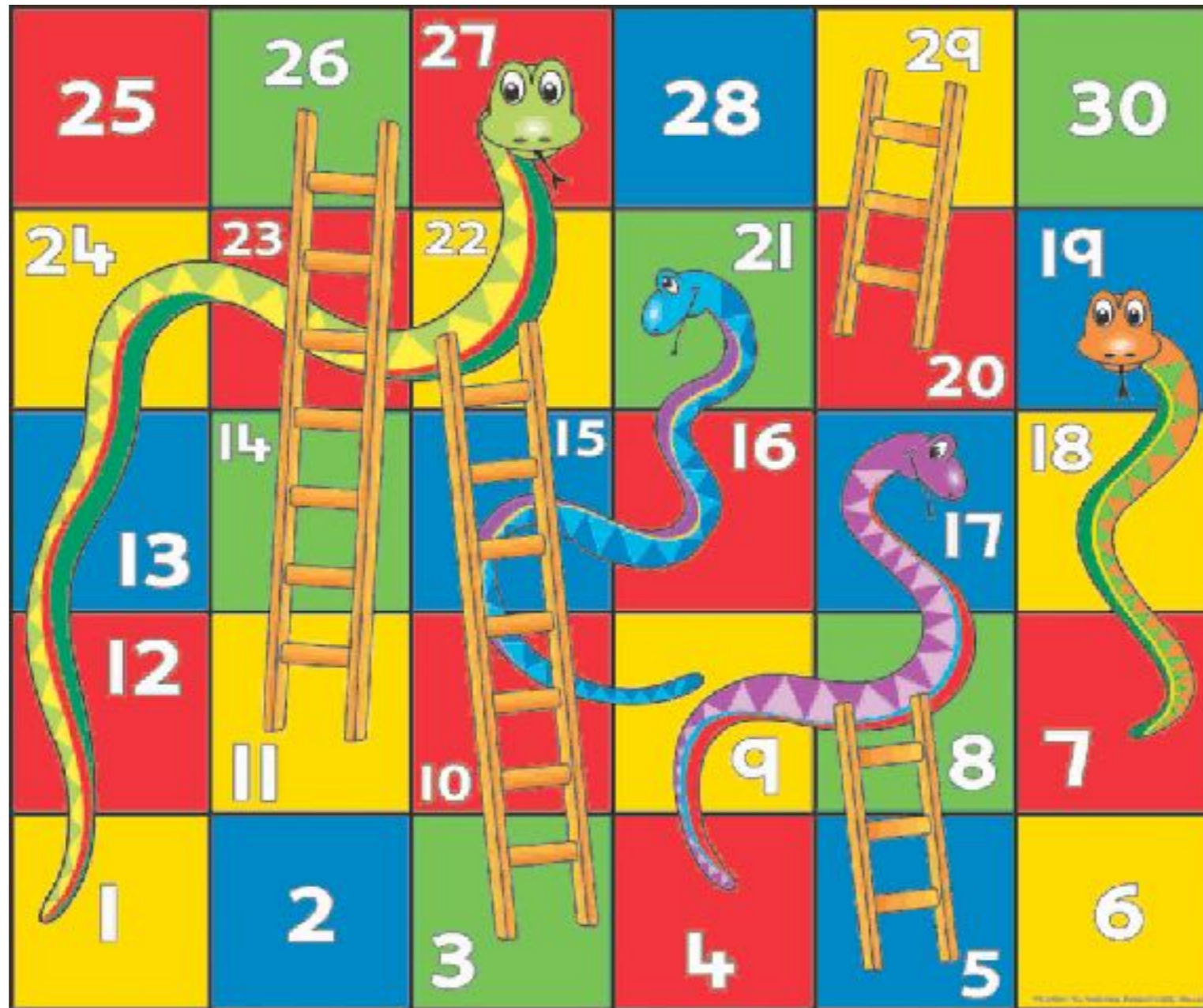


Machine Learning for Data Science (CS4786)

