CS485

Feb 22nd

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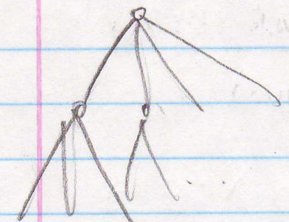
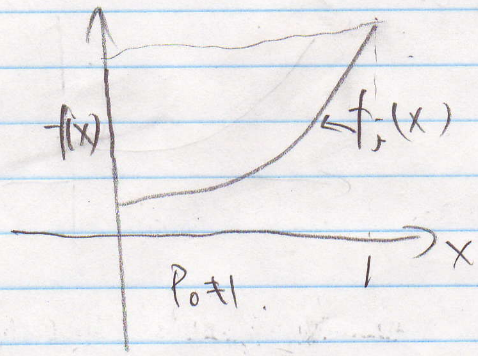


$P_0, P_1, P_2, \dots$

$f(x) = \sum_{i=0}^{\infty} P_i x^i$

iterates: $f_1(x) = f(x)$

$f_j(x) = f_{j-1}(f(x))$



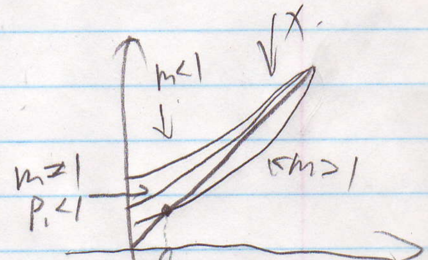
$P_0 + P_1 + P_2 + \dots = 1$

root of equation $f(x) = x$

$x = 1$ is always a root

$f(x) = P_0 + P_1 x + P_2 x^2 + \dots$

$f(1) = P_0 + P_1 + P_2 + \dots = 1$

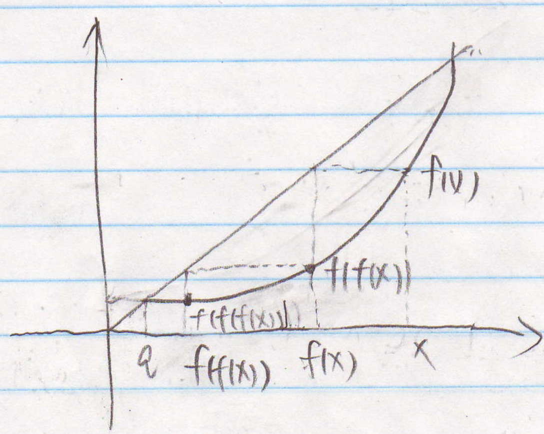


probability of mortality

$m = f'(1) = P_0 + P_1 + 2P_2 + 3P_3 + \dots$

$\lim_{j\to\infty} f_j(x) = \overset{q}{a_0} + \overset{0}{a_1} x + \overset{0}{a_2} x^2 + \dots$

$1-q$ prob of immortality



$q < x < 1$, $x > f(x) > f_2(x) > f_3(x) > \dots > q$

$0 < x < q$ $x < f(x) < f_2(x) < \dots < q$

$x \in [0,1)$ $\lim_{j\to\infty} f_j(x) = q$

if $m<1$ then process dies out with probability 1
if $m>1$ then process dies out with probability q, where q is unique root of $f(x)=x$ in interval $[0,1)$

Expected Size of component is finite

Example: Let X be a random variable
Let p_i be probability that $|X|=i$

$$p_i = \frac{6}{\pi}\frac{1}{i^2} \qquad \sum_{i=0}^{\infty} p_i = 1$$

$$E(X) = \sum_{i=0}^{\infty} i p_i = \frac{6}{\pi}\sum_{i=0}^{\infty}\frac{1}{i} = \infty \quad \text{(doesn't exist)}$$

Lemma: If the slop $m=f'(1)\neq 1$, then the expected size of an extinct family is finite

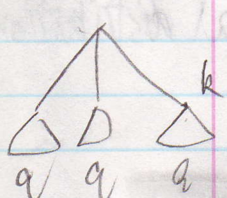
what happens if $m=1$ $p_1<1$, I don't know the answer

Let z_i be the random variable denoting the size of the ith generation.

