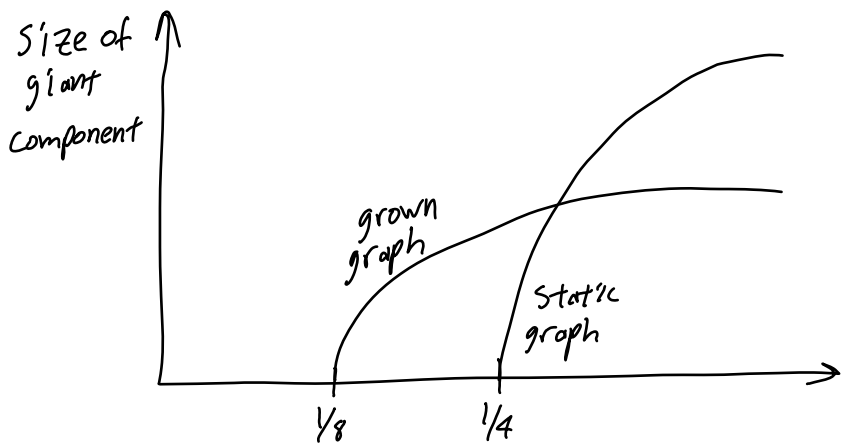
Among the 3 possible solutions ($\frac{1}{2\delta}, \frac{1+\sqrt{1-8\delta}}{4\delta}, \frac{1-\sqrt{1-8\delta}}{4\delta}$), g' = $\frac{1-\sqrt{1-8\delta}}{4\delta}$ gives such result.

$\delta_c = \frac{1}{8}$ => g' = $\frac{1-\sqrt{1-8\delta}}{4\delta} = 2$

So:

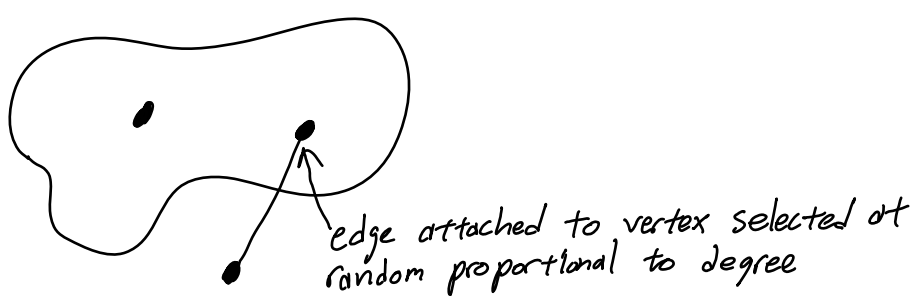


Static Graph:



$$S_{static} = \begin{cases} 0, & \delta \le 1/4 \\ 1 - \frac{1}{\delta + \sqrt{\delta^2 + 2\delta}}, & \delta > 1/4 \end{cases}$$

Grown Graph Preferential Attachment:



Now we're talking about the degrees of vertices.

Let $d_i(t)$ be degree of ith vertex at time t. The probability that an edge is connected to vertex i is:

$$\frac{d_i(t)}{2\delta t}$$

We want to ensure that all the probabilities sum to 1, so we need a normalizing factor (the above). Here, δ is the rate at which edges are added per unit time.

$$\frac{d}{dt}\left[d_i(t)\right] = \cancel{\delta}\frac{d_i(t)}{2\cancel{\delta}t}$$

Solution: $d_i(t) = at^{1/2}$

Check:

$$\frac{1}{2}at^{-1/2} = \frac{at^{1/2}}{2t} \checkmark$$

Now,

We'll soon get to the point where we'll be able to answer questions like: what is the distribution of degree sizes?

$$at_i^{1/2} = \delta \rightarrow a = \frac{\delta}{\sqrt{t_i}}$$

$$d_i(t) = \delta\sqrt{\frac{t}{t_i}}$$