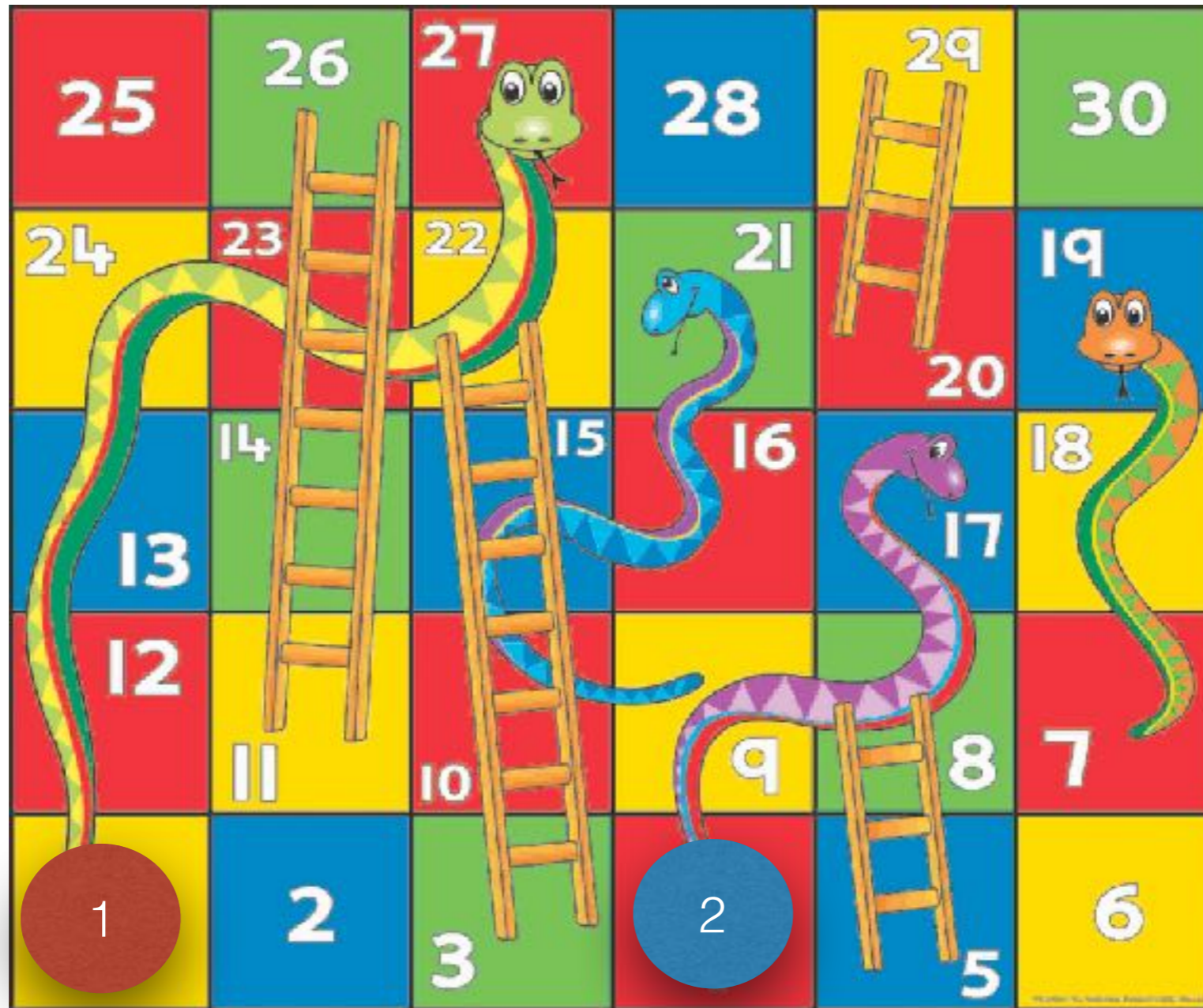
Lecture 22

HMM Via Particle Filtering
+
General Bayesian Networks

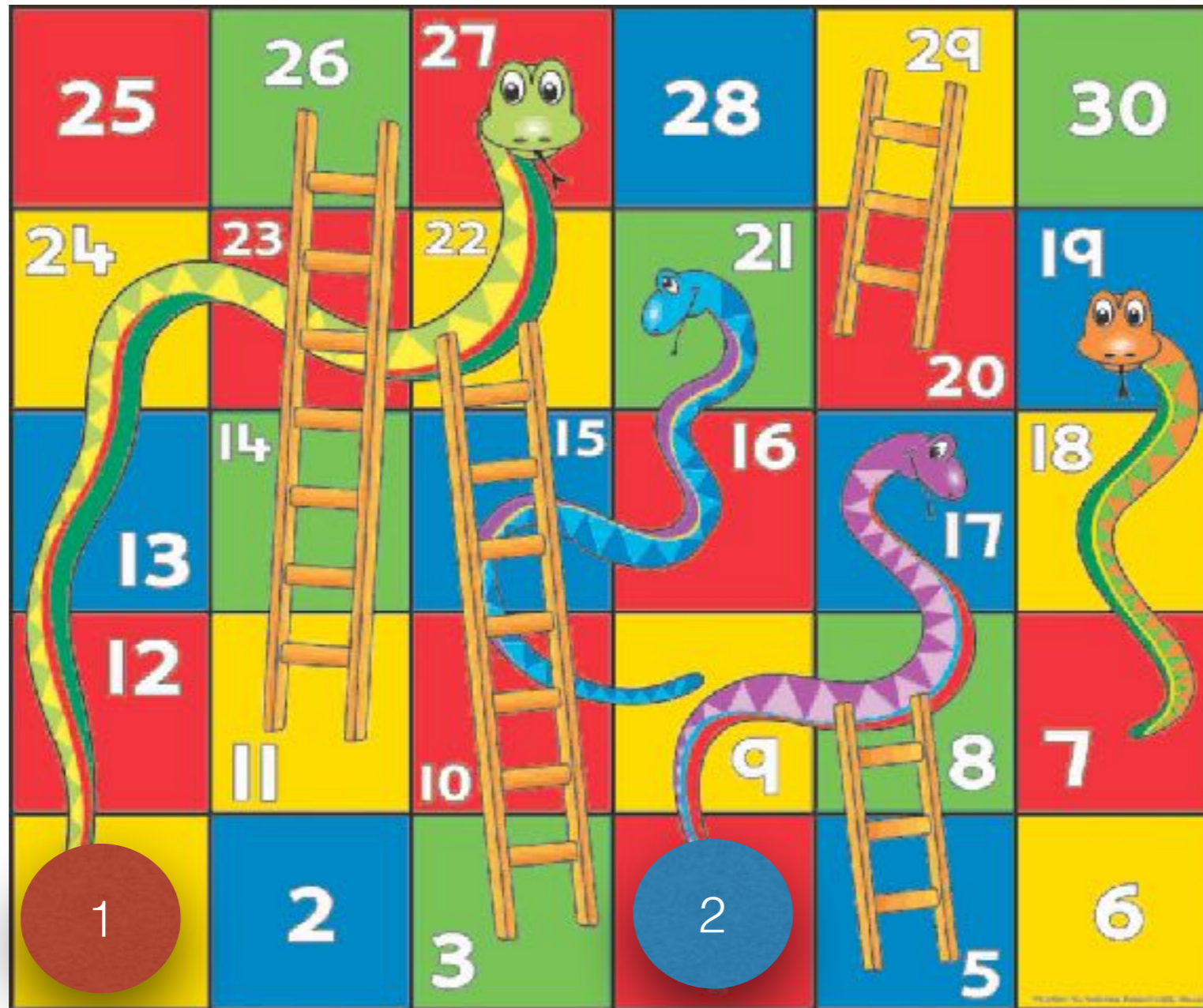
INFERENCE



INFERENCE

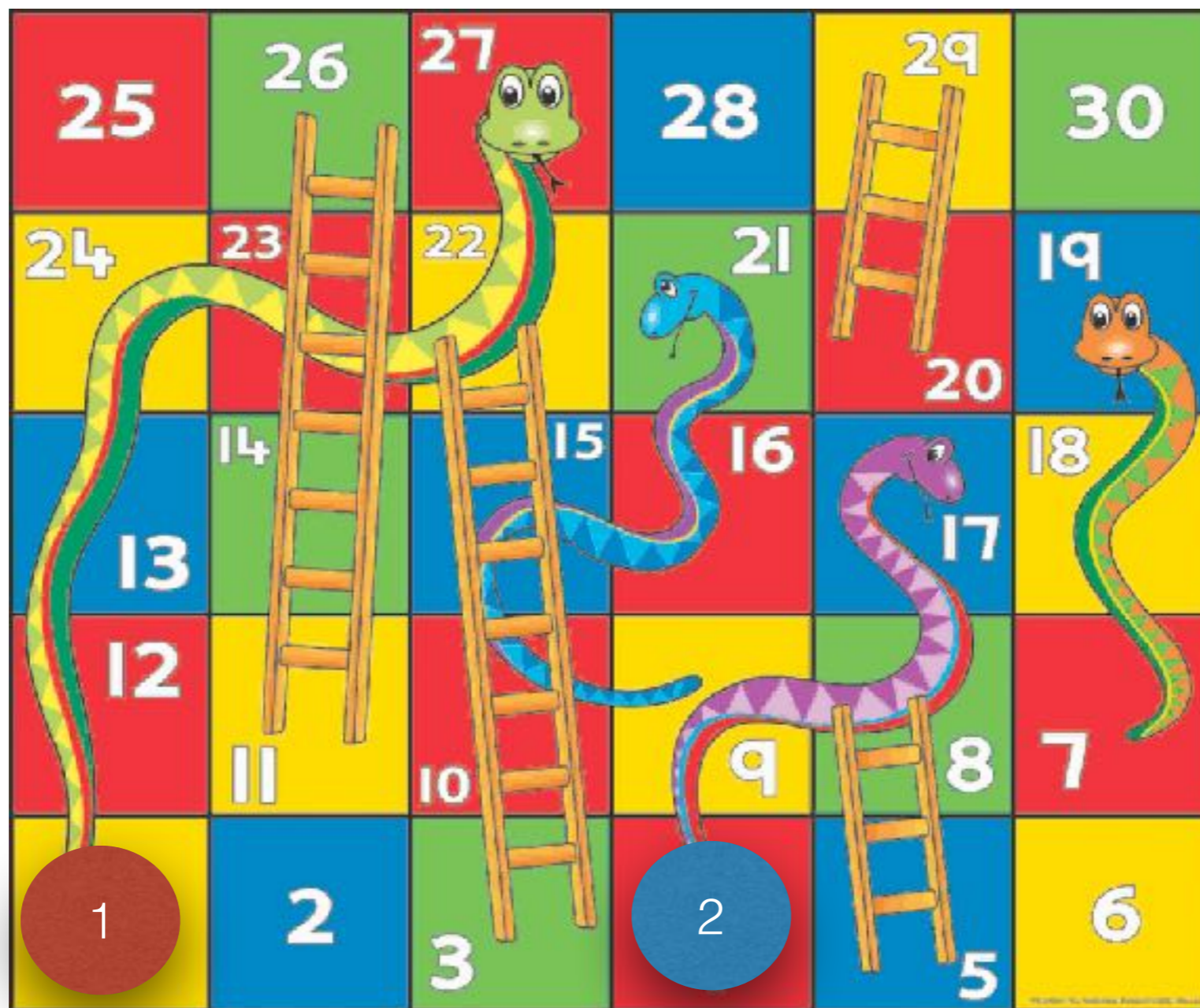


INFERENCE



Who is more likely to win the game?

INFERENCE



Who is more likely to win the game?

