CS485 Spring 2007
Homework 5

Due Date: Feb 28 2007

NOTE: To speed up homework grading, please submit each homework problem on a separate sheet of paper, with you name and NetID on the top. Thank you!

1. Write a proof that every increasing property of $G(n,p)$ has a threshold.

2. Let p_i be probability of i children in a branching process. Prove that if $p_1 + 2p_2 + 3p_3 + \cdots = 1$ and $p_1 < 1$, then $p_0 > 0$.

3. For the probability distributions given below, compute the extinction probability and the expected size assuming the branching process is finite.

   (a) $p_0 = \frac{1}{4}, p_1 = \frac{1}{2}, p_2 = \frac{1}{4}$

   (b) $p_0 = \frac{1}{8}, p_1 = \frac{3}{8}, p_2 = \frac{3}{8}, p_3 = \frac{1}{8}$

4. Let $F(x)$ be a generating function for some sequence $a_0, a_1, \ldots$ (not necessarily a probability distribution).

   (a) Treat $F(x)$ as a "black box" evaluatable on the interval $[0,1)$. How do you compute values of $a_0, a_1, \ldots$? You can use common operations on F (evaluation, algebraic operations, derivations, integration, etc.)

   (b) Let $F(x) = \frac{x+1}{(1-x)^3}$ for $x \in [0,1)$. What sequence is F a generating function of? Compute some (4,5) values, and guess the sequence.

   (c) **Bonus:** prove that your guess is correct, i.e. start with the sequence you guessed above, and work out its generating function. Show that it is equivalent to F.