Low-Precision Arithmetic

CS4787 Lecture 21 — Spring 2020

The standard approach
Single-precision floating point (FP32)

- 32-bit floating point numbers

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

sign　　8-bit exponent　　23-bit mantissa

- Usually, the represented value is

$$\text{represented number} = (-1)^{\text{sign}} \cdot 2^{\text{exponent}-127} \cdot 1.b_{22}b_{21}b_{20}\ldots b_0$$

Three special cases

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

sign 8-bit exponent 23-bit mantissa

- When the exponent is all 0s, and the mantissa is all 0s
  - This represents the real number **0**
  - Note the possibility of "negative 0"

- When the exponent is all 0s, and the mantissa is nonzero
  - Called a "denormal number" — degrades precision gracefully as 0 approached

$$\text{represented number} = (-1)^{\text{sign}} \cdot 2^{-126} \cdot 0.b_{22}b_{21}b_{20} \ldots b_0.$$

Three special cases (continued)

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

sign 8-bit exponent 23-bit mantissa

- When the exponent is all 1s (255), and the mantissa is all 0s
  - This represents **infinity** or **negative infinity**, depending on the sign
  - Indicates overflow or division by 0 occurred at some point
    - Note that these events usually do not cause an exception, but sometimes do!

- When the exponent is all 1s (255), but the mantissa is nonzero
  - Represents something that is **not a number**, called a **NaN** value.
  - This usually indicates some sort of compounded error.
  - The bits of the mantissa can be a message that indicates how the error occurred.

DEMO

Measuring Error in Floating Point Arithmetic

If $x \in \mathbb{R}$ is a real number within the range of a floating point representation, and $\tilde{x}$ is the closest representable floating-point number to it, then

$$|\tilde{x} - x| = |\,\mathrm{round}(x) - x| \leq |x| \cdot \varepsilon_{\mathrm{machine}}.$$

Here, $\varepsilon_{\mathrm{machine}}$ is called the *machine epsilon* and bounds the relative error of the format.

Error of Floating-Point Computations

If x and y are real-numbers representable in a floating-point format, $\circ$ denotes an (infinite-precision) binary operation (such as $+$, $\cdot$, etc.) and $\bullet$ denotes the floating-point version of that operation, then

$$x \bullet y = \mathrm{round}(x \circ y) \quad \text{and} \quad |(x \bullet y) - (x \circ y)| \leq |x \circ y| \cdot \varepsilon_{\mathrm{machine}},$$

as long as the result is in range.

Exceptions to this error model

- If the exact result is larger than the largest representable value
  - The floating-point result is **infinity** (or minus infinity)
  - This is called **overflow**

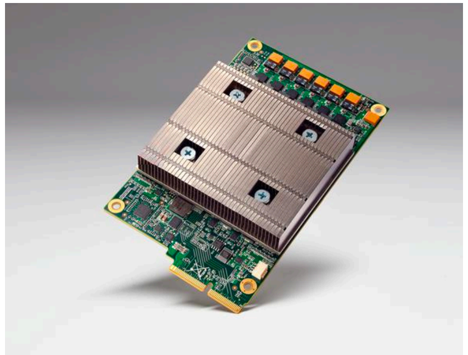
- If the exact result falls in the range of denormal numbers, there may be more error than the model predicts

- If there is an invalid computation
  - e.g. the square root of a negative number, or infinity + (-infinity)
  - the result is **NaN**

How can we use this info to make our ML systems more scalable?

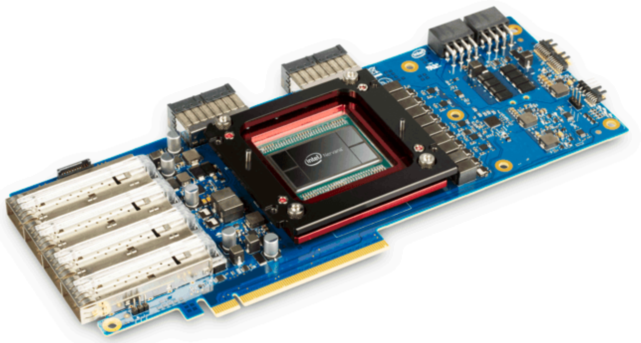
Low-precision compute

- Idea: replace the 32-bit or 64-bit floating point numbers traditionally used for ML with **smaller numbers**
  - For example, 16-bit floats or 8-bit integers
- New specialized **chips for accelerating ML training**.
  - Many of these chips leverage **low-precision compute**.



