RECALL. Reducing VERTEX COVER to HAM CYCLE
Will represent $G$ as a HAM CYCLE input
with 3 types of gadgets.

EDGE GADGET

\[
\begin{array}{cc}
  & v \\
  u & \rightarrow
\end{array}
\]

VERTEX GADGET

\[
\begin{array}{ccc}
  & v_1 \\
  u & \rightarrow & v_2 \\
 & \rightarrow & v_3
\end{array}
\]

COUNTER GADGET

\[
1 \rightarrow 2 \rightarrow 3 \rightarrow \ldots \rightarrow n
\]
If you enter a vertex gadget on an orange edge, you must use each of its red edges, and exit on another orange.
If you use any red edge of a vertex gadget, you must use all of them, and you must enter and exit that vertex using orange edges.

2 types of vertices:

A. The cycle uses all of their red edges and 2 of their orange edges.

B. The cycle uses none of their red/orange edges.

≤k vertices of type A.

To get into/out of an edge gadget we must cross a red/orange edge, hence visiting the 4 vertices of the edge gadget requires having a type A endpoint.
When trying to reduce Problem A to Problem B, think about matching up "variables" and "constraints."

**Variable type**

<table>
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<tr>
<th>Problem</th>
<th>Type</th>
<th>Example</th>
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<tbody>
<tr>
<td>Vertex cover</td>
<td>2-valued</td>
<td>(v∈S or v∉S)</td>
</tr>
<tr>
<td>Ham. cycle</td>
<td>2-valued</td>
<td>(e∈C or e∉C)</td>
</tr>
<tr>
<td>3 SAT</td>
<td>2-valued</td>
<td>(xᵢ=T or xᵢ=F)</td>
</tr>
</tbody>
</table>

(Graph 3-colorability has 3-valued variables.)

**Constraints**

- **Vertex cover**: one global counting constraint \((≤K)\)
- many 2-variable constraints \((v \text{ or } w)\)
- **Ham. cycle**: one global connectivity constraint,
many local counting constraints (in-degree = 1, out-degree = 1)

3SAT: many 3-variable constraints

The "alternating shared-fate gadget"

\[ \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \]

(2-dim matching is another example)

Useful when reducing FROM a problem whose variables are in unbounded # constrs.

to those where they are in just a few.