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g' = 1/2d * (1-g/x)/(1-g)

If g(1) != 1, there exists an infinite component containing 1- g(1) fractions of vertices. Average size of finite components is g'(1) = 1/2d

If g(1) = 1 then there is no infinite component, g'(x) is indeterminate, so we use L' Hoptal rule to get a quadratic for g'(1). Solve it to get: g'(1) = [1 +- sqrt(1-8d)]/4d

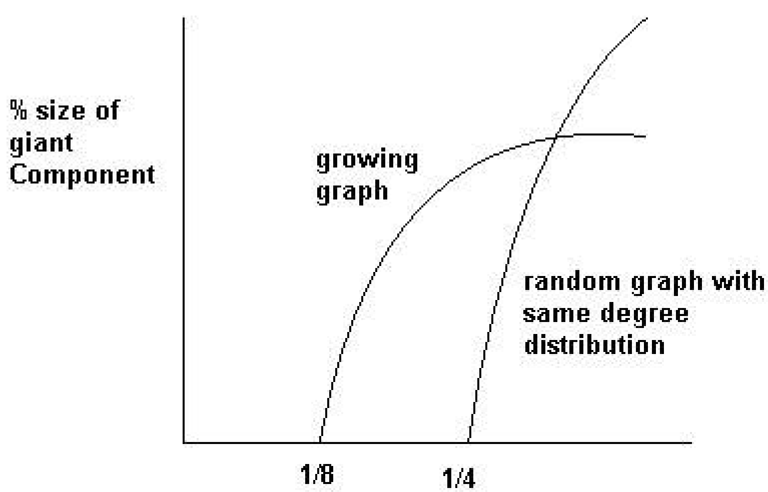
If d > 1/8 average size of finite component is 1/2d and there exists an infinite component.
If d → 0 there is no infinite component, $d_c = 1/8$

$G'(1) = 1 – [1 – 0.5(8d) – 0.25(4d)^2 - \ldots] / 4d$

$= (4d + 16d^2 + \ldots) / 4d \rightarrow 1$

Average size of finite component:

Before d_c:	[1 – sqrt(1-8d)] / 4d	at d = 4	→ 2
After d_c:	1/2d		→ 4



Four Generating Functions

$g_0(x)$ = degree of vertex selected at random
$g_1(x)$ = out degree of vertex reached by following random edge
$h_0(x)$ = size of component of random vertex
$h_1(x)$ = size of component by following edge to end point and exploring outgoing edges

Let P_k be the probability that vertex selected at random has degree k
$g_0(x) = \sum P_k x^k$

So,
$g_1(x) = 1/x * [x\, g_0'(x) / g_0'(1)]$
$g_{1'}(x) = g_0''(x) / g_1'(1)$
Hence,
$g_0''(1) = g_0'(1) g_1'(1)$

<u>Size of Components</u>
Let q_k be probability that a vertex reached has k outgoing edges, then
$h_1(x) = x\, q_0 + x\, q_1\, h_1(\mathrm{x}) + x\, q_2\, h_1^2(x) + x\, q_3\, h_1^3(x) + \ldots$
q^k is coefficient of x^k in $g_1(x) = q_0 + q_1\, x + q_2\, x^2 + \ldots$
Hence,
$h_1(x) = x\, q_1(h_1(\mathrm{x}))$

Using the same P_k defined earlier, we have
$h_0(x) = x\, P_0 + x\, P_1\, h_1(x) + x\, P_1\, h_1^2(x) + \ldots = x\, g_0(h_1(x))$

Average size of component of a randomly chosen vertex, assuming no giant component
$h_0'(1) = h_0'(x)|_{x=1} = g_0(h_1(x)) + x\, g_0'(h_1(x))\, h_1'(x)|_{x=1}$
$= 1 + g_0'(1)\, h_1'(1)$
$h_1'(x) = g_1(h_1(x)) + x\, g_1'(h_1(x))\, h_1'(x)|_{x=1}$

Therefore,
$h_1'(1) = 1 + g_1'(1)\, h_1'(1)$
$h_0'(1) = 1 + g_0'(1)\, h_1'(1)$