Google's TPU

NVIDIA's GPUs



Intel's NNP

A low-precision alternative

FP16/Half-precision floating point

- 16-bit floating point numbers

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

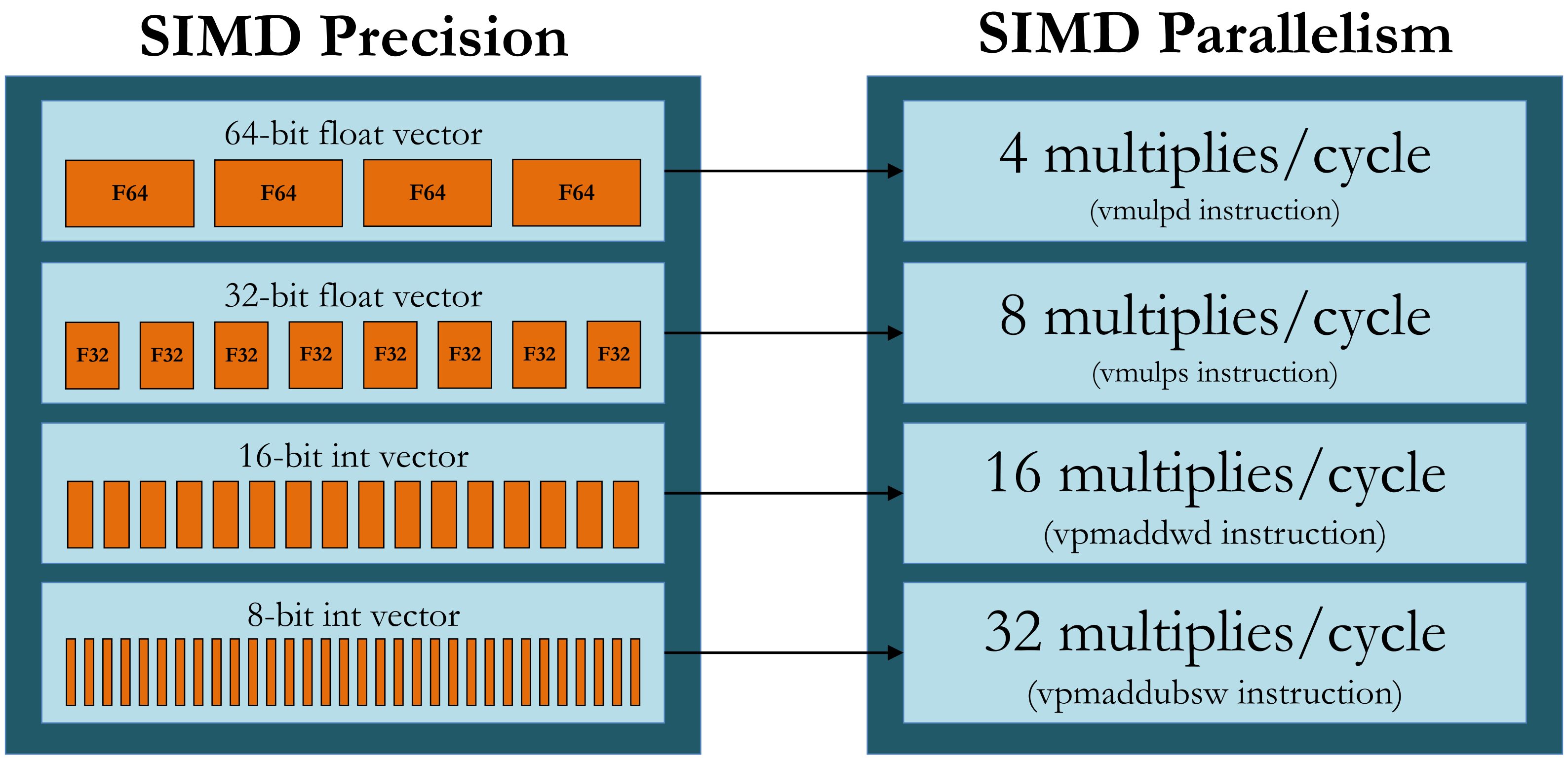
sign 5-bit exp 10-bit mantissa

- Usually, the represented value is

$$x = (-1)^{\text{sign bit}} \cdot 2^{\text{exponent}-15} \cdot 1.\text{significand}_2$$

