The following table gives the number of respondents who obtained each score. 22 of 40 took the quiz.

<table>
<thead>
<tr>
<th>score</th>
<th>number</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
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</tr>
<tr>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
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<tr>
<td>10</td>
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<tr>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

The numbers in parentheses below show the number of people who missed each question.

1. Recall:
   - A TM is total if it halts on all inputs
   - A set is r.e. if it is \( L(M) \) for some TM \( M \)
   - A set is recursive if it is \( L(M) \) for some total TM \( M \)
   - The halting problem is the set
     \[
     HP = \{ M#x \mid M \text{ is a TM, } x \text{ is a string over } M\text{'s input alphabet, } M \text{ halts on input } x \}.
     \]

   True or false?
   (i) Every CFL is recursive. true (1)
   (ii) There exists a recursive set that is not a CFL. true (1)
   (iii) All recursive sets are r.e. true (3)
   (iv) \( \{ a^p \mid p \text{ is a prime number} \} \) is a recursive set. true (3)
   (v) If \( L(M) \) is recursive, then \( M \) is total. false (10) A machine can loop and still accept a recursive set. For example, a machine that loops on all inputs accepts \( \emptyset \). For a set to be recursive, there must exist a total machine accepting it.
   (vi) If \( M \) is total, then \( L(M) \) is recursive. true (0)
   (vii) TMs with two tapes accept more sets than TMs with one tape. false (1)
   (viii) Every Turing machine accepts a nonregular set. false (1)
   (ix) It is decidable for a given TM \( M \) and string \( x \) whether \( M \) accepts \( x \). false (6)
   (x) It is decidable for a given TM \( M \) whether \( L(M) = \sim HP \). (\( \sim \) denotes set complement.) true (17)
   \( \sim HP \) is not r.e., so the answer is always “no”.

2. In the following TM, the input alphabet is \( \{ a, b \} \), the left endmarker is \( \vdash \), and the blank symbol is \( \sqsubset \).
   The transitions are given in the following table.

   \[
   \begin{array}{cccccccc}
   \vdash & a & b & \sqsubset \\
   \hline
   \text{start state} & s & s, \vdash, R & t, b, L & r, a, L & s, \sqsubset, L \\
   \text{accept state} & t & t, \vdash, R & t, b, L & t, a, L & t, \sqsubset, L \\
   \text{reject state} & r & r, \vdash, R & r, b, L & r, a, L & r, \sqsubset, L \\
   \end{array}
   \]

   What language does it accept?
   (a) strings beginning with \( a \) (1)
   (b) strings containing only \( a \)'s
   (c) strings containing at least one \( a \)

3. True or false?
   (i) The machine of question 2 is total. false (8) The machine loops on input \( \epsilon \).
   (ii) The language accepted by the machine of question 2 is recursive. true (7)