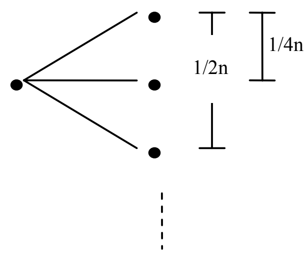
CS 485 Lecture Notes
February 6, 2006

$G(n,1/2)$
Looking for a clique $(2-\varepsilon)\log n$ $\varepsilon>0$




$\log_2 n$ times

Review

$N = \{1,2,\ldots,n\}$ $n\to\infty$

$N_p$ = subset of N where each integer is selected independently with probability p
Arithmetic Progression: a, a+b, a+2b, a+3b, …

*For what p does an arithmetic progression of length k appear?*
$n^{-2/k}$
Proof:

$n^2$ potential progressions of length k
let x be the number of progressions of length k
$E(x)= n^2 p^k$
If $p \ll n^{-2/k}$ $\lim_{n\to\infty} E(x)=0$ $\therefore \lim_{n\to\infty} P(x=0)=1$

If $p \gg n^{-2/k}$ $\lim_{n\to\infty} E(x)=\infty$ would like to show $\lim_{n\to\infty} P(x=0)=0$

(use second moment method)

Second Moment Method

If $\lim_{n\to\infty} \frac{Var(x)}{E^2(x)} = 0 \Rightarrow P(x>0)=1$

Indicator Variable $I_i = \begin{cases} 0 & \\ 1 & \text{if the ith arithmetic progression is there} \end{cases}$

$x = I_1 + I_2 + \ldots + I_{n2}$

$Var(x) = \sum_i \sum_j Cov(I_i, I_j)$ * $Cov(I_i, I_j) = 0$ if $I_i$ and $I_j$ are disjoint

$n^2$ possible arithmetic progressions

$n^4$ possible "pairs" of arithmetic progressions

1) almost all pairs are disjoint
2) $n^3$ pairs overlap in one integer

   $n^2$ possibilities for 1st arithmetic progression

   kn for 2nd, since an integer is contained in n arithmetic progressions

   $Cov(I_i, I_j) \leq E(I_i, I_j) = p^{2k-1}$

3) $n^2$ pairs overlap in 2 integers

   $Cov(I_i, I_j) \leq E(I_i, I_j) = p^{2k-2}$

$Var(x) \leq n^3 p^{2k-1} + n^2 p^{2k-2}$

$$\frac{Var(x)}{E^2(x)} \leq \frac{n^3 p^{2k-1}}{n^4 p^{2k}} + \frac{n^2 p^{2k-2}}{n^4 p^{2k}} = \frac{1}{np} + \frac{1}{(np)^2} \to 0 \quad \text{as } n \to \infty$$

$p = n^{-2/k}$ $\quad \lim_{n\to\infty} \frac{1}{np} + \frac{1}{(np)^2} = 0 \qquad k>2$

$\therefore P(x=0)=0$

<u>Other Structures</u>

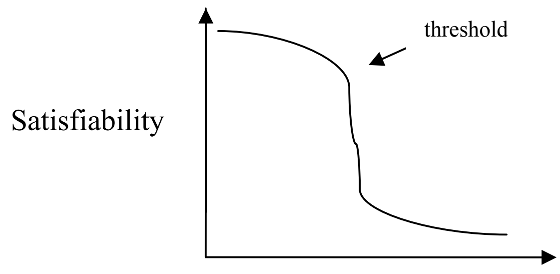
Su Doku: Can you generalize to parameter n?

What additional constraints are needed?

What properties appear?

Boolean formulas in CNF

n = number of variables




thresholds appear for structures other than G(n,p)