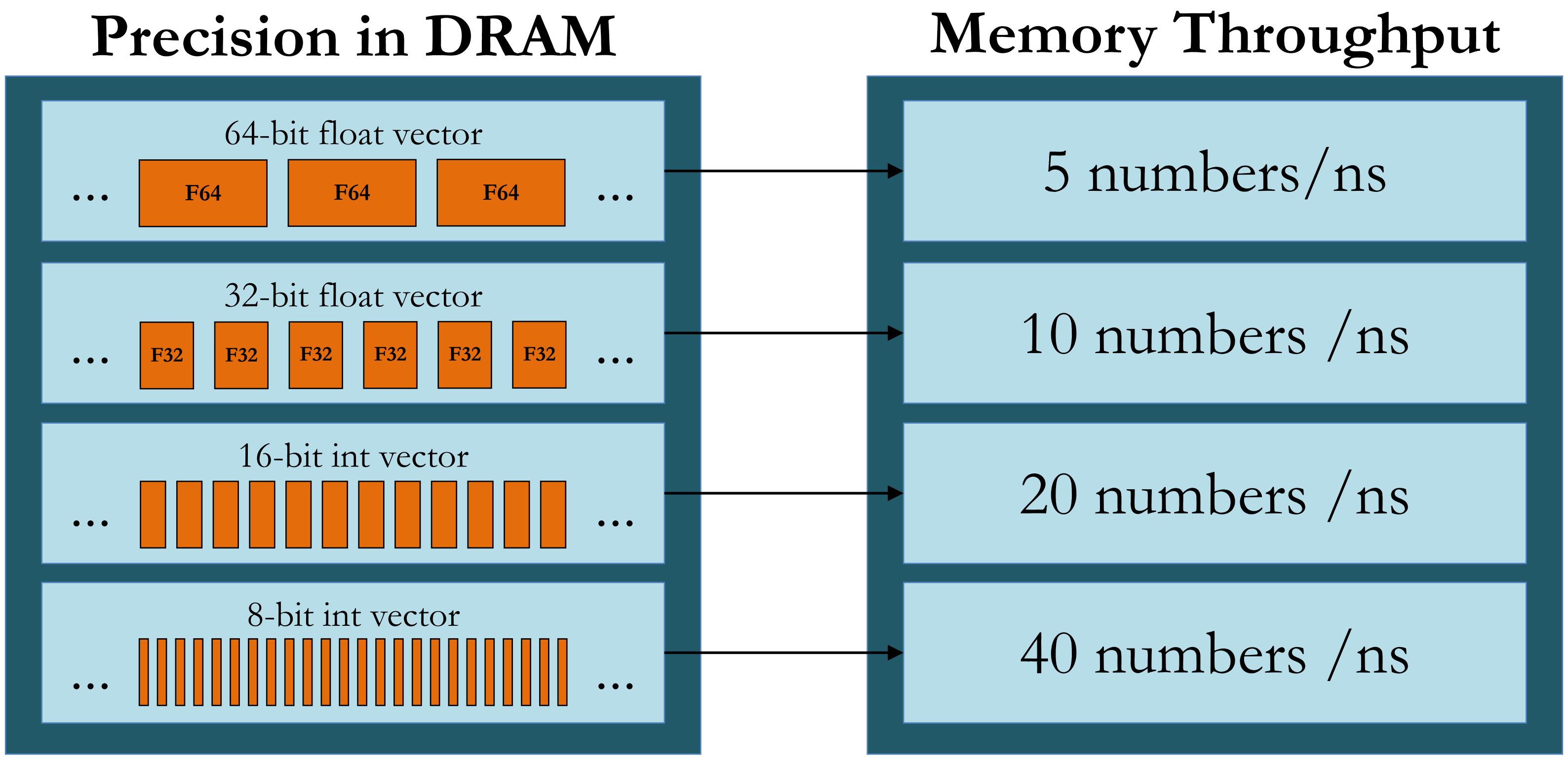
Benefits of Low-Precision: Compute

- Use **SIMD/vector instructions** to run more computations at once



Benefits of Low Precision: Memory

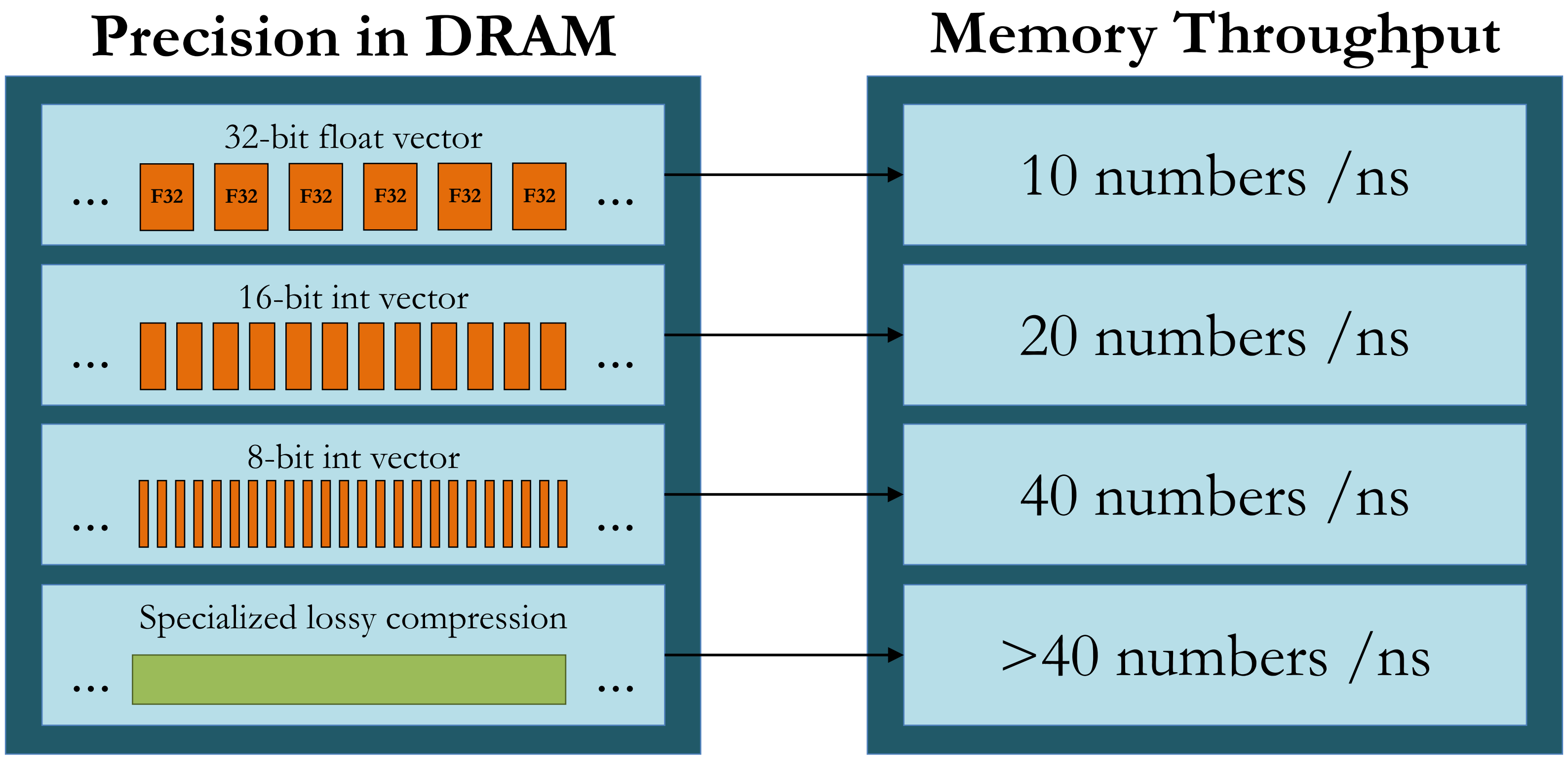
- Puts **less pressure** on memory and caches