Compute sum of exact probabilities of all possible sequence of moves leading to Player 1's victory

INFERENCE

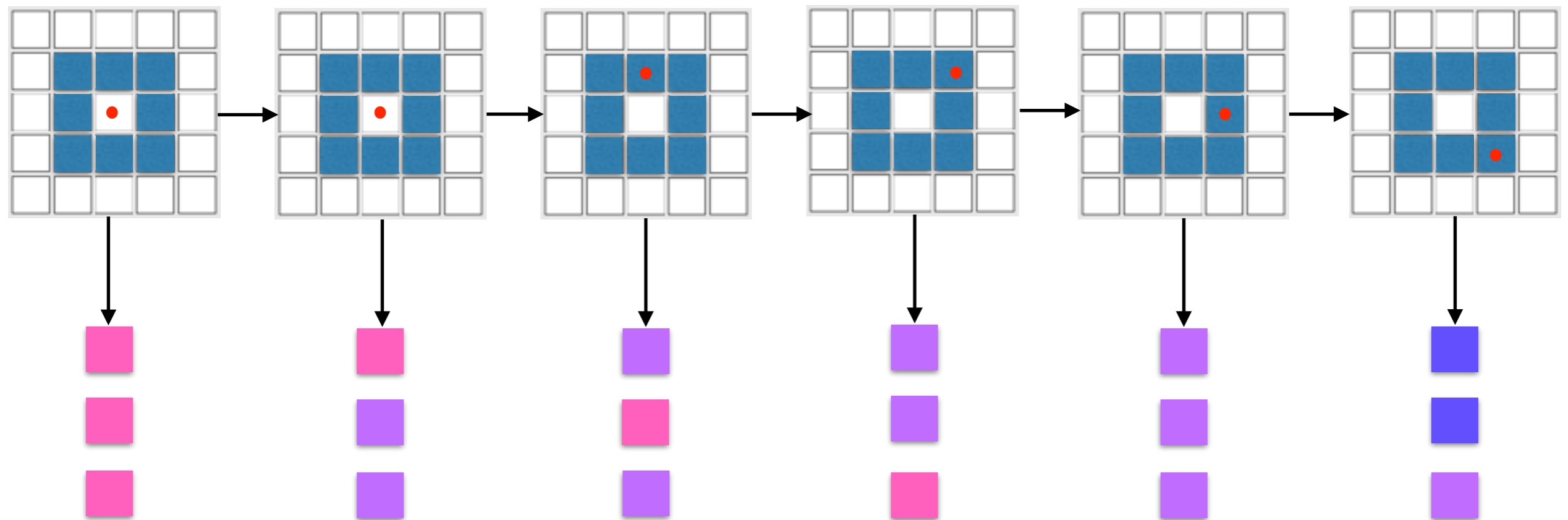
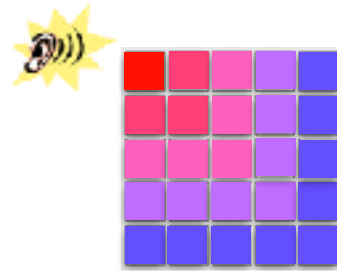


Who is more likely to win the game?

Throw dice and simulate multiple games, see who wins more often

HIDDEN MARKOV MODEL (HMM)

Eg: say observations were



Rejection sampling: Reject samples that don't match observations

We can do this sequentially!

IMPORTANCE SAMPLING

- We really want to draw from distribution P .
- But we can only draw from distribution Q easily
- Trick:
 - Draw $x_1, \dots, x_n \sim Q$
 - Re-weight each sample x_t by $P(X = x_t)/Q(X = x_t)$

IMPORTANCE SAMPLING

- We had the problem of too many rejections because probability of getting our sample to match exactly the observation is very low!
- How do we fix this?
- Fix observations and sample only states from the markov chain!
- Desired distribution P: $P(S_1, \dots, S_N | X_1 = x_1, \dots, X_N = x_N)$
- Sampling distribution Q: $P(S_1, \dots, S_N)$

IMPORTANCE SAMPLING

For a given sample s_1, \dots, s_N , importance weight given by:

$$\frac{P(S_1 = s_1, \dots, S_N = s_n | X_1 = x_1, \dots, X_N = x_N)}{P(S_1 = s_1, \dots, S_N = s_N)}$$

IMPORTANCE SAMPLING

For a given sample s_1, \dots, s_N , importance weight given by:

$$\begin{aligned} & \frac{P(S_1 = s_1, \dots, S_N = s_n | X_1 = x_1, \dots, X_N = x_N)}{P(S_1 = s_1, \dots, S_N = s_N)} \\ &= \frac{P(X_1 = x_1, \dots, X_N = x_N | S_1 = s_1, \dots, S_N = s_n)}{P(X_1 = x_1, \dots, X_N = x_N)} \end{aligned}$$

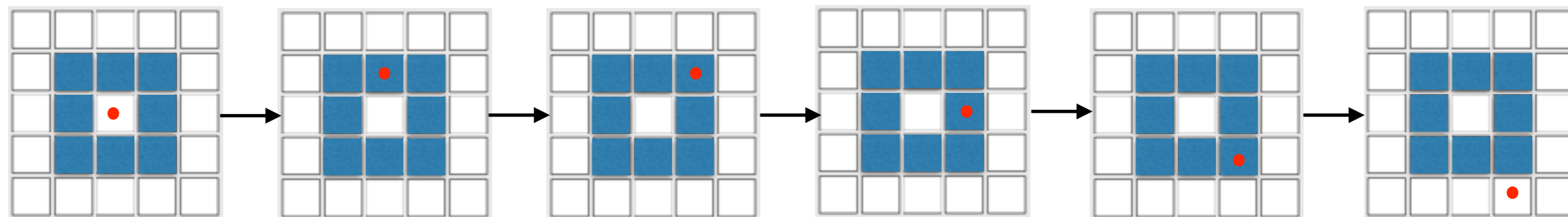
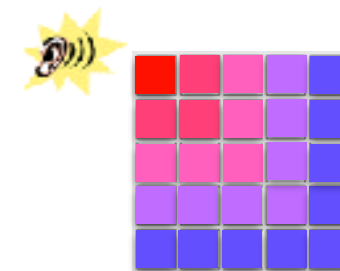
IMPORTANCE SAMPLING

For a given sample s_1, \dots, s_N , importance weight given by:

$$\begin{aligned} & \frac{P(S_1 = s_1, \dots, S_N = s_n | X_1 = x_1, \dots, X_N = x_N)}{P(S_1 = s_1, \dots, S_N = s_N)} \\ &= \frac{P(X_1 = x_1, \dots, X_N = x_N | S_1 = s_1, \dots, S_N = s_n)}{P(X_1 = x_1, \dots, X_N = x_N)} \\ &= \frac{\prod_{t=1}^N P(X_t = x_t | S_t = s_t)}{P(X_1 = x_1, \dots, X_N = x_N)} \propto \prod_{t=1}^N P(X_t = x_t | S_t = s_t) \end{aligned}$$

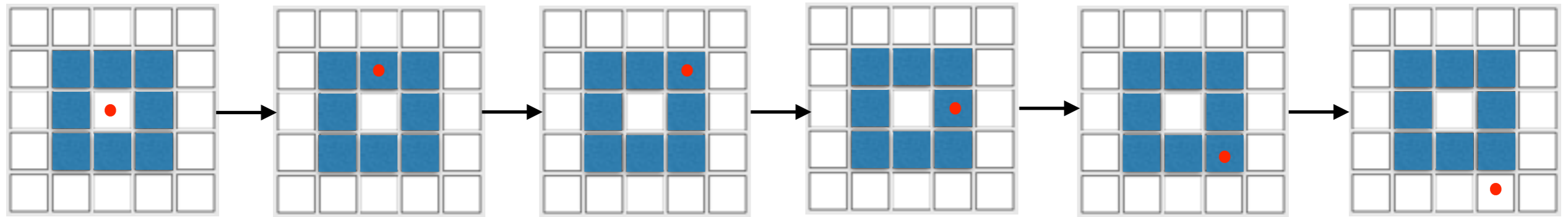
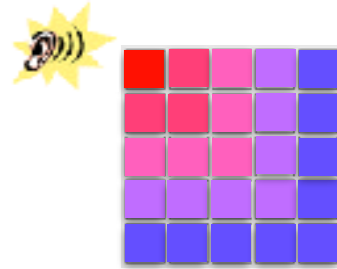
HIDDEN MARKOV MODEL (HMM)

Eg: say observations were



HIDDEN MARKOV MODEL (HMM)

