CS 485 Assignment 8, due March 31

1. Prove that $\|AB\|_F^2 \le \|A\|_F^2 \|B\|_F^2$.

2. Let Q be an orthonormal matrix. Prove that $\|QA\|_F = \|A\|_F$.

3. Consider the convergence of the iterative process for computing the stationary probabilities of a random walk for the following graphs and starting probabilities.

a. Undirected square with probability one on one vertex, on two opposite vertices, on all four vertices.

b. Undirected triangle starting with probability ½ on two of its vertices.

c. Directed square with probability one on one vertex.

d. Characterize the stationary probabilities for a random walk on undirected hexagon for arbitrary distribution of initial probability. Hint: Consider initial probability of one on one vertex and then use superposition.

4. Show that if A is a symmetric matrix and λ_1 and λ_2 are distinct eigenvalues, then their corresponding eigenvectors x_1 and x_2 are orthogonal. **Hint**: Start with $\lambda_1 \left(x_1^T x_2 \right)$ and show that it is equal to $\lambda_2 \left(x_1^T x_2 \right)$. Then show that if λ_1 and λ_2 are distinct that x_1 and x_2 must be orthogonal.