Introduction to Theory of Computing **Final Info and Practice Questions**
CS 481 Fall 2004

Final will be in class, Wednesday December 15 at 9:00-11:30 in Phillips 219). This is a closed book, closed notes test. On the test you may use any fact that was proved in the text, lectures, homeworks, or this practice sheet, but you should be able to use them.

Material. The Final is cumulative, but it is emphasis the material since the last prelim. It covers all problem sets 1-10. (The last two lectures may be covered only at a quiz level.)

Office Hours. The office hour schedule for the two weeks of finals is as follows.

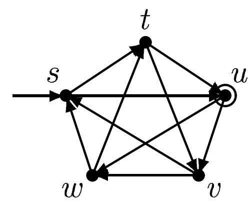
Monday, December 6		no office hour
Tuesday, December 7	Thanh	1-2pm in Upson 328
Wednesday, December 8	Thanh	4-5pm in Upson 328
Thursday, December 9	Eva	2-3pm in Upson 5135
Friday, December 10	Joel	1-2pm in Upson 328
Monday, December 13	Eva	1:30-3:30pm in Upson 5135
Tuesday, December 14	Eva	2-3 pm in Upson 5135
Tuesday, December 14	Joel	3-4 pm in Upson 328

Practice Questions. The following questions all have been on past finals (though there are too many of them here). There are also many great questions in the back of our textbook. Feel free to ask about any of these, or other questions from the book in office hours. Solutions to the practice questions will be posted on the Web Sunday, December 12th, and available in Upson 303 Monday and Tuesday.

1. Show which states are equivalent (if any are) on the following DFA over the alphabet $\Sigma = \{a, b\}$. Here the arrow marks the the starting state 0, F marks the two accepting states 0 and 2, and transitions are given by the table below.

		a	b
$\rightarrow$	0F	5	3
	1	1	5
	2F	3	5
	3	0	2
	4	4	3
	5	2	0

2. Consider the following nondeterministic finite automaton over input alphabet $\{a\}$ (edge labels a are omitted in the picture and the arrow marks the start state s).



Construct an equivalent deterministic automaton using the subset construction. Show clearly which subset of $\{s, t, u, v, w\}$ corresponds to each state of the deterministic automaton. Omit inaccessible states.

3. Give a regular expression for the set of strings in $\{a, b\}^*$ not of the form $(ab)^*$.

4. Give a context free grammar for the set in question 3.

5. True or false? No proofs necessary.

   (i) Every subset of a regular set is regular.

   (ii) There exist CFLs whose complements are not recursive.

   (iii) Every infinite CFL contains an infinite regular subset.

   (iv) If $A \le B$ then $\sim A \le \sim B$.

   (v) If $A \le B$ then $A \le \sim B$.

   (vi) The family of regular sets is closed under intersection, union, complement, and homomorphic image.

   (vii) The family of recursive sets is closed under homomorphic image.

   (viii) The family of r.e. sets is closed under homomorphic image.

   (ix) There is a set accepted by a nondeterministic TM that is accepted by no deterministic TM.

   (x) Enumeration machines and Turing machines describe the same class of sets.

6. Consider the following TM M with input alphabet $\{a, b\}$, left endmarker $\vdash$, blank symbol $\sqcup$, start state s, accept state t, and reject state r, and one other state u.

	$\vdash$	a	b	$\sqcup$
s	$s, \vdash, R$	s, b, R	s, a, R	$u, \sqcup, L$
u	$t, \vdash, R$	r, b, L	u, a, L	$r, \sqcup, L$
t	$t, \vdash, R$	t, b, L	t, a, L	$t, \sqcup, L$
r	$r, \vdash, R$	r, b, L	r, a, L	$r, \sqcup, L$

   What is $L(M)$?