(assuming ~40 GB/sec memory bandwidth)

Benefits of Low Precision: Communication

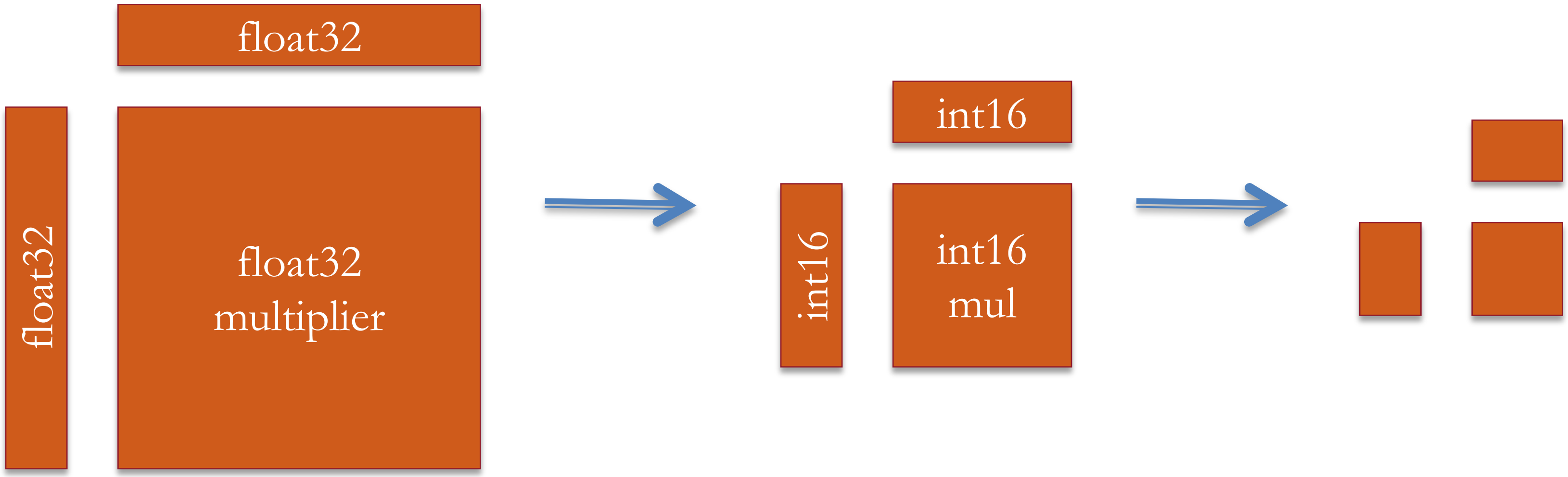
- Uses **less network bandwidth** in distributed applications



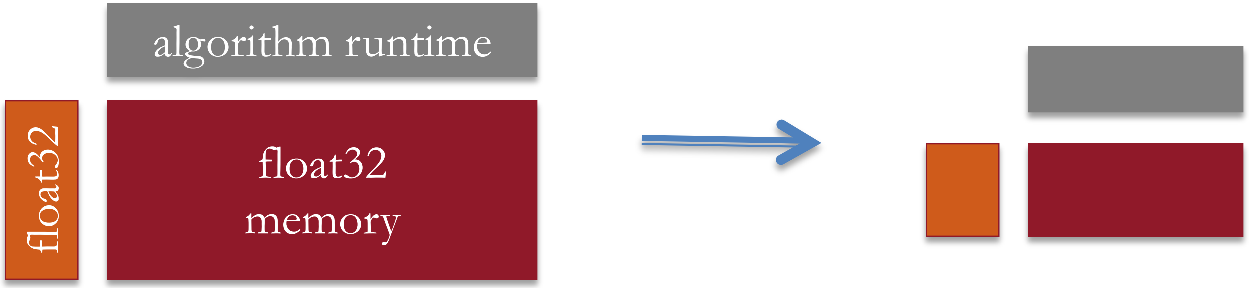
(assuming ~40 GB/sec network bandwidth)

