Introduction to Algorithms
CS 482 Summer 2006

Problem Set 8
Due: Tuesday, August 8

Reading: Chapters 8.1 - 8.5.

Question 1

Suppose you’ve been asked to build a computer network for an up-and-coming telecom. There are \( n \) cities that the network needs to connect using the new prototype cable that the company has been developing. This cable has effectively unlimited bandwidth, but unfortunately it’s difficult to make, so currently only \( M \) miles of cable are available. In addition to having limited amounts of cable, it also costs money to lay down the cable. Given that you only have \( k \) miles of cable to use, the company would like you to build a network that connects all \( n \) cities as cheaply as possible.

More concretely, assume the cities are represented by nodes in a graph \( G = (V, E) \). Each edge \( e \) represents a pair of cities that can be connected by cables, and has two parameters; a cost \( c_e \) indicating how expensive it would be to install a cable between the two endpoints of \( e \), and a distance \( d_e \), indicating how much cable would be required. The company wants you to find a minimum cost network that spans all nodes in \( V \) uses at most \( M \) miles of cable.

After a few abortive attempts, you begin to suspect that the optimization you are attempting might not be possible (unless \( P = NP \)). Prove that this is the case. In other words, prove that if you did have an algorithm to solve this problem, you would be able to solve some NP-Complete problem.

Question 2

After some explaining, you convince the company that it will probably not be able to find an efficient algorithm for their problem. They spend some time trying to pick a new task for you, and they settle on the following.

Since it seemed that part of the problem lay in the fact that we had two parameters per edge, the telecom tells you to forget about the cost. After all, they just had their IPO. Instead, they simply want to know whether the \( k \) miles of cable they have is sufficient to connect all \( n \) cities. You quickly point out that this is easy; all you need to do is find the MST with respect to distances. Unfortunately, once you do this, it turns out that there isn’t enough cable. So the folks down at the corporate headquarters pick a subset \( S \subseteq V \) of \( k \) cities to act as a test-bed for their new cable technology.

The question is now, given the graph \( G \) with distances \( d_e \) for all edges \( e \), the set of nodes \( S \), and an integer \( M \), determine whether there is a subgraph of \( G \) that connects all nodes in \( S \) and has a total length of at most \( M \).

Review for Problem 2

The following problems are only suggestions for review for the prelim on Wednesday. You should not hand in solutions to these problems, and solutions will not be given out. However, we can go over
any of these problems in review session if requested. Remember to review notes and assigned readings, and if you feel you still need practice, there are tons of good problems beyond the ones listed here.

**Network Flow:** Problems 7.18, 7.34, 7.39.

**NP-Completeness:** Problems 8.2, 8.33, 8.39, 8.41.