CS485 Spring 2007
Homework 6

Due Date: March 2 2007

NOTE: To speed up homework grading, please submit each homework problem on a separate sheet of paper, with you name and NetID on the top. Thank you!

1. Find the generating function for the recurrence $a_i = 2a_{i-1} + i$, and $a_0 = 0$.

2. Consider a branching process with geometric degree distribution, defined as: $p_k = (1 - p_0)^k p_0$ for some fixed $p_0 \in (0, 1]$, and any $k \geq 1$. This corresponds to a branching process where at each node, you flip a coin with tail bias p_0 and if it comes up "heads", you crete a new child and repeat, and if it comes up "tails", you stop creating children for this node. Compute analytically, or find empirically, the following:

   - What is the value of p_0 at which this branching process starts having a possibility of generating an infinite tree? Is the probability of generating an infinite tree decreasing or increasing with p_0?
   - What is the expected size of *finite* trees for a given p_0? Does this expected size decrease or increase with p_0?

   If you decide to do empirical calculations, please provide a plot of average size of the generated trees vs. p_0 for several values of p_0 (define some size at which you declare the tree "infinite"). For the second part, do the same but only consider trees that are "finite".