Benefits of Low Precision: Power

- Low-precision computation can even have a super-linear effect on energy



- Memory energy can also have quadratic dependence on precision



Effects of Low-Precision Computation

- **Pros**
  - Fit more numbers (and therefore more training examples) in memory
  - Store more numbers (and therefore larger models) in the cache
  - Transmit more numbers per second
  - Compute faster by extracting more parallelism
  - Use less energy

- **Cons**
  - Limits the numbers we can represent
  - Introduces **quantization error** when we store a full-precision number in a low-precision representation

Numeric properties of 16-bit floats

- A larger machine epsilon (**larger rounding errors**) of $\epsilon_{\text{machine}} \approx 9.8 \times 10^{-4}$
  - Compare 32-bit floats which had $\epsilon_{\text{machine}} \approx 1.2 \times 10^{-7}$.
- A smaller overflow threshold (**easier to overflow**) at about 6.5×10^4
  - Compare 32-bit floats where it's 3.4×10^{38}
- A larger underflow threshold (**easier to underflow**) at about 6.0×10^{-8}.
  - Compare 32-bit floats where it's 1.4×10^{-45}

With all these drawbacks, does anyone use this?

Half-precision floating point support

- Supported on most **modern machine-learning-targeted GPUs**
  - Including efficient implementation on NVIDIA Pascal GPUs

Pascal Hardware Numerical Throughput

GPU	DFMA (FP64 TFLOP/s)	FFMA (FP32 TFLOP/s)	HFMA2 (FP16 TFLOP/s)	DP4A (INT8 TIOP/s)	DP2A (INT16/8 TIOP/s)
GP100 (Tesla P100 NVLink)	5.3	10.6	21.2	NA	NA
GP102 (Tesla P40)	0.37	11.8	0.19	43.9	23.5
GP104 (Tesla P4)	0.17	8.9	0.09	21.8	10.9

Table 1: Pascal-based Tesla GPU peak arithmetic throughput for half-, single-, and double-precision fused multiply-add instructions, and for 8- and 16-bit vector dot product instructions. (Boost clock rates are used in calculating peak throughputs. TFLOP/s: Tera Floating-point Operations per Second. TIOP/s: Tera Integer Operations per Second. https://devblogs.nvidia.com/parallelforall/mixed-precision-programming-cuda-8/

- Good empirical results for **deep learning**

Micikevicius et al. "Mixed Precision Training." on arxiv, 2017.

One way to address limited range: more exponent bits
Bfloat16 — "brain floating point"

- Another 16-bit floating point number

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

sign 8-bit exp 7-bit mantissa

Q: What can we say about the range of bfloat16 numbers as compared with IEEE half-precision floats and single-precision floats? How does their machine epsilon compare?

Bfloat16 (continued)

- Main benefit: numeric range is **now the same as single-precision float**
  - Since it looks like a truncated 32-bit float
  - This is useful because ML applications are **more tolerant to quantization error** than they are to overflow

- Also supported on specialized hardware

AI & MACHINE LEARNING

BFloat16: The secret to high performance on Cloud TPUs

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August 23, 2019

Machine learning workloads are computationally intensive and often run for hours or days. To help organizations significantly improve the running time of these workloads, Google developed custom processors called Tensor Processing Units, or TPUs, which make it possible to train and run cutting-edge deep neural networks at higher performance and lower cost.

The second- and third-generation TPU chips are available to Google Cloud customers as Cloud TPUs. They deliver up to 420 teraflops per Cloud TPU device and more than 100

An alternative to low-precision floating point
Fixed point numbers

- **p + q + 1** –bit fixed point number

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

sign fixed-point number

- The represented number is

$$x = (-1)^{\text{sign bit}} \left(\text{integer part} + 2^{-q} \cdot \text{fractional part}\right)$$
$$= 2^{-q} \cdot \text{whole thing as signed integer}$$

Arithmetic on fixed point numbers

- **Simple and efficient**
  - Can just use preexisting integer processing units
  - **Lower power** than floating point operations with the same number of bits
- **Mostly exact**
  - Underflow impossible
  - Overflow can happen, but is easy to understand
  - Can always convert to a higher-precision representation to avoid overflow
- Can represent a **much narrower range of numbers than float**

Support for fixed-point arithmetic

- **Anywhere integer arithmetic is supported**
  - CPUs, GPUs
  - Although not all GPUs support 8-bit integer arithmetic
  - And AVX2 does not have all the 8-bit arithmetic instructions we'd like
- Particularly effective on **FPGAs and ASICs**
  - Where floating point units are costly
- Sadly, very **little support for other precisions**
  - **4-bit operations** would be particularly useful

