CS481F01 Homework 1 – FAs and Regular Sets

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10 Sep – due 17 Sept

In all the following problems, let $\Sigma = \{0, 1\}$.

1. Let $L \subset \Sigma^*$ be the set of all strings in which the third symbol from the right end is 1; that is,

$$L \;=\; \{\; x1ab \mid x \in \Sigma^* \;\wedge\; a, b \in \Sigma \;\}$$

For example, L contains 100 and 0101 but not ε or 1011.

(a) Construct a NFA that recognizes L. Give a formal description as a 5-tuple

$$N \;=\; (Q, \Sigma, \Delta, S, F)$$

(with Δ presented as a table) and draw the transition diagram as well.

(b) Prove formally that the NFA you constructed in part (a) recognizes L.

(c) Using the subset construction described in lecture and the text, convert your NFA from part (a) to a DFA recognizing L.

2. Let $L \subset \Sigma^*$ be a regular language. Consider the following three languages:

$$\begin{array}{lrcl}
(a) & L_a & = & \{\; 0^i x \mid (|x| = i) \wedge (x \in L) \;\} \\
(b) & L_b & = & \{\; 0^i \mid (\exists x \in \Sigma^*)\; ((|x| = i) \wedge (x \in L)) \;\} \\
(c) & L_c & = & \{\; 0^i \mid (\exists x \in \Sigma^*)\; ((|x| = i) \wedge (0^i x \in L)) \;\}
\end{array}$$

Which of these languages is (are) regular? Prove your answers.

3. Prove: for every $k \geq 0$ there exists a regular language L_k with the property that any DFA recognizing L_k has at least k final states.

Hint: observe that

$$(\forall x, y, z \in \Sigma^*)((\hat{\delta}(q,x) \;=\; \hat{\delta}(q,y)) \;\;\Rightarrow\;\; (\hat{\delta}(q,xz) \;=\; \hat{\delta}(q,yz)))$$

so that any two strings x and y leading to the same final state must behave identically under extension by z. Exploit this fact in the construction of L_k.