Reading. To review the material covered so far, read sections 1-6.

For each problem set please write both your name and your cornell.edu email address on top.

When designing a DFA or NFA provide the required definitions (a transition diagram is good enough), and also a brief English explanation of the main idea (e.g., what the states represent, etc). In designing NFA you may use \( \varepsilon \)-transitions defined on page 36 of Kozen, but it is not necessary.

(1) Let \( \Sigma = \{0, 1\} \).

(a) In class we gave a nondeterministic finite automata with \( k+1 \) states that accepts the language \( L_k = \Sigma^* \cdot 1 \cdot \Sigma^{k-1} \) (all words where the letter \( k \) from the end is a 1). Give a deterministic finite automata that accepts the language \( L_k \). How many states did you use?

(b) Give a deterministic finite automata that accepts the language \( L_k = \Sigma^* \cdot 1^k \) (the last \( k \) characters are all 1). Do this with less than \( 2^k \) states.

(2) Show that for any regular language \( \mathcal{L} \subseteq \Sigma^* \) the language \( \text{Even}(\mathcal{L}) \) is also regular, where \( \text{Even}(\mathcal{L}) \) is defined as

\[
\text{Even}(\mathcal{L}) = \{ \sigma_2 \sigma_4 \ldots \sigma_{2n} \mid \exists \sigma_1, \sigma_3, \ldots, \sigma_{2n-1} \in \Sigma \text{ such that } \sigma_1 \sigma_2 \sigma_3 \sigma_4 \ldots \sigma_{2n} \in \mathcal{L} \}
\]

(3) Let \( \Sigma \) be an alphabet with \( k \) characters. Define the language

\[
\text{Incomplete}(\Sigma) = \{ w \in \Sigma^* \mid \exists a \in \Sigma \text{ such that } w \text{ does not contain } a \}\.
\]

Design a NFA using at most \( k \) states that accepts \( \text{Incomplete}(\Sigma) \).

(4) The Hamming distance between two equal length strings \( x \) and \( y \) is the number of places they differ. We will use \( H(x, y) \) to denote this distance. For any language \( \mathcal{L} \subseteq \Sigma^* \) we say that \( H(x, \mathcal{L}) = \min_{y \in \mathcal{L}} H(x, y) \) (where the Hamming distance of strings \( x \) and \( y \) with \( |x| \neq |y| \) is infinite. Now let

\[
N_k(\mathcal{L}) = \{ x \in \Sigma^* \mid H(x, \mathcal{L}) \leq k \}.
\]

Prove that if \( \mathcal{L} \) is regular than so is \( N_1(\mathcal{L}) \). Hint: use two copies of the machine \( M \) accepting \( \mathcal{L} \), and use nondeterministic transitions to guess where the error occurs.