

The following table gives the number of respondents who obtained each score. 22 of 40 took the quiz.

|               |    |    |    |    |   |   |
|---------------|----|----|----|----|---|---|
| <i>score</i>  | 13 | 12 | 11 | 10 | 9 | 8 |
| <i>number</i> | 1  | 5  | 4  | 6  | 2 | 4 |

The numbers in parentheses below show the number of people who missed each question.

1. Recall:

- A TM is *total* if it halts on all inputs
- A set is *r.e.* if it is  $L(M)$  for some TM  $M$
- A set is *recursive* if it is  $L(M)$  for some total TM  $M$
- The *halting problem* is the set

$$\text{HP} = \{M\#x \mid M \text{ is a TM, } x \text{ is a string over } M\text{'s input alphabet, } M \text{ halts on input } x\}.$$

True or false?

- Every CFL is recursive. **true (1)**
  - There exists a recursive set that is not a CFL. **true (1)**
  - All recursive sets are r.e. **true (3)**
  - $\{a^p \mid p \text{ is a prime number}\}$  is a recursive set. **true (3)**
  - If  $L(M)$  is recursive, then  $M$  is total. **false (10)** **A machine can loop and still accept a recursive set. For example, a machine that loops on all inputs accepts  $\emptyset$ . For a set to be recursive, there must *exist* a total machine accepting it.**
  - If  $M$  is total, then  $L(M)$  is recursive. **true (0)**
  - TMs with two tapes accept more sets than TMs with one tape. **false (1)**
  - Every Turing machine accepts a nonregular set. **false (1)**
  - It is decidable for a given TM  $M$  and string  $x$  whether  $M$  accepts  $x$ . **false (6)**
  - It is decidable for a given TM  $M$  whether  $L(M) = \sim\text{HP}$ . ( $\sim$  denotes set complement.) **true (17)**  
 **$\sim\text{HP}$  is not r.e., so the answer is always “no”.**
2. In the following TM, the input alphabet is  $\{a, b\}$ , the left endmarker is  $\vdash$ , and the blank symbol is  $\sqcup$ . The transitions are given in the following table.

|                            |     |                |           |           |                |
|----------------------------|-----|----------------|-----------|-----------|----------------|
|                            |     | $\vdash$       | $a$       | $b$       | $\sqcup$       |
| start state $\rightarrow$  | $s$ | $s, \vdash, R$ | $t, b, L$ | $r, a, L$ | $s, \sqcup, L$ |
| accept state $\rightarrow$ | $t$ | $t, \vdash, R$ | $t, b, L$ | $t, a, L$ | $t, \sqcup, L$ |
| reject state $\rightarrow$ | $r$ | $r, \vdash, R$ | $r, b, L$ | $r, a, L$ | $r, \sqcup, L$ |

What language does it accept?

- strings beginning with  $a$  **(1)**
  - strings containing only  $a$ 's
  - strings containing at least one  $a$
3. True or false?
- The machine of question 2 is total. **false (8)** **The machine loops on input  $\varepsilon$ .**
  - The language accepted by the machine of question 2 is recursive. **true (7)**