Breakout Questions

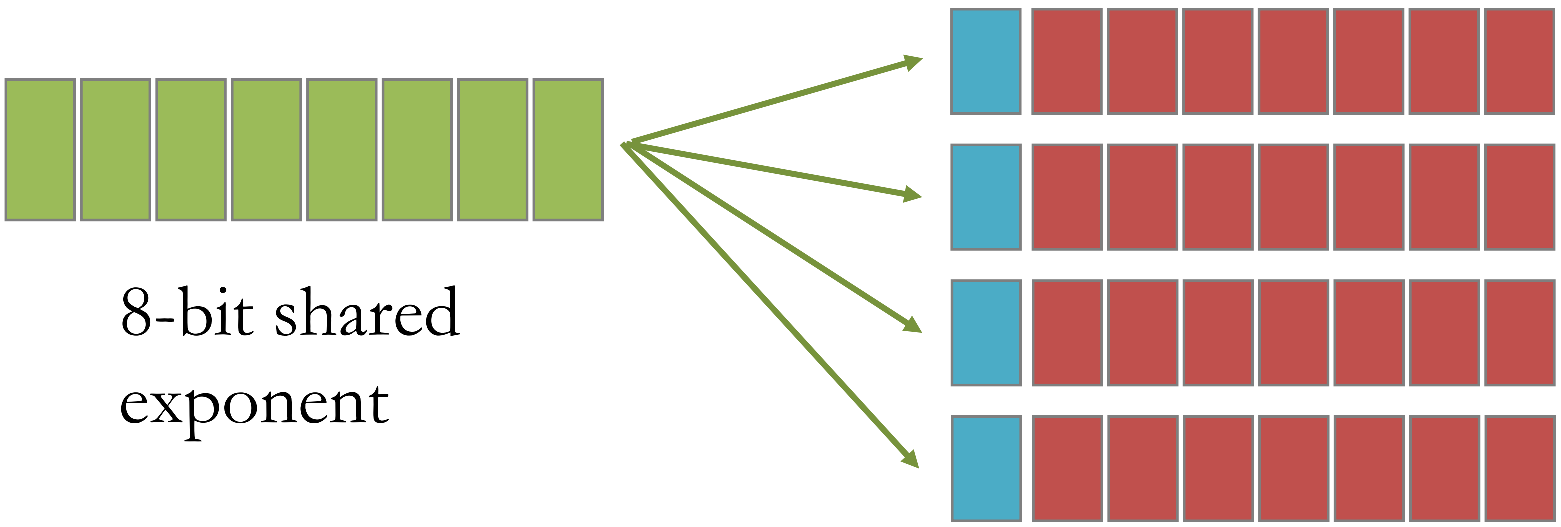
- **Q: What are the upsides/downsides of using fixed-point numbers for ML?**
  - **Compared to floating-point?**

- **Q: Can you think of a place where you've already used something like fixed-point numbers in a programming assignment?**

A powerful hybrid approach
Block Floating Point

- Motivation: when storing a vector of numbers, often these numbers all lie in the same range.
  - So they will have the same or similar exponent, if stored as floating point.

- **Block floating point** shares a single exponent among multiple numbers.

