Spectral Embedding

Consider an undirected graph $G = (V, E)$ consisting of $|V| = n$ vertices and $E$ is the set of edges that indicate which pairs of vertices are connected. Let $A$ be the $n \times n$ adjacency matrix of the graph where $A[i, j] = 1$ if node $i$ and node $j$ have an edge between them. Let $D$ be the $n \times n$ diagonal matrix with entry $D[i, i]$ indicating the degree of node $i$ in the graph.

Question 1: Let $y$ be any $n$ dimensional vector (one coordinate for every node in the graph). Show that

$$\sum_{(u, v) \in E} (y_u - y_v)^2 = 2 \, y^\top (D - A) y$$
Question 2: This is a simple one given what we showed in question 1. Define the matrix $L = D - A$ (this is called the Laplacian matrix)

1. Show that $L$ is positive semidefinite.
2. If we take $1$ to be the all ones vector, show that $L \ 1 = 0$