CS 485 Assignment 9, due April 14

1. Let G be a regular degree d graph. Prove that G is bipartite if and only if $\lambda_n = -d$. A graph is bipartite if its vertices can be partitioned into two sets L and R such that all edge have one end point in L and the other in R.

2. What is the spectrum of a clique? The spectrum is the set of eigenvalues. **Hint**: express the adjacency matrix in terms of the all ones square matrix and the identity matrix.

3. What is the expansion parameter for the following graphs.

(a) The complete graph on n vertices.

(b) A torus consisting of a square grid folded to form a cylinder and then the two circular ends connected to form a torus.

(c) A graph with at least two connected components.

4. Prove the following

a) In d dimensions $|v|_1 \le \sqrt{d}\,|v|_2$.

b) If p is a probability distribution on the integers 1 to n and $u = (1,1,\cdots,1)$ is the all ones vector then $\left|p - \frac{u}{n}\right|_2 \le 1$.