CS481F01 HW 6 – PDAS

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Please remember to turn in each problem on a separate page, *put your name on each page*, and turn in the pages in three separate piles!

1. Give a NPDAs that recognize the following languages:

(a) The set of all strings in $\{0, 1\}^*$ that contain twice as many 1s as 0s.

(b) The set

$$\{\ x^r\$y \mid (\exists n)(x = \mathbf{binary}(n)\ \wedge\ y = \mathbf{binary}(n+1)\ \}$$

where $\mathbf{binary}(n)$ is the binary encoding of natural number n. For example, this set contains 0\$1, 1101\$1100 and 001\$101 but not 1\$1 or 11\$10.

You may use whichever form of acceptance – empty stack or final state – is convenient. In each case prove your machine is correct.

2. An NPDA

$$M\ =\ (\ Q, \Sigma, \Gamma, \delta, s, \bot, F\ )$$

is a *Binary-Stack* NPDA if $|\Gamma| = 2$. M is a *Unary-Stack* NPDA if $|\Gamma| = 1$.

(a) Prove that every CFL is $L_{es}(M)$ for some Binary-Stack NPDA M.

Hint: You don't need a grammar for this – you can do it entirely with machines – but think about the way a bottom-up recognizer as discussed in lecture implements "reduce" actions.

(b) Give a language L that is not regular but is $L_{es}(M)$ for some Unary-Stack NPDA M.

(c) Does there exist a CFL L that is not $L_{es}(M)$ for any Unary-Stack NPDA M? Argue convincingly for your answer. A detailed proof is not necessary.

3. This is a "cumulative" problem – each part develops on the previous parts. We derive some properties of the Deterministic CFLs (DCFLs), i.e. languages that are $L(M)$ for some Deterministic PDA M.

(a) Show the set

$$\{ \ a^i b^i c^i \ | \ i > 0 \ \}$$

is not a CFL.

(b) Show the set

$$\{ \ a^i b^j c^i \ | \ i, j > 0 \ \}$$

is a DCFL.

(c) Show the DCFLs are not closed under intersection: give DCFLs L_1 and L_2 such that $L_1 \cap L_2$ is not a DCFL.

(d) In lecture and in the text we show that DCFLs are closed under complement:

$$L \text{ is a } DCFL \quad \Rightarrow \quad \bar{L} \ \equiv \ (\Sigma^* - L) \text{ is a } DCFL$$

It follows that the DCFLs cannot be closed under union (Why?). Give an example; that is, give two DCFLs L_1 and L_2 such that $L_1 \cup L_2$ is not a DCFL. (Note that $L_1 \cup L_2$ is certainly a CFL; it's just not a deterministic one). Prove your answer.

(e) Let the *tagged union* of two languages be defined by

$$L_0 \cup_t L_1 \quad \equiv \quad \{ \ 0w \ | \ w \in L_0 \ \} \ \cup \ \{ \ 1w \ | \ w \in L_1 \ \}$$

Prove the DCFLs are closed under tagged union.

(f) Are the DCFLs closed under homomorphism? Explain your answer.