Reading: Chapters 4.1, 4.2, 4.4 and 4.5.

Question 1

Suppose, perchance, that you have a class that meets at an absurdly early time each morning. So ridiculously early, in fact, that you decide to get the class together one fine summer afternoon and barbecue on the dean’s front lawn in protest. To do this you’ll need to call everyone in the class and tell them to get on over (and maybe bring something to drink).

Ideally, you’d like to have everyone there as early as possible. Some of your classmates live closer to the dean than others, so for each classmate $i$, you quickly estimate $t_i$, the travel time needed for $i$ to get to the dean’s place. But you also need to take the time to actually call everyone in the class (sorry, no pyramid calling schemes, you’re organizing this shin-dig). And you know from experience that with some of your classmates, ending a conversation can be difficult. So you estimate the time you’ll need to spend on the phone with each of them convincing them to go. For classmate $i$, we’ll call this $i$’s phone time, and denote this $p_i$.

You need to pick an order in which to call your classmates. Given such an order, the arrival time $A_i$ of $i$ is the total phone time of all people called before $i$, plus $i$’s phone time $p_i$ and travel time $t_i$. You would like to find a schedule for calling all $n$ classmates that minimizes the latest arrival time, i.e.

$$\max_i A_i.$$ 

Give an algorithm to solve this problem.

Question 2

Do problem 4.12 from the textbook.

Question 3

As we will soon see, we can use a number of greedy algorithms to efficiently find the minimum-cost spanning tree (MST) for a graph $G$ with edge costs $c_e \geq 0$ for each edge $e \in G$. Now suppose that you have just finished computing an MST $T$ for $G$, when you discover that the cost you had been using for one of the edges was too large. In particular, for some edge $e \in E$ its real cost $c'_e \geq 0$ is strictly smaller than $c_e$. You could just throw out $T$, reduce $c_e$ to $c'_e$, and run your algorithm again.

But suppose that this sort of thing seems to happen often. Could you be more efficient, and somehow take advantage of the fact that we know what the minimum spanning tree looks like for almost the right set of edge costs? Show how to use $T$ to construct an MST $T'$ for the correct set of edge costs in $O(n)$ time.