Introduction to Algorithms
CS 482 Summer 2006

Problem Set 9
Due: Tuesday, August 15


Question 1
Do problem 11.6 from the textbook.

Question 2
The vertex cover optimization problem is as follows. Given a graph $G$, find a minimum sized vertex cover $S$. As we know, this problem is NP-Complete, so we will attempt to find an approximation algorithm for it. In this problem you are given two greedy approximation algorithms. For parts (a) and (b), you should do one of the following:

- Prove that the algorithm returns a vertex cover at most twice the size of an optimal one, or
- Show that the algorithm may return a vertex cover that is a $\Theta(n)$ factor larger than optimal.

(a) Algorithm 1:

Construct a vertex cover $S$ as follows:
While there is an edge $e = (x, y)$ in $G$
    Pick one endpoint $x$ of $e$
    Add $x$ to $S$
    Remove $x$, along with all incident edges, from $G$
Endwhile
Return $S$

(b) Algorithm 2:

Construct a vertex cover $S$ as follows:
While there is an edge $e = (x, y)$ in $G$
    Add $x$ and $y$ to $S$
    Remove $x$, $y$, and incident edges from $G$
Endwhile
Return $S$

(c) We now know that one of these two algorithms does indeed find a 2-approximation for the vertex cover problem. We have also seen a polytime reduction from the vertex cover problem to the independent set problem. Is it safe to conclude then that we can use this algorithm to get a 2-approximation for the independent set problem? Explain your answer.