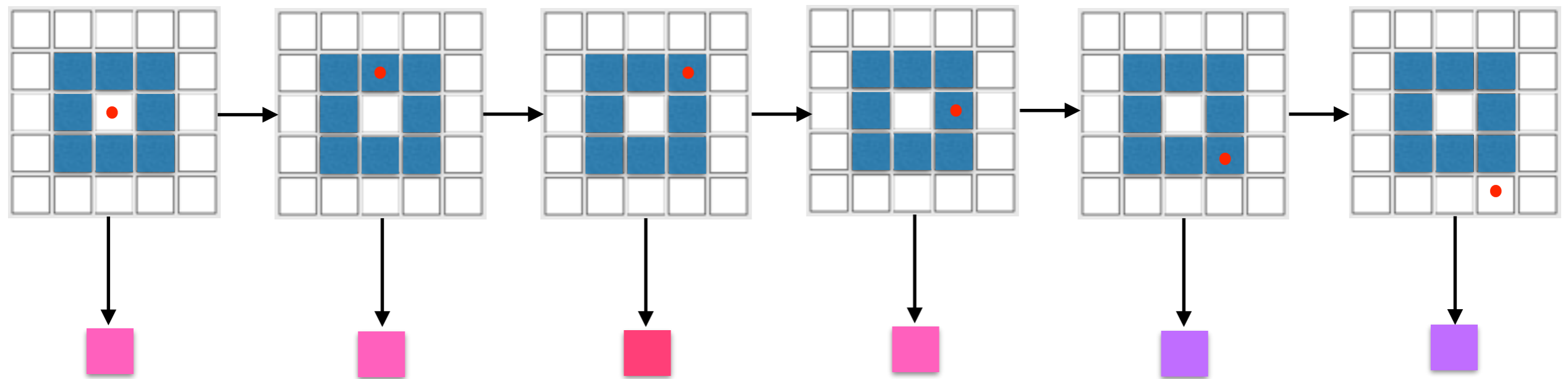
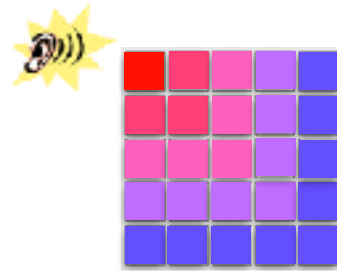
Eg: say observations were



Importance weighting: weight samples

HIDDEN MARKOV MODEL (HMM)

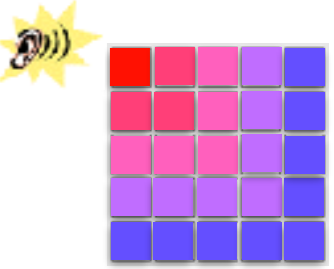
Eg: say observations were



$$P(\text{pink} | S_1=13) \times P(\text{pink} | S_2=8) \times P(\text{red} | S_3=9) \times P(\text{pink} | S_5=24) \times P(\text{purple} | S_6=19) \times P(\text{purple} | S_7=14)$$

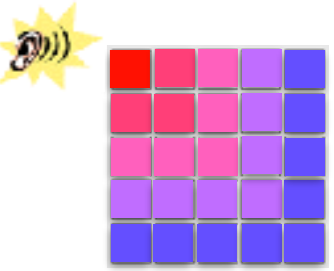
Importance weighting: weight samples

HMM PARTICLE FILTER

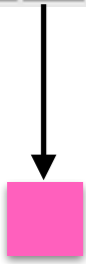
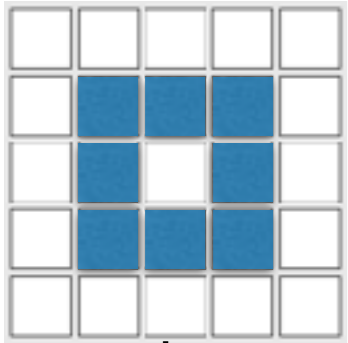


- Use multiple samples and track each ones weights.

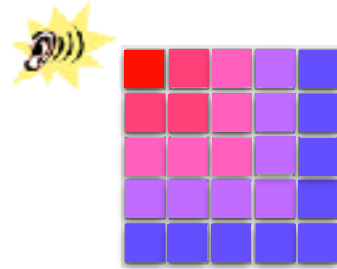
HMM PARTICLE FILTER



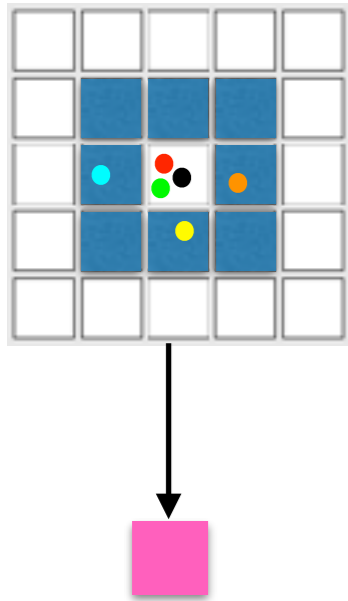
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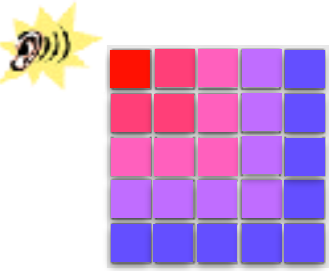
HMM PARTICLE FILTER



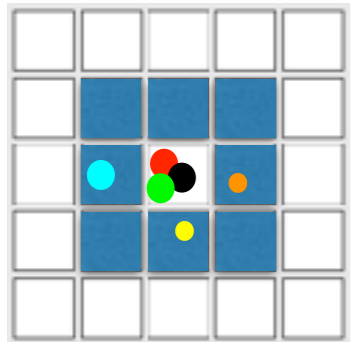
- Use multiple samples and track each ones weights.



HMM PARTICLE FILTER

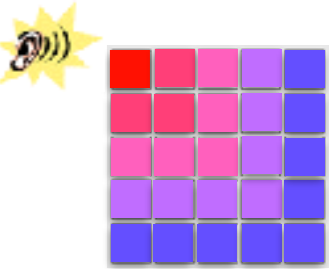


- Use multiple samples and track each ones weights.

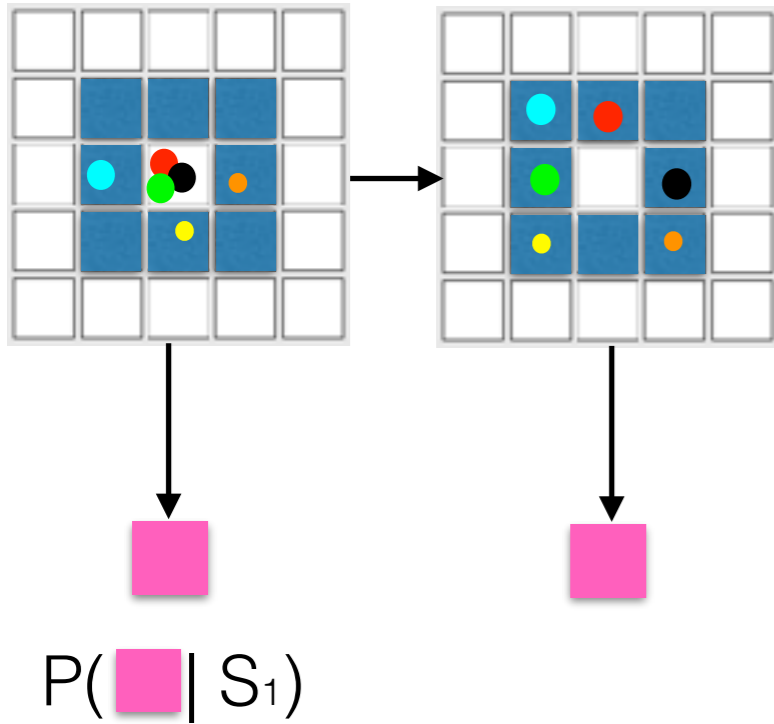


$$P(\text{pink} \mid S_1)$$

HMM PARTICLE FILTER



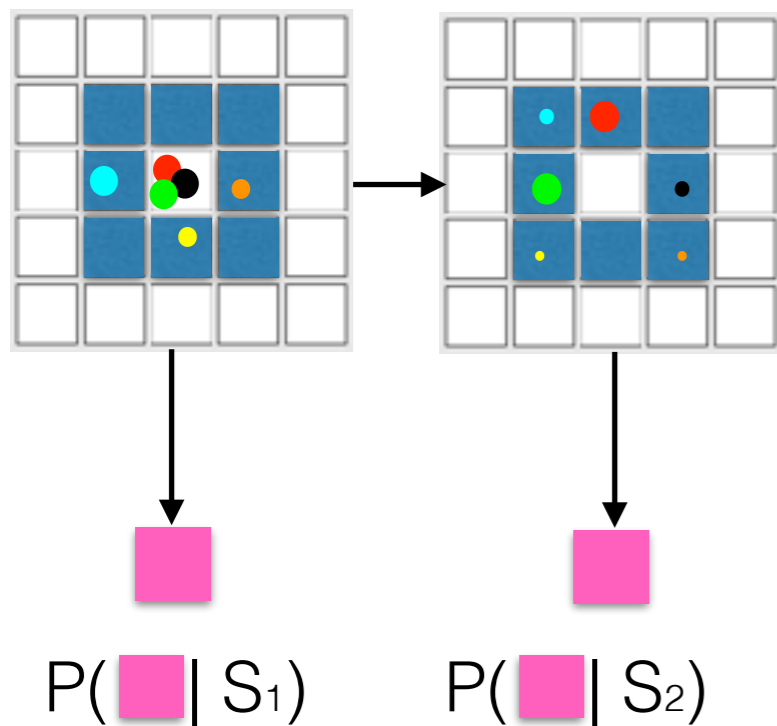
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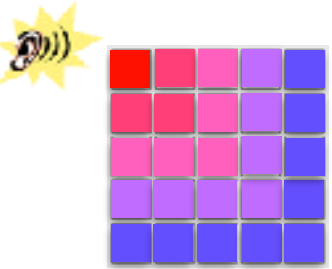
HMM PARTICLE FILTER



