7 April 2021  3-Dimensional Matching

Announcement: Prelim 2 coverage
- Chapter 5, 7, 8.1-8.4
- Can use material from this week if you want, but it won’t be needed.

Prelim 2 review sessions
- Sat 8-9:30 pm  
  Zoom
- Sun 2-8:30 pm  
  Hollister 314
- Mon 9:05-9:55  
  in this lecture (Zoom)

See Ed post "Prelim 2 Review Sessions"
Remains to show: if this digraph has a Hamilton cycle, then the 3SAT formula has a satisfying truth assignment.
Lemma: Each blue node has only two neighbors (above and below it).

If a Hamilton cycle goes through any blue node & neighbors in top-to-bottom (resp. B-to-T) order then it must go through every blue node of that variable gadget & neighbors in the same order.

Using the lemma we conclude each variable gadget is traversed T-to-B or B-to-T. If T-A-B assign true, if B-b-T assign false. This must be a satisfying truth assignment because otherwise if \( \exists \) an unsatisfied clause, there is no way for the Hamilton cycle to visit its node.

\[
\text{Input to 3SAT} \xrightarrow{\text{reduction}} \text{Input to HAM CYCLE}
\]
Proof

1. Solution of A → Solution of B
2. Reduce from another NP-Complete problem (e.g., 3SAT) to your problem
3. Analyze running time of reduction
4. Show the problem belongs to NP

Five steps to show an NP-Completeness Proof:
1. Solution
2. Proof Part 1
3. Proof Part 2
4. HAM CYCLE
5. Solution
Three Dimensional Matching

Input:
- Three sets \( R, G, B \) each with some \( n \) of elements, \( n \).
- Collection of \( m \) 3-element sets each containing one element from each of \( R, G, B \).

Question: Among the given \( m \) 3-elt sets are there \( n \) of them which constitute a partition of \( R \cup G \cup B \)?

To show NP-Hard: reduce from IND-SET.
Choosing 3-element sets to cover all red & green nodes in this picture corresponds to choosing a k-element set of vertices.

Q1. How to constrain the k vertices to be non-neighboring?

Q2. There are left over blue nodes. How to cover them?