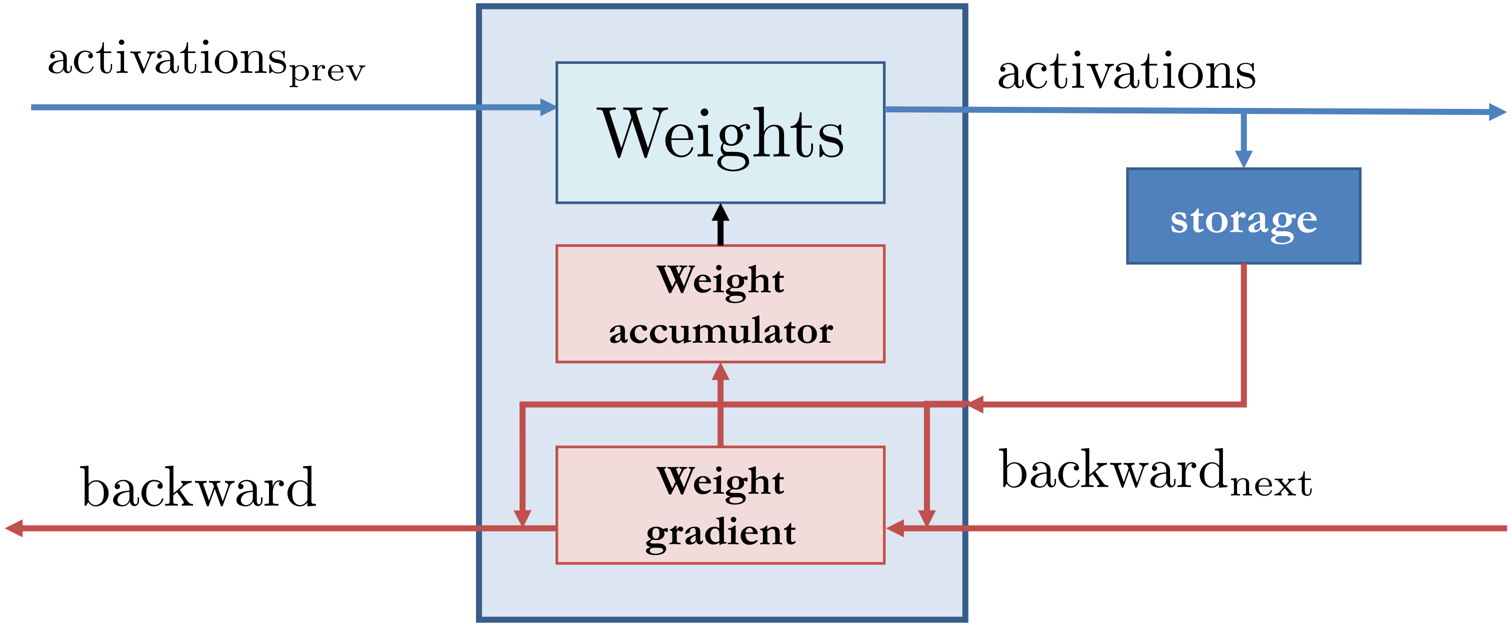


A more specialized approach
Custom Quantization Points

- Even more generally, we can just have a list of 2^b numbers and say that these are the numbers a particular low-precision string represents
  - We can think of the bit string as indexing a number in a dictionary
- Gives us total freedom as to range and scaling
  - But **computation can be tricky**
- Some **recent research into using this with hardware support**
  - "*The ZipML Framework for Training Models with End-to-End Low Precision: The Cans, the Cannots, and a Little Bit of Deep Learning*" (Zhang et al 2017)

How is precision used for DNN training

- Signals flow through network in backpropagation
  - Generally, we assign a precision to each of the types of signals, and different types of signals can have different precisions



Types of signals in backpropagation:
- Training dataset
- Vectors that store weights/parameters
- Gradients
- Communication among parallel workers
- Activations
- Backward pass signals
- Weight accumulators
- Momentum/ADAM vectors

Low-precision formats in general

- These are some of the **most common formats used in ML**
  - …but we're not limited to using only these formats!

- There are **many other things we could try**
  - For example, floating point numbers with different exponent/mantissa sizes
  - Block floating point numbers with different block sizes/layouts
  - Fixed point numbers with nonstandard widths

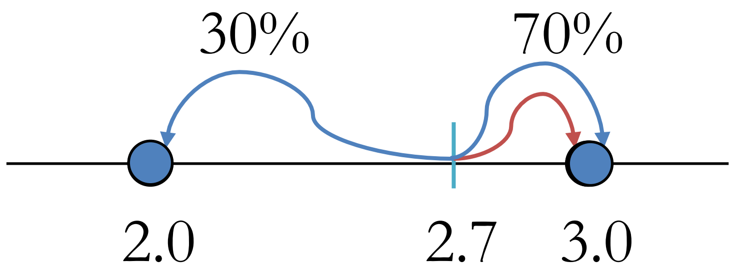
- Problem: there's **no hardware support** for these other things yet, so it's hard to get a sense of how they would perform.

Theoretical Guarantees for Low Precision

- Reducing precision adds noise in the for

- Two approaches to rounding:
  - **biased rounding** – round to nearest number
  - **unbiased rounding** – round randomly: $\boldsymbol{E}[Q(x)] = x$

Using this, we can prove **guarantees** that SGD works with a low precision model…since a low-precision gradient is an unbiased estimator.



Why unbiased rounding?

- Imagine running SGD with a low-precision model with update rule

$$w_{t+1} = \tilde{Q}\left(w_t - \alpha_t \nabla f(w_t; x_t, y_t)\right)$$

  - Here, **Q** is an unbiased quantization function

- In expectation, this is **just gradient descent**

$$\begin{aligned}
\mathbf{E}[w_{t+1} | w_t] &= \mathbf{E}\left[\tilde{Q}\left(w_t - \alpha_t \nabla f(w_t; x_t, y_t)\right) \middle| w_t\right] \\
&= \mathbf{E}\left[w_t - \alpha_t \nabla f(w_t; x_t, y_t) | w_t\right] \\
&= w_t - \alpha_t \nabla f(w_t)
\end{aligned}$$

Implementing unbiased rounding

- To implement an unbiased to-integer quantizer:

$$\text{sample } u \sim \text{Unif}[0,1], \quad \text{then set } Q(x) = \lfloor x + u \rfloor$$

- Why is this unbiased?