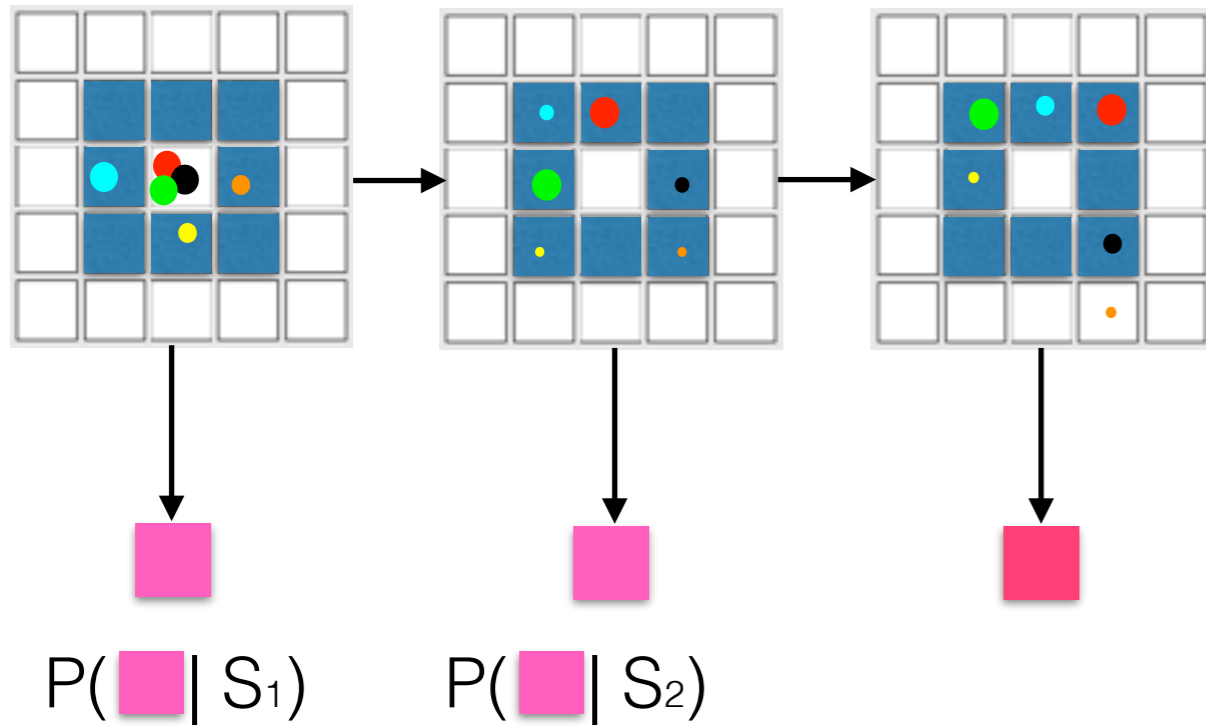
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HMM PARTICLE FILTER



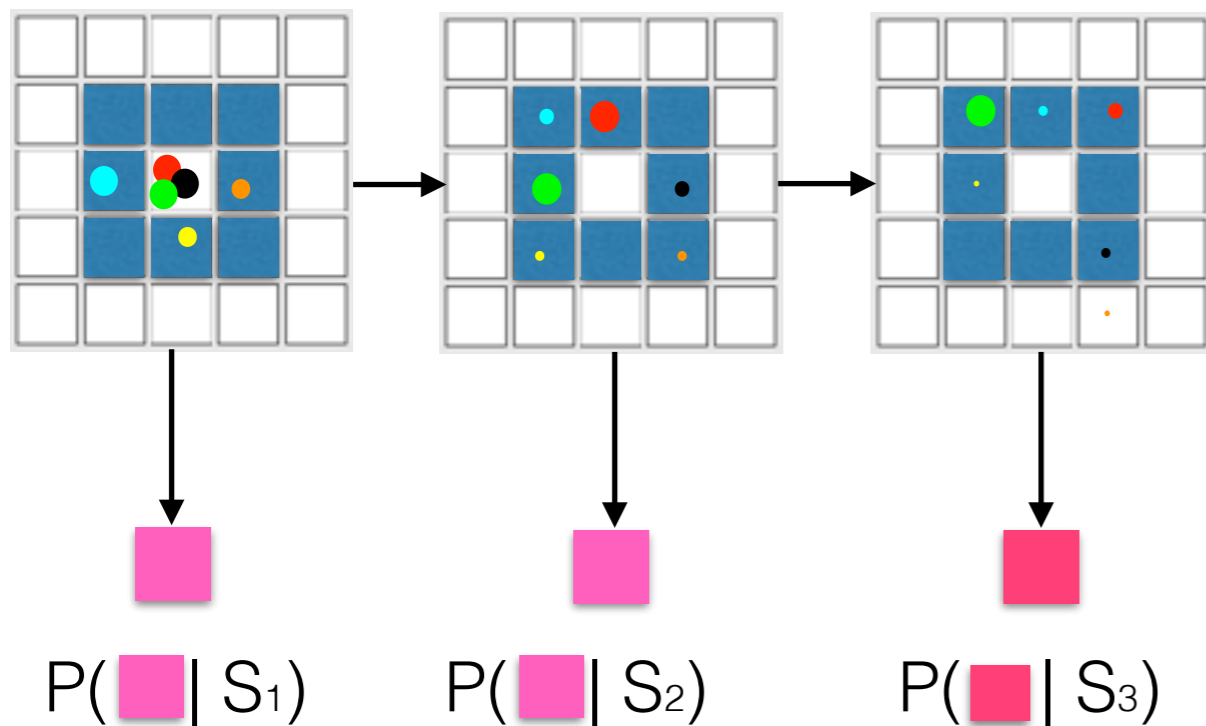
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HMM PARTICLE FILTER

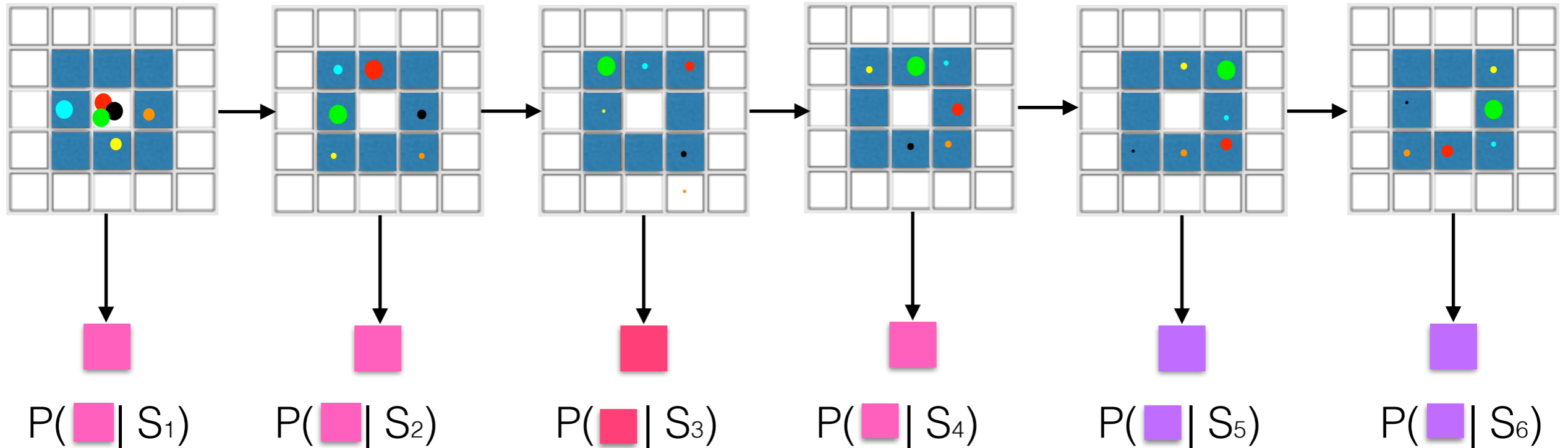
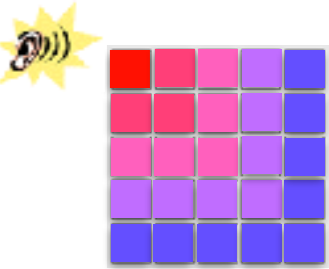


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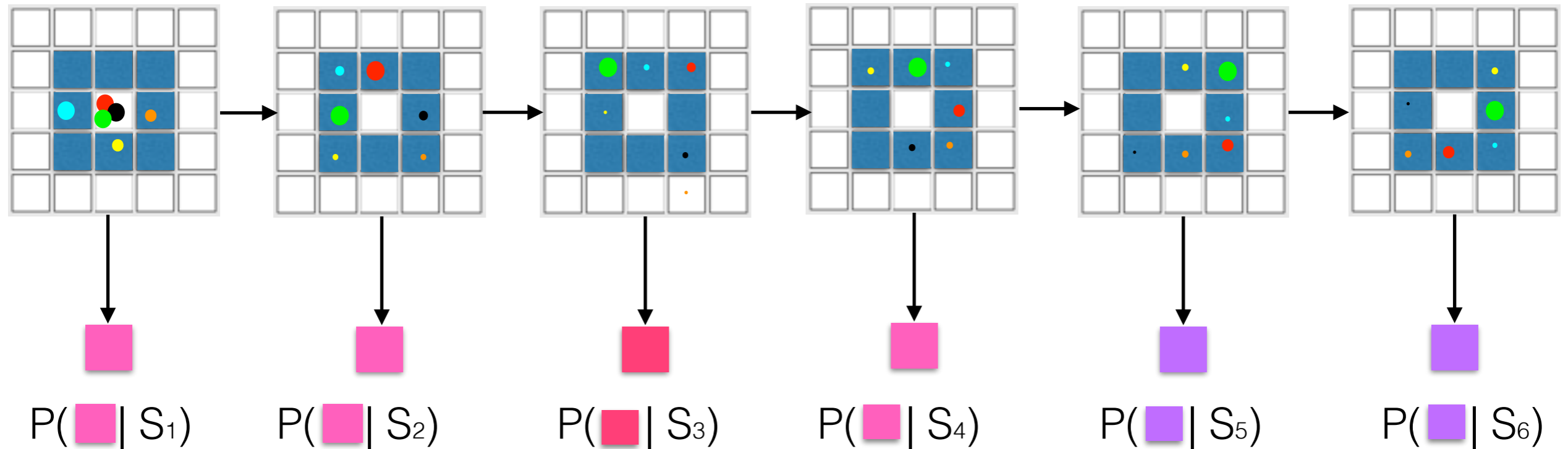
HMM PARTICLE FILTER

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HMM PARTICLE FILTER

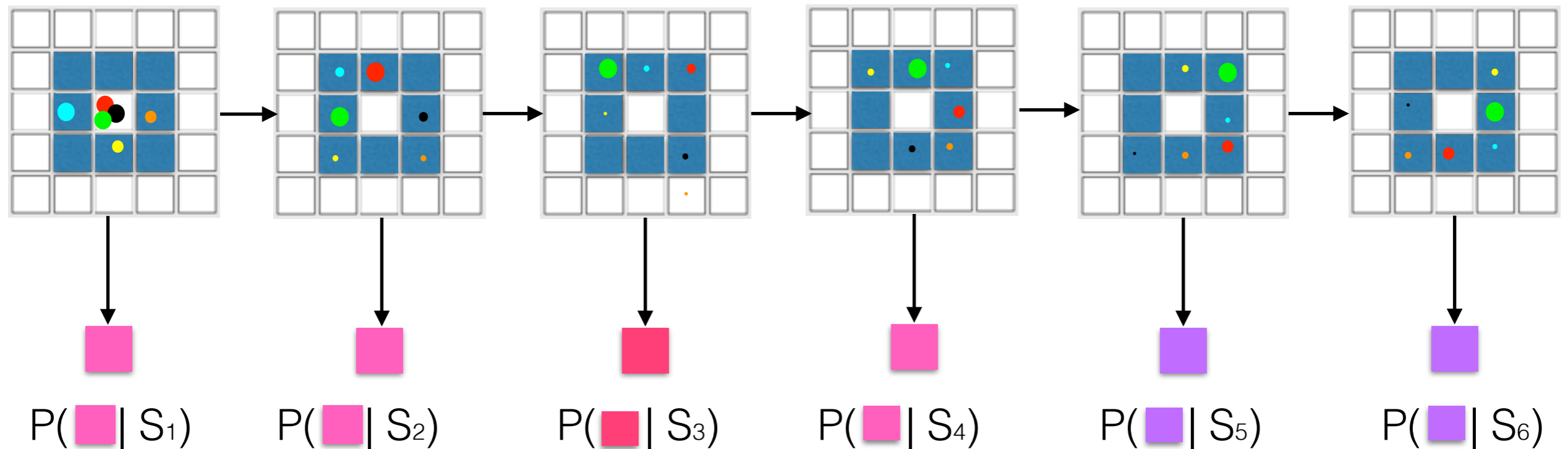
- Use multiple samples and track each ones weights.



- This is same as 6 separate samples

HMM PARTICLE FILTER

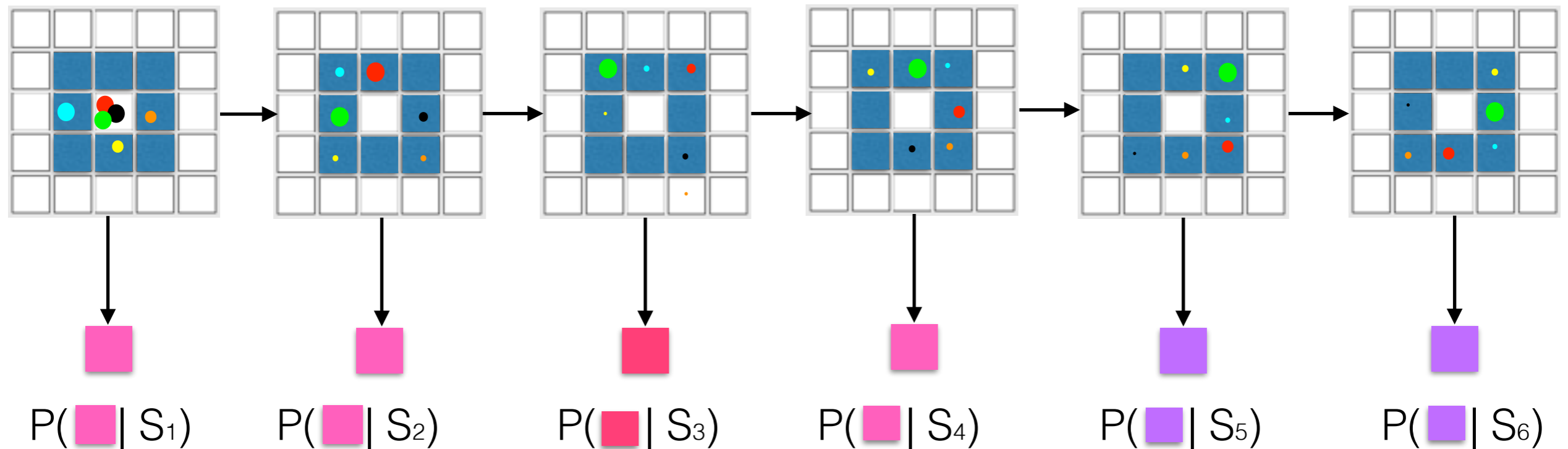
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- This is same as 6 separate samples
- Instead of tracking each sample's weight, resample according to weights

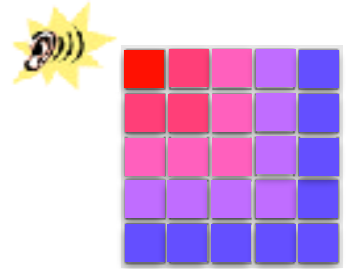
HMM PARTICLE FILTER

- Use multiple samples and track each ones weights.



- This is same as 6 separate samples
- Instead of tracking each sample's weight, resample according to weights
- Problem: Too many samples have negligible weight!

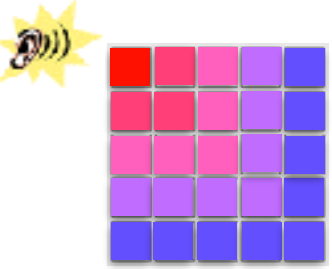
HMM PARTICLE FILTER



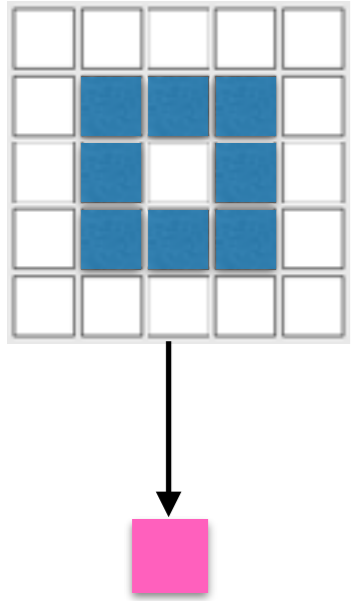
Instead of tracking each one, resample!



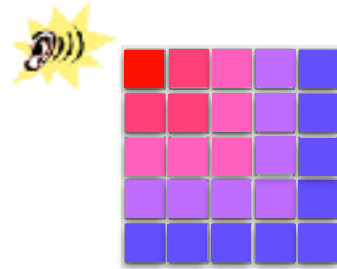
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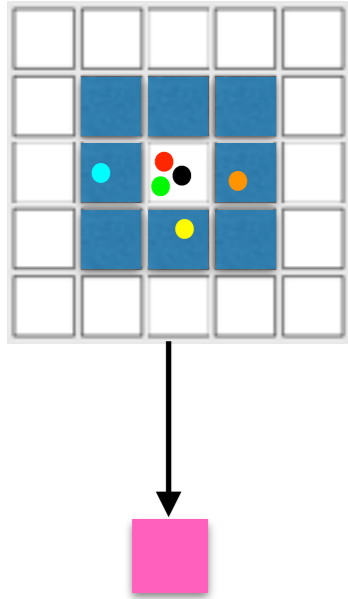
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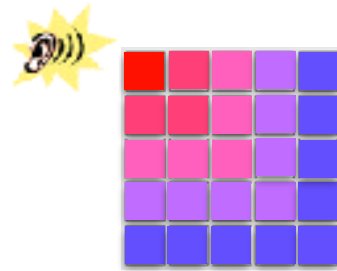
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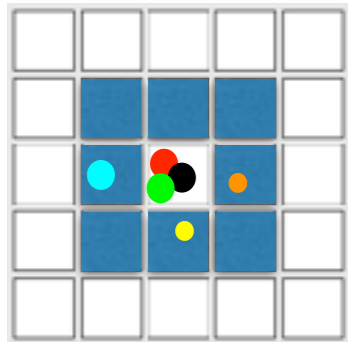
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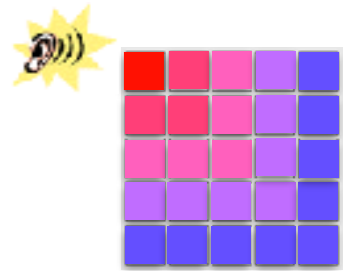
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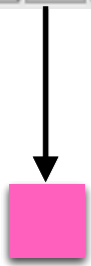
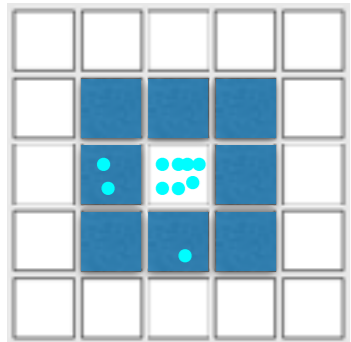
$$P(\text{pink} \mid S_1)$$



HMM PARTICLE FILTER

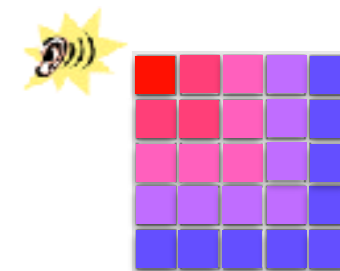


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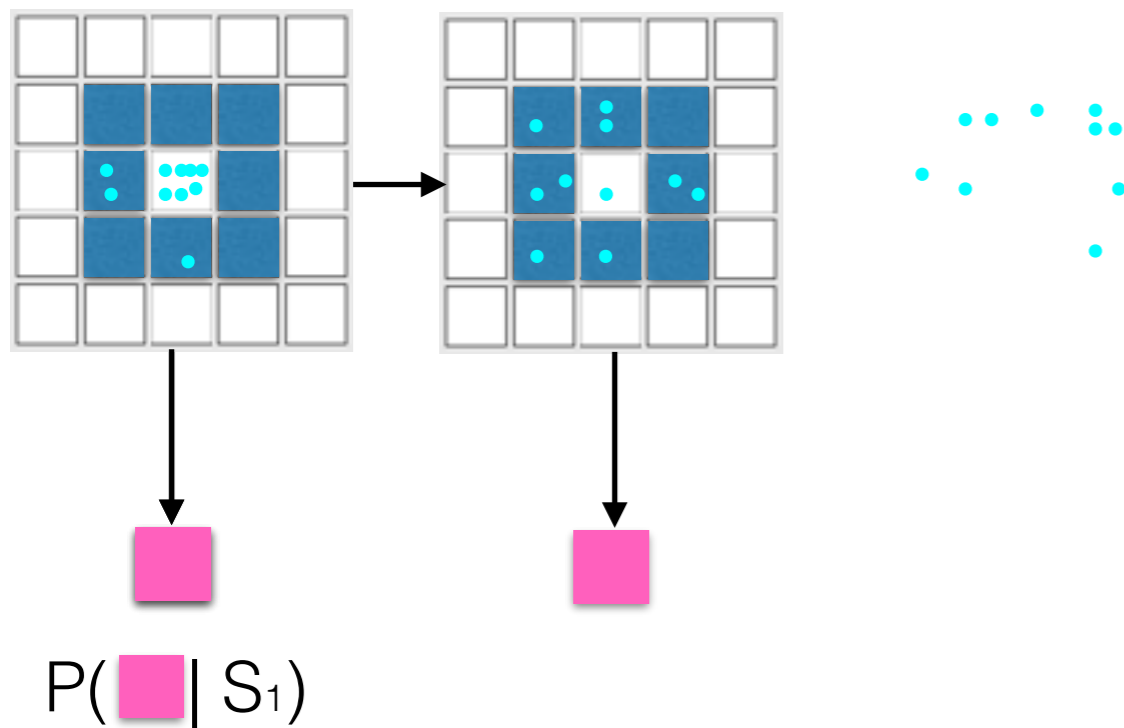


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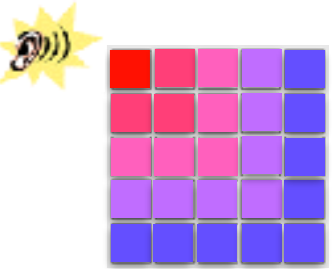
HMM PARTICLE FILTER



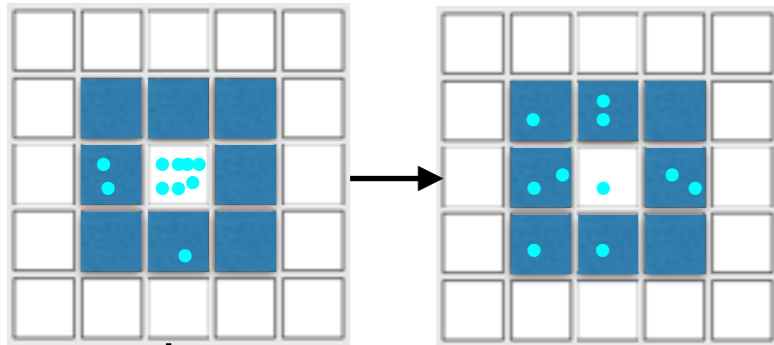
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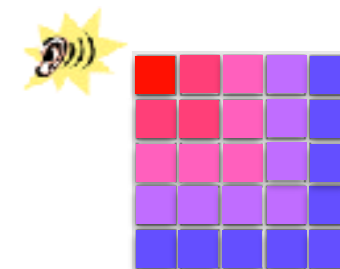


$$P(\text{pink} | S_1)$$

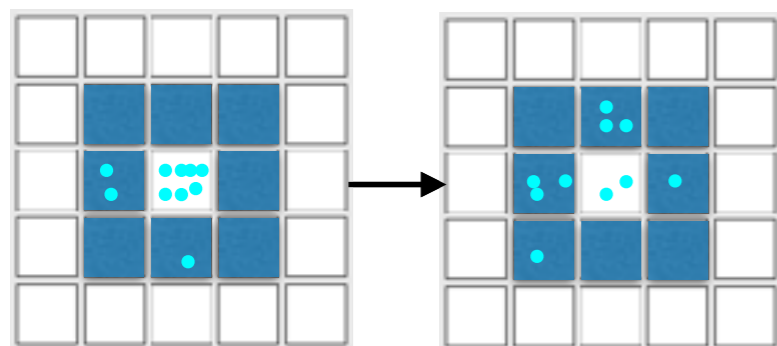


$$P(\text{pink} | S_2)$$

HMM PARTICLE FILTER



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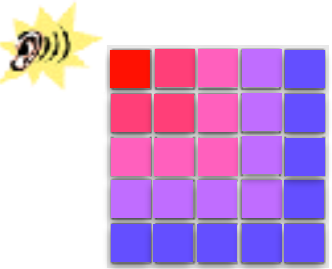


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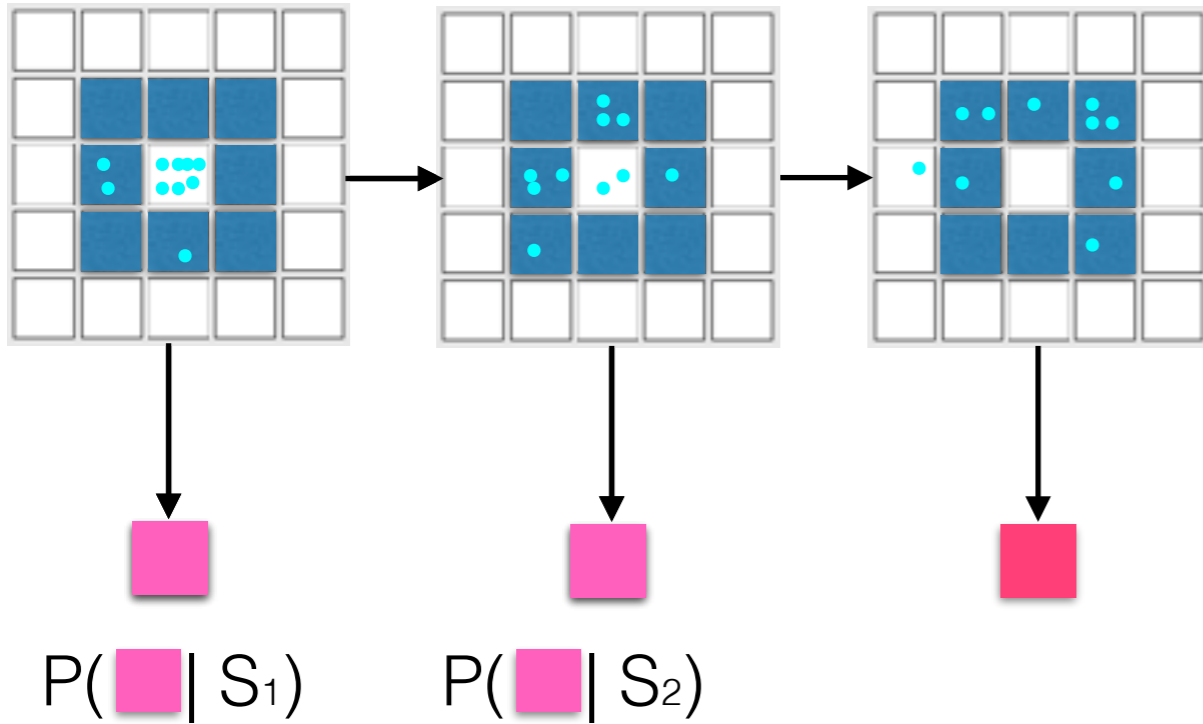


$$P(\text{pink} | S_2)$$

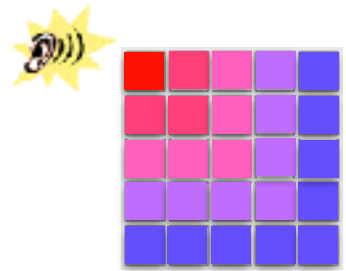
HMM PARTICLE FILTER



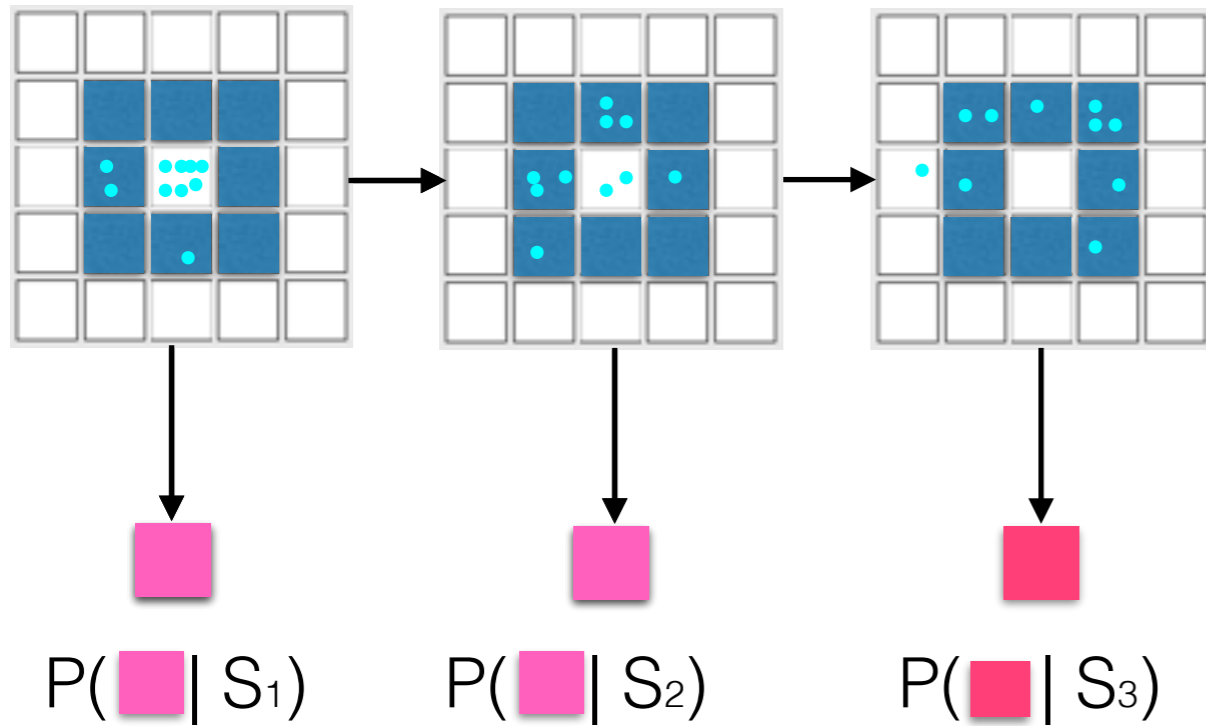
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HMM PARTICLE FILTER