7. Say whether the following problems are decidable or undecidable. No proofs necessary.

   (a) whether a given NFA accepts Σ^*

(b) whether a given PDA accepts Σ^*

(c) whether a given TM runs for more than $10^{42,000,000,000}$ steps on some input

(d) whether a given TM accepts the set $\{a^p \mid p \text{ is prime}\}$

(e) whether the intersection of two given CFLs is a CFL.

(f) whether the intersection of two given r.e. sets is r.e.

(g) whether two given Java programs compute the same function

(h) whether a given Java program will ever get into an infinite loop on some input

8. A *one-counter automaton* is a two-way finite automaton equipped with an integer counter in addition to its finite control. The tape head is two-way read-only and may not leave the input area. In each move, it can add one to the counter, subtract one, or test for zero.

   (a) Taking into account only the state and head position and ignoring the contents of the counter, there are $k(n+2)$ configurations of a one-counter automaton on inputs of length n, where k is the number of states. If the counter ever exceeds this value on an input of length n, then it must be in an infinite loop, since it must have repeated a configuration of state and head position since the last time the counter contained 0. Using this fact, argue that halting is decidable for one-counter automata.

   (b) Prove that it is undecidable whether a given one-counter automaton accepts $\varnothing$. (Hint: use VALCOMPS).

9. Prove that it is undecidable whether $L(M) = L(N) \cdot L(O)$ for given Turing machines M, N, and O.

10. Classify the sets (i)–(xiii) according to the the hierarchy: (a) regular; (b) context-free but not regular; (c) recursive but not context-free; (d) r.e. but not recursive; (e) not r.e. No proofs necessary.

    (i) $\{M \mid L(M) \text{ is finite}\}$

    (ii) $\{M \mid L(M) \text{ is infinite}\}$

    (iii) $\{M \mid L(M) \neq \varnothing\}$

    (iv) $\{M \mid L(M) \text{ contains a string with more 1's than 0's}\}$

    (v) $\{M \mid \text{every string in } L(M) \text{ contains more 1's than 0's}\}$

    (vi) $\{w\#w \mid w \in \{0,1\}^*\}$

    (vii) the complement of the set in (vi)

    (viii) $\{a^i b^j c^k d^\ell \mid i+k = j+\ell\}$

    (ix) $\{a^i b^j c^k d^\ell \mid i+k \equiv j+\ell \bmod 481\}$

    (x) $\{a^i b^j c^k d^\ell \mid i = 2\ell \text{ and } j = 7k\}$

(xi) $\{a^i b^j c^k d^\ell \mid i = 2\ell \text{ or } j = 7k\}$

(xii) $\{x \in \{a,b,c\}^* \mid \#a(x) + \#b(x) = \#c(x)\}$

(xiii) $\{x \in \{a,b,c\}^* \mid \#a(x) \cdot \#b(x) = \#c(x)\}$

11. Consider the following TM M with input alphabet $\{a,b\}$, left endmarker $\vdash$, blank symbol $\sqcup$, start state s, accept state t, and reject state r.

	$\vdash$	a	b	$\sqcup$
s	$s, \vdash, R$	s, b, R	r, a, L	$t, \sqcup, L$
t	$t, \vdash, R$	t, b, L	t, a, L	$t, \sqcup, L$
r	$r, \vdash, R$	r, b, L	r, a, L	$r, \sqcup, L$

What is $L(M)$?

12. Answer and provide a short explanation.

(i) Suppose G is a a context free grammar in Chomsky normal form with $k = 5$ production rules, and it can generate a string of size 1000. Is it possible, necessary, or not possible that $L(G)$ is infinite.

(ii) Is the language $\{G\#w | w \in L(G)\}$ recursive, where G is a context free grammar in Chomsky normal form.

(iii) Is the language $\{G_1, G_2 | L(G_1) = L(G_2)\}$ recursive, where G_1 and G_2 are context free grammars in Chomsky normal form.

13. Show that the language L of all Turing machines M such that $L(M)$ contains only strings with an even number of 1's, is not recursive. (*Hint.* Use Rice's theorem.)