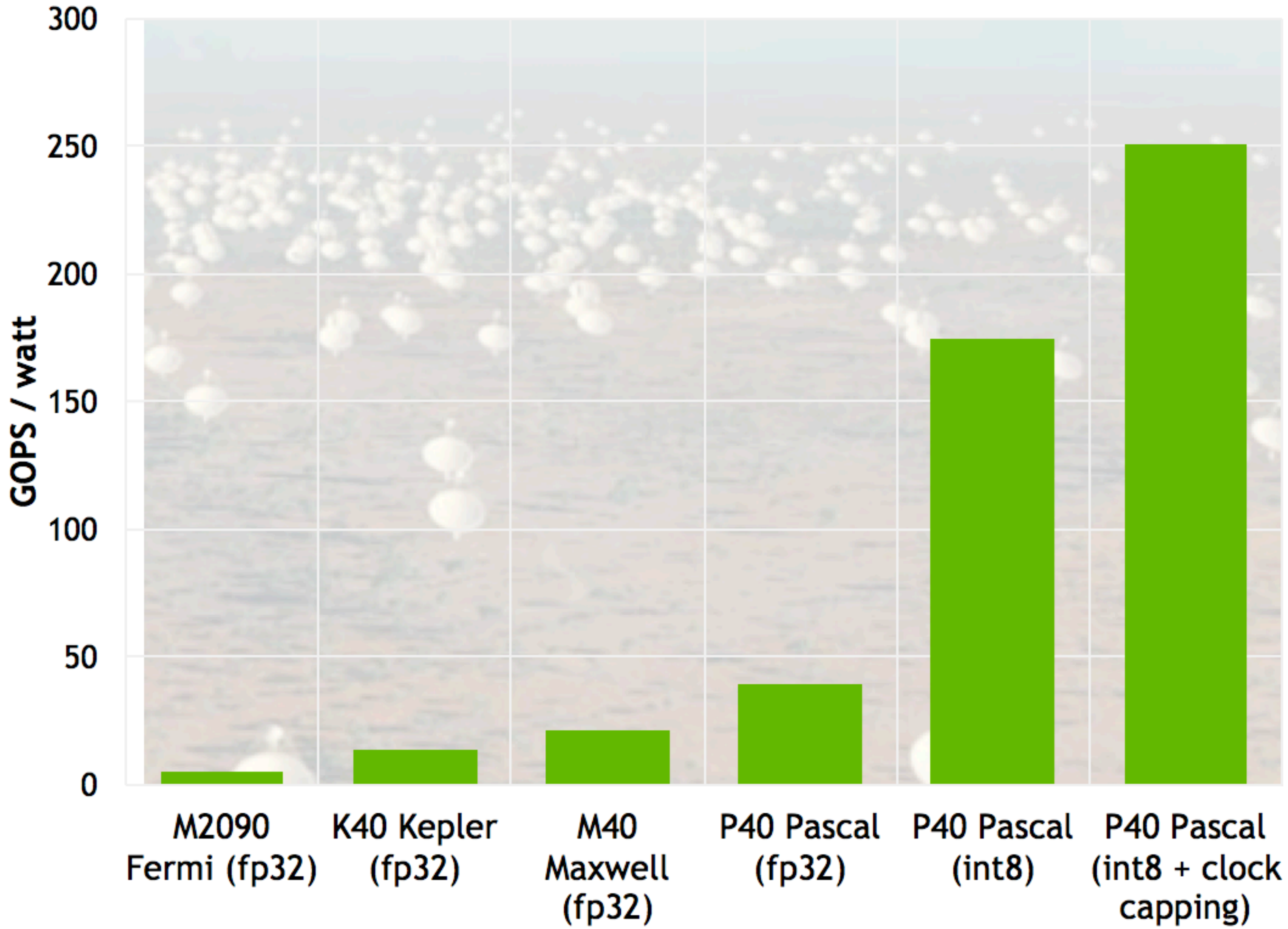
$$\begin{aligned}
\mathbf{E}[Q(x)] &= \lfloor x \rfloor \cdot \mathbf{P}(Q(x) = \lfloor x \rfloor) + (\lfloor x \rfloor + 1) \cdot \mathbf{P}(Q(x) = \lfloor x \rfloor + 1) \\
&= \lfloor x \rfloor + \mathbf{P}(Q(x) = \lfloor x \rfloor + 1) = \lfloor x \rfloor + \mathbf{P}(\lfloor x + u \rfloor = \lfloor x \rfloor + 1) \\
&= \lfloor x \rfloor + \mathbf{P}(x + u \geq \lfloor x \rfloor + 1) = \lfloor x \rfloor + \mathbf{P}(u \geq \lfloor x \rfloor + 1 - x) \\
&= \lfloor x \rfloor + 1 + (\lfloor x \rfloor + 1 - x) = x.
\end{aligned}$$

DEMO

Doing unbiased rounding efficiently

- We still need an efficient way to do unbiased rounding
- **Pseudorandom number generation can be expensive**
  - E.G. doing C++ rand or using Mersenne twister takes many clock cycles
- Empirically, we can use **very cheap** pseudorandom number generators
  - And still get good statistical results
  - For example, we can use XORSHIFT which is just a cyclic permutation

Benefits of Low-Precision Computation



From https://devblogs.nvidia.com/parallelforall/mixed-precision-programming-cuda-8/

Conclusion and Drawbacks of low-precision

- The draw back of low-precision arithmetic is the **low precision**!
- Low-precision computation means we accumulate **more rounding error** in our computations
- These rounding errors can add up throughout the learning process, resulting in **less accurate learned systems**
- The trade-off of low-precision: **throughput/memory vs. accuracy**