The expected value of z_i over extinct families. it is likely to be small than expected value of z_i over all cases

$$\text{Prob}[z_1=k \text{ and extinction}] = \text{Prob}[z_1=k/\text{extinction}]\,\text{Prob}[\text{extinction}]$$

$$=\text{Prob}[\text{extinction}/z_1=k]\,\text{Prob}[z_1=k]$$



Bayes Rule

$$\text{Prob}[z_1=k/\text{extinction}] = \frac{\text{Prob}[\text{extinction}/z_1=k]\,\text{Prob}[z_1=k]}{\text{Prob}[\text{extinction}]}$$

$$= \frac{q^k p_k}{q} = q^{k-1} p_k$$

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m is slope at 1.

Expected size of z_i given extinction

$f(x)=\sum_{i=0}^{\infty} p_i x^i$

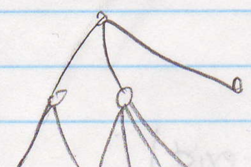
$f'(x)=\sum_{i=0}^{\infty} i p_i x^{i-1}$

$f'(q)=\sum_{i=0}^{\infty} i p_i q^{i-1}$

$$E(z_1/\text{extinction})=\sum_{k=0}^{\infty} k q^{k-1} p_k = f'(q)$$

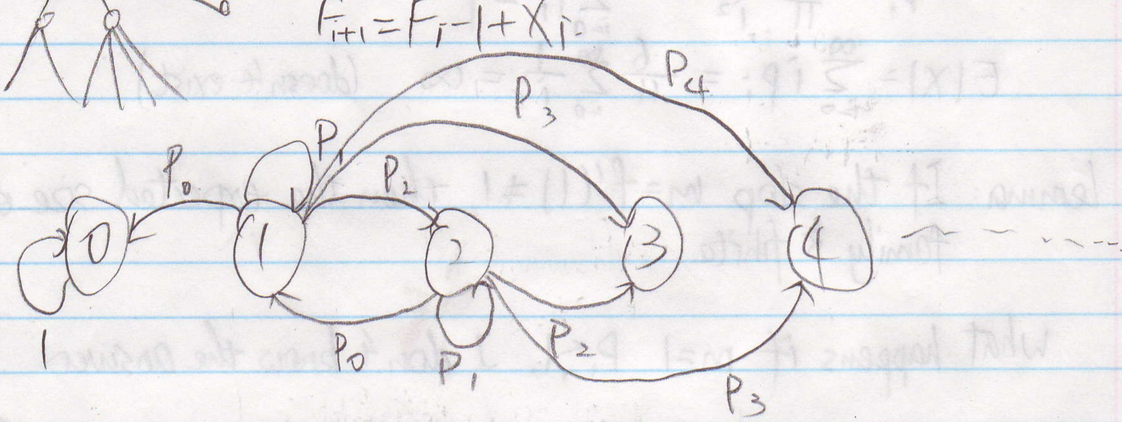
$$E(z_i/\text{extinction})=[f'(q)]^i$$

$$E(\text{tree}/\text{extinction})=\sum_{i=0}^{\infty}[f'(q)]^i=\frac{1}{1-f'(q)}$$



$F_1=1$

$F_{i+1}=F_i-1+X_i$



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p_0 p_1 p^2 p^3 p^4

$p_0+p_1p_0$ $p_1p_1+p_2p_0$

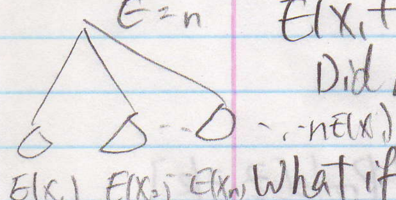
not necessary to 1

$\nearrow q$ 0 0 0 - - - - -

Expected value of sum of n random variables with identical distribution

$E(x_1+x_2+\cdots+x_n)=E(x_1)+E(x_2)+\cdots+E(x_n)$

Did not use independence



What if the number n of random variables is itself a random variable

$E(\sum_{i=1}^{n} x_i)=E(n)(x_i)$ provided you have independent