CS481F01 HW 5 – CNF, GNF, Pumping Lemma

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Please remember to turn in each problem on a separate page, *put your name on each page*, and turn in the pages in three separate piles!

1. Problem 73 from p. 334 of the text: Let $\Sigma = \{0, 1\}$. Let $\bar{x}$ denote the Boolean complement of x; that is, the string obtained from x by changing all $0's$ to $1's$ and all $1's$ to $0's$. Let x^r denote the reverse of x; that is, the string x written backwards. Consider the set

$$A \;=\; \{\, x \mid x^r \;=\; \bar{x} \,\}$$

For instance, the strings 011001 and 010101 are in A but 101101 is not.

(a) Give a CFG generating this set.

(b) Give grammars in Chomsky and Greibach normal form for $A - \{\epsilon\}$.

2. Prove the following closure properties, where L and L' are CFLs and h is a homomorphism:

(a) (union): $L \cup L'$ is a CFL.

(b) (concatenation): $L \cdot L' \;=\; \{\, xy \mid x \in L, y \in L' \,\}$ is a CFL.

(c) (homomorphic image): $h(L) \;=\; \{\, h(x) \mid x \in L \,\}$ is a CFL.

(d) (inverse homomorphism): $h^{-1}(L) = \{ x \mid h(x) \in L \}$ is a CFL.

For part (d) you might want to assume you have a grammar in GNF that generates $L - \{\epsilon\}$.

3. In this question you will show that the CFLs are not closed under intersection.

(a) Show that the set

$$\{ 0^i 1^{i+1} \mid i > 0 \}^*$$

is a CFL.

(b) Show that the set

$$\{ 0^{i^2} \mid i > 0 \}$$

is *not* a CFL.

(c) Now show that the same set

$$\{ 0^{i^2} \mid i > 0 \}$$

is a homomorphic image of the intersection of two CFLs.

Conclude that CFLs are not closed under intersection.