1. For each of the following languages, give a context-free grammar that generates the language.

   (a) \{0^i1^i \mid i \geq 0\}^*  
   (b) \{0^i1^j0^k \mid i = j \lor j = k\}  
   (c) \{0^i1^j \mid i \leq j \leq 2i\}  
   (d) \{ w \in \{0,1\}^* \mid \#1(w) = \#0(w)\}  
   (e) \{ x \in \{0,1\}^* \mid \neg (\exists i, j)(x = (0^i1^j)^i)\}  

The language described in part (e) is bit subtle: it is strings over \{0,1\} that do not consist of \(j\) repetitions of \(0^i1^i\) for any fixed \(i\). Examples: the empty string is not in the language, since it corresponds to the case \(i = j = 0\). The strings 1\(w\) and \(w0\) are in the language for any \(w\). The string 001100110011 is not in the language, since it corresponds to the case \(i = 2, j = 3\). The strings 01110011 and 0011000111 are both in the language. This is getting near the limit of what can be checked by a CFG.

In each case, make sure the grader can understand why your answer generates the specified language. Your answers to (e), (d) and possibly (c) may require informal proofs.

2. Recall from lecture that a right linear grammar is one in which the productions are of the form

   \[ S \to \varepsilon \]
   \[ A \to aB \quad \text{or} \]
   \[ A \to a \]

   1
These grammars are often called regular grammars. In this question you show why.

(a) Suppose you are given an arbitrary union-dot-star regular expression $E$. Show (by induction on the structure of $E$) how to construct a right linear grammar $G$ such that $L(G) = L(E)$. Explain your answer.

(b) Suppose you are given an arbitrary right linear grammar $G$. Show how to construct an NFA $M$ such that $L(M) = L(G)$. Argue that your solution is correct.

Now you can conclude that the right linear languages are exactly the regular sets.

3. Say grammar $G$ is a symmetric linear grammar if its productions are of the form

$$A \rightarrow aBc$$
$$A \rightarrow a \quad \text{or}$$
$$A \rightarrow \varepsilon$$

A language $L$ is a symmetric linear language if $L = L(G)$ for some symmetric linear grammar $G$.

(a) Give an example of a symmetric linear language that is not regular.

(b) Prove every regular language is a symmetric linear language. For example, given a DFA $M$, show how to construct a symmetric linear grammar $G$ generating $L(M)$.

(c) Show that every symmetric linear language over a single-letter alphabet is regular.

**Hint:** Part (b) may be harder than you think. Note that states (or sets of states, or pairs of states, or ...) of a FA can be encoded into the nonterminal symbols of a grammar.