CS481F01 HW 8 – Decidability

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Please remember to turn in each problem on a separate page, *put your name on each page*, and turn in the pages in three separate piles!

1. Prob. 111 from p. 344 of the text. One of the following sets is r.e. and the other is not. Which is which?

(a) $\{ i \mid L(M_i) \text{ contains at least 481 elements } \}$
(b) $\{ i \mid L(M_i) \text{ contains at most 481 elements } \}$

Prove your answers.

2. Suppose P is any property of *pairs* of r.e. sets. We define

$$L_P \quad = \quad \{ \langle i, j \rangle \mid P(L(M_i), L(M_j)) \}$$

We say such a property is *nontrivial* if it is neither identically true nor identically false; i.e.,

$$P \text{ nontrivial} \quad \Leftrightarrow \quad (\exists \langle i, j \rangle \in L_P) \ \wedge \ (\exists \langle i, j \rangle \notin L_P)$$

Prove the following extension of Rice's Theorem:

No nontrivial property of pairs of r.e. sets is decidable.

Hint: Consider $P(\emptyset, \emptyset)$ and $P(L(M), L(M'))$ for suitably chosen M and M'. Also, recall that a property is decidable iff its negation is decidable.

3. Let L and L' denote CFLs (presented as CFGs), and let R denote a regular set (presented as a regular expression or right-linear grammar). Which of the following are decidable and which undecidable?

(a) $L = R$
(b) $L \subseteq R$
(c) $L \supseteq R$
(d) $L = L'$
(e) $L \subseteq L'$
(f) $L \supseteq L'$
(g) $L = L\ L$

Prove your answers.