CS481F01 HW 3 – Non-Regular Sets and State Minimization

A. Demers

26 Sep – due 3 Oct

Please remember to turn in each problem on a separate page. This week, please remember to *put your name on each page*, and turn in the pages in three separate piles!

1. For this question, you may assume the languages

$$L_{01} \quad = \quad \{\ 0^i 1^i \mid i \geq 0\ \}$$

and

$$L_{010} \quad = \quad \{\ 0^i 1 0^i \mid i \geq 0\ \}$$

are not regular, since we proved this in lecture.

Show that each of the languages given below is not regular by using closure properties to show that, if it *were* regular, then either L_{01} or L_{010} would have to be regular, leading to a contradiction.

(a) $\{\ 0^{2i} 1^{3i} \mid i \geq 0\ \}$
(b) $\{\ 0^i 1^j 2^k \mid i = j \vee j = k\ \}$
(c) $\{\ 0^i 1^j \mid j = (i + 481)\ \}$
(d) $\{\ ww \mid w \in \{0,1\}^*\ \}$
(e) $\{\ w \in \{0,1\}^* \mid \sharp 0(w) = \sharp 1(w)\ \}$

(Recall $\sharp a(w)$ is the number of a's in w).

2. Consider the DFA

$$M = (Q, \Sigma, \delta, s, F)$$

where

$$\begin{aligned} Q &= \{ q_0, q_1, q_2, q_3, q_4, q_5 \} \\ s &= q_0 \\ F &= \{ q_0, q_5 \} \end{aligned}$$

and δ is given by the table:

q	$\delta(q,0)$	$\delta(q,1)$
q_0	q_1	q_2
q_1	q_3	q_4
q_2	q_3	q_4
q_3	q_0	q_5
q_4	q_0	q_5
q_5	q_2	q_1

a. Compute the state indistinbuishability relations $\equiv_0$, $\equiv_1$, $\ldots$, $\equiv_\infty$ for this machine.

b. Construct the minimal machine $M_{\equiv_\infty}$ equivalent to this machine. A state transition diagram is sufficient.

c. For any $k \geq 0$ show how to construct a DFA for which the indistinguishability relations do not converge until the k^{th} step. That is, show how to construct a DFA such that

$$(\equiv_{i+1} \;=\; \equiv_i) \;\Rightarrow\; (i \geq k)$$

Convince us that your construction is correct; a formal inductive proof is not necessary.

3. Question 29 from p 323 of the text:

For A a set of natural numbers, define

$$\begin{aligned} \textbf{binary}A &= \{ \text{ binary representations of numbers in } A \} \subseteq \{0,1\}^* \\ \textbf{unary}A &= \{ 0^n \mid n \in A \} \subseteq \{0\}^* \end{aligned}$$

For example, if A is $\{2, 3, 5\}$ then

$$\begin{aligned} \mathbf{binary}A &= \{\ 10, 11, 101\ \} \\ \mathbf{unary}A &= \{\ 00, 000, 00000\ \} \end{aligned}$$

Consider the following two propositions:

(i) $\forall A,$ if $\mathbf{binary}A$ is regular then so is $\mathbf{unary}A$

(ii) $\forall A,$ if $\mathbf{unary}A$ is regular then so is $\mathbf{binary}A$

One of (i) and (ii) is true and the other is false. Which is which? Give a proof and a counterexample.