CS 485 Notes
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Dan FitzGerald
dpf7

Back to Wigner's Semicircular Law:

Where we left off:
Think of A^k as representing all paths of length k in a graph. How many of these are there?
--> k/2.

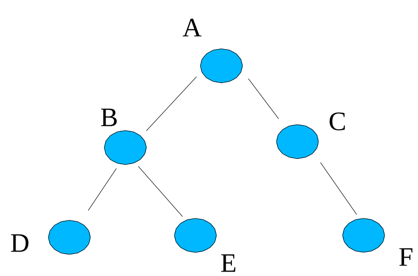
To see this, imagine that you are on a path in the graph. Each time you go through an edge the first time, you must go to a new vertex.

How many trees are rooted at vertex? :=
(# of types of dfs trees with k/2 vertices)(# of ways of embedding it= n/2)

Dfs trees can be represented as a string of balanced parentheses. To see this, take a graph. The first time you are at a node, form an open parenthesis as you leave it. When you return to that node at some other point in the dfs, add a closed parenthesis. Recursively do this on subtrees.

For instance, imagine the DFS path A-B-D-B-E-B-A-C-F-C

=> ((()())(())))



We know that the number of pairs of parentheses is equal to $C_{k/2}$(the k/2[nd] Catalan number). Thus the # of dfs trees is equal to $C_{k/2}n^{k/2}$

$$C_k = \frac{1}{k+1}\binom{2k}{k} \text{ so...}$$

$$C_{k/2} = \frac{1}{k/2+1}\binom{k}{k/2}$$

thus,

$$trace(A^k) = n\frac{1}{k/2+1}\binom{k}{k/2}n^{k/2}$$

$$p(l) = \frac{1}{2^k n^{1+k/2}} trace(A^k)$$

$$= \frac{1}{2^k}\frac{1}{k/2+1}\binom{k}{k/2}$$

$$= \frac{1}{k2^{k-1}+2^k}\binom{k}{k/2}$$

$$= \frac{1}{2^{k-1}(k+2)}\binom{k}{k/2}$$

given k odd, we have (k-1)/2 free vertices and thus n^((k-1)/2) trees of a given type, which is independent of n. As n goes to infinity, the moment goes to 0.