Reading: Chapters 8.1 - 8.5.

Question 1
As you may have noticed, Ithaca doesn’t have much in the way of a zoo. So after enduring many years without wombats, wildebeest or walruses (walri?), you decide to take matters into your own hands and open your own. You’ve gathered $k$ donors to fund the creation of your new park, and you’ve picked the location (just a little ways behind Wegmans), so now you just need to choose the inhabitants.

Ideally you’d get every animal imaginable, except you just don’t have space. As it is, you have room to comfortably fit $n$ animals. Since you’d like to ensure that your funding doesn’t dry up, you figure your donors should get to make requests about which animals will be kept. After sending out a few emails, you collect from each donor $i$ a list of animals $A_i$ (of varying size) which donor $i$ would like to have at the zoo. As you suspected, the total number of different animals appearing on the $k$ lists exceeds $n$, so you won’t be able to satisfy all of their requests.

You decide the fairest thing to do is to select a number $\ell_i$ for each donor $i$ (based on their contributions, bigger donors have a larger $\ell_i$ value), and try to ensure that at least $\ell_i$ animals from $A_i$ are chosen. Given all these lists and values, we want to know whether we can select a set $A$ of at most $n$ animals to put in the zoo such that each donor $i$ will be able to see at least $\ell_i$ of the animals from her list $A_i$.

You should either give an efficient algorithm to solve this problem, or prove that it is NP-Complete.

Question 2
In class, we saw the NP-Complete problem known as INDEPENDENT SET. As you may recall, in this problem we are given a graph $G = (V, E)$ and a natural number $k$, and want to determine whether there is a set of vertices $S \subseteq V$ with size $|V| = k$ such that no two vertices in $S$ are joined by an edge in $G$.

Consider the following related problem, which we will call REALLY INDEPENDENT SET. Again, we are given a graph $G = (V, E)$ and a number $k$, and asked to find a set of $k$ vertices. However, the condition on this set is now stronger; we’d like to make sure that no two nodes in $S$ are joined by either an edge or a length 2 path in $G$.

Prove that REALLY INDEPENDENT SET is NP-Complete.

Question 3
Do problem 8.30 from the textbook.