

1. Compare the Markov and Chebyshev bounds for the following probability distributions

a)
$$p(x) = \begin{cases} 1 & x = 1 \\ 0 & \text{otherwise} \end{cases}$$

b)
$$p(x) = \begin{cases} \frac{1}{2} & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

2. Let $N = \{1, 2, \dots, n\}$ and let S_k be a random subset of N of size k . Consider the property P on S_k that S_k contains a 3-term arithmetic progression. Show that $n^{\frac{1}{3}}$ is a threshold for property P . An arithmetic progression is a sequence of integers of the form $a_0, a_0 + b, a_0 + 2b, \dots$

3. Let M be the multi set formed by drawing pn integers from the set $\{1, 2, \dots, n\}$ with repetition.

- a) How large must p be in order to have some integers appear twice?
- b) Does a sharp transition occur?
- c) How large must p be in order for every integer to occur in M_p ?
- d) How does the frequency of occurrences of integers evolve with increasing p ?

4. Explain why two giant components cannot exist simultaneously.