CS485 lecture notes for 3/15/06
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Recall:

dmin and dmax are the minimum and maximum degree of vertices.

λ_1 is the 1st eigenvalue.

Lemma:

$$max\{dmin, \sqrt{dmax}\} \leq \lambda_1 \leq min\{dmax, 2*\sqrt{2*|E|}\}$$

Proof:

let u be a vector of all ones: u=(1,1,.....1)

Au yields the degree distribution of the graph: $(d1, d2, ..., dn)^{T.}$. We know this is >= than dmin*u.

$$u^T Au \geq dmin * u^T u$$

$$\lambda_1 = \max_x \frac{x^T Ax}{x^T x} \geq \frac{u^T Au}{u^T u} \geq dmin$$

Highest degree vertex:

Get Gs be a star of the higest degree edge, i.e. Gs is simply the highest degree node and the edges coming out of it.

$$\lambda_1(Gs) = \sqrt{dmax}$$

$$Gs \subset G,\ \lambda_1(e) \geq \sqrt{dmax}$$

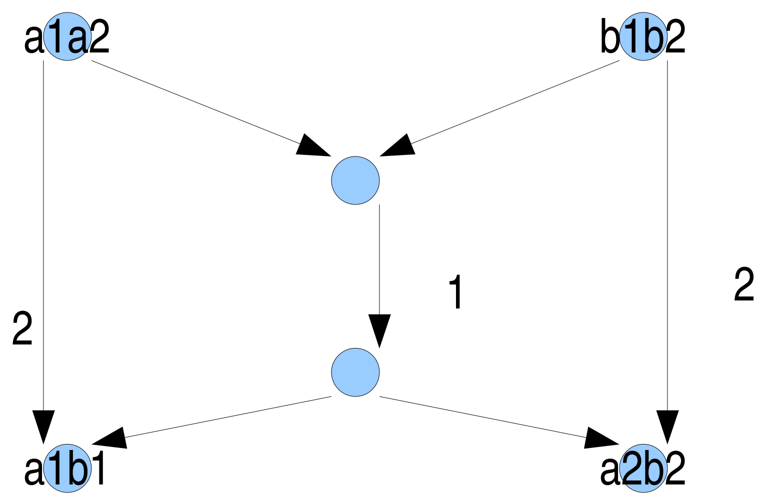
Let v be the eigenvector associated with λ_1. Normalize it so that its max coordinated is 1.

$$\lambda_{1v} = Av \leq Au \leq dmax * u$$

$$remember,\ \lambda_1 \leq dmax$$

$$\lambda_1 = |A|_2 \leq |a|_F = \sqrt{\sum_{ij} a_{ij}^2} = 2|E|$$

Given bits a_1, a_2 and b_1,b_2, how can we compute a_1b_1 and a_2b_2 using these constraints:



Answer: Send a_1a_2 and b_1b_2 down the edges with capacity 2. Through the capacity 1 edge, send a_2 xor $b_{1.}$ This will result in a_1b_1 and a_2b_2 ending up in their correct places while still following the capacity constraints.

But can we do better? Imagine if we had bits a1..a8 and b1...b8. Using a sort of divide and conquer approach, we can achieve a1b1, ..., a8b8 in O(n log n) edges. We would simply send a1 through a4 and b1 through b4 to the second level of our graph, then split those inputs and send a1 and a2 with b1 and b2, etc.

Expanders:

A graph is an expander if for every subset S of vertices V, |S| <= n/2, S will be adjacent to <= |S|

vertices not in S.

Define ∂s to be the set of edges from vertices in S to vertices not in S.

Define the expansion parameter for G as $h(G) = \min_{|S| \leq \frac{n}{2}} \frac{|\partial S|}{|S|}$

A family of graphs is an edge expander if h(G) is bounded away from 0.

Consider a d-regular graph G= (V,E), S subset of V.

for a $v \in S$ there will be $\frac{\partial |S|}{n}$ adjacent vertices in S'.

$$\partial s = \frac{\partial |S||S'|}{n}$$

$$h(G) = \min_{|S| \leq \frac{n}{2}} \frac{\partial |S'|}{|n|} = \frac{d}{n} \min_{|S| \leq \frac{n}{2}} |S'| = \frac{d}{2}$$