CS 485 Assignment 6, due March 10

1. Find the generating function for the recurrence $a_i = 2a_{i-1} + i$.

2. Define increasing property and threshold. Then write up in your own words a clear proof that any increasing property for N_p has a threshold. If you type your answer we will grade on clarity of exposition.

3. For each of the following probability distributions what is the extinction probability? the expected size assuming the branching process is finite?
a) $p_0 = \frac{1}{4}$ $p_1 = \frac{1}{2}$ $p_2 = \frac{1}{4}$
b) $p_0 = \frac{1}{8}$ $p_1 = \frac{3}{8}$ $p_2 = \frac{3}{8}$ $p_3 = \frac{1}{8}$

4. Consider a branching process with probabilities p_i and generating function

$$f(x) = \sum_{i=0}^{\infty} p_i x^i.$$ What is the expected size of an extinct family if $f'(1) = 1$?