CS481F01 HW 5 – CNF, GNF, Pumping Lemma

A. Demers

17 Oct – due 24 Oct

Please remember to turn in each problem on a separate page, put your name on each page, and turn in the pages in three separate piles!

1. Problem 73 from p. 334 of the text: Let $\Sigma = \{0, 1\}$. Let $\overline{x}$ denote the Boolean complement of $x$; that is, the string obtained from $x$ by changing all 0’s to 1’s and all 1’s to 0’s. Let $x^r$ denote the reverse of $x$; that is, the string $x$ written backwards. Consider the set

$$A = \{ x \mid x^r = \overline{x} \}$$

For instance, the strings 011001 and 010101 are in $A$ but 101101 is not.

(a) Give a CFG generating this set.

(b) Give grammars in Chomsky and Greibach normal form for $A - \{\varepsilon\}$.

2. Prove the following closure properties, where $L$ and $L'$ are CFLs and $h$ is a homomorphism:

(a) (union): $L \cup L'$ is a CFL.

(b) (concatenation): $L \cdot L' = \{ xy \mid x \in L, y \in L' \}$ is a CFL.

(c) (homomorphic image): $h(L) = \{ h(x) \mid x \in L \}$ is a CFL.
(d) (inverse homomorphism): $h^{-1}(L) = \{ x \mid h(x) \in L \}$ is a CFL.

For part (d) you might want to assume you have a grammar in GNF that generates $L - \{\epsilon\}$.

3. In this question you will show that the CFLs are not closed under intersection.

(a) Show that the set

$$\{ 0^i1^{i+1} \mid i > 0 \}^*$$

is a CFL.

(b) Show that the set

$$\{ 0^i^2 \mid i > 0 \}$$

is not a CFL.

(c) Now show that the same set

$$\{ 0^i^2 \mid i > 0 \}$$

is a homomorphic image of the intersection of two CFLs.

Conclude that CFLs are not closed under intersection.