Question 1

Imagine this. You are part of a highly talented comedy troupe. After years of hard work, you have perfected two acts that you will take on the road, traveling through \( n \) cities over the course of a year. In each city, you would like to select one of the two acts to perform. You already know which act will be better received in which city (some cities lean more towards the dryer British style of humor, while in other cities, involving any form of primate in a skit is the surest way to draw applause).

You have the list of \( n \) cities in the order you plan to visit them. You also have estimated how much value people will get from the first act \( (a_i) \) and from the second act \( (b_i) \) in city \( i \). At first, things seem pretty straight forward; just pick the act with the higher value in each city. But there is one complication. You seem to have attracting a small and dedicated band of groupies, who come along to every act you perform. And while they can be a bit tiresome, they also tend to buy you food after each show, so you don’t really want to have them go.

The reason this is a complication is that to keep them interested, you shouldn’t ever perform the same act three times in a row. Given this constraint and the values \( a_i \) and \( b_i \) for all cities \( i \), give an algorithm to determine the maximum possible value that can be generated.

For example, suppose \( n = 4 \), the \( a_i \) values are 8, 3, 1, 6 and the \( b_i \) values are 2, 5, 4, 9. Then the optimal schedule is to perform the first act in the first two cities, and the second act in the last two cities. Performing the second act on all but the first night would gather more total value, but you’d lose the groupies.

Question 2

You run a surprisingly lucrative chain of ice-cream stands that operate at the various beaches all along the coast. In total, you have \( n \) stands, one at each beach. Sadly, the little musical ditty that your stands play periodically to attract children has driven some beach-goers bonkers, and under pressure from these irate ice-cream-haters, the mayor has demanded that you either get rid of the music, or scale back your operation. You’re not to eager to get rid of the tune, so you press the mayor for details of this scaling back.

The mayors demands (assuming you keep the music) that you make it easy for any beach-goers to find a beach that is free of your stands. In particular, for any beach \( i \) along the coast, at least one of the stands located at beaches \( i - 1 \), \( i \) and \( i + 1 \) should be closed.

Of course, you’d like to lose as little money as possible, so you estimate the profit \( p_i \) that the stand at each beach \( i \) is currently bringing in. Give an algorithm to determine the minimum profit you would
need to lose in order to satisfy these demands.

For example, suppose that \( n = 5 \), and the stands have \( p_i \) values of 2, 7, 6, 8 and 9. Then the optimal solution would be to close stands 1 and 4.

**Question 3**

Do problem 6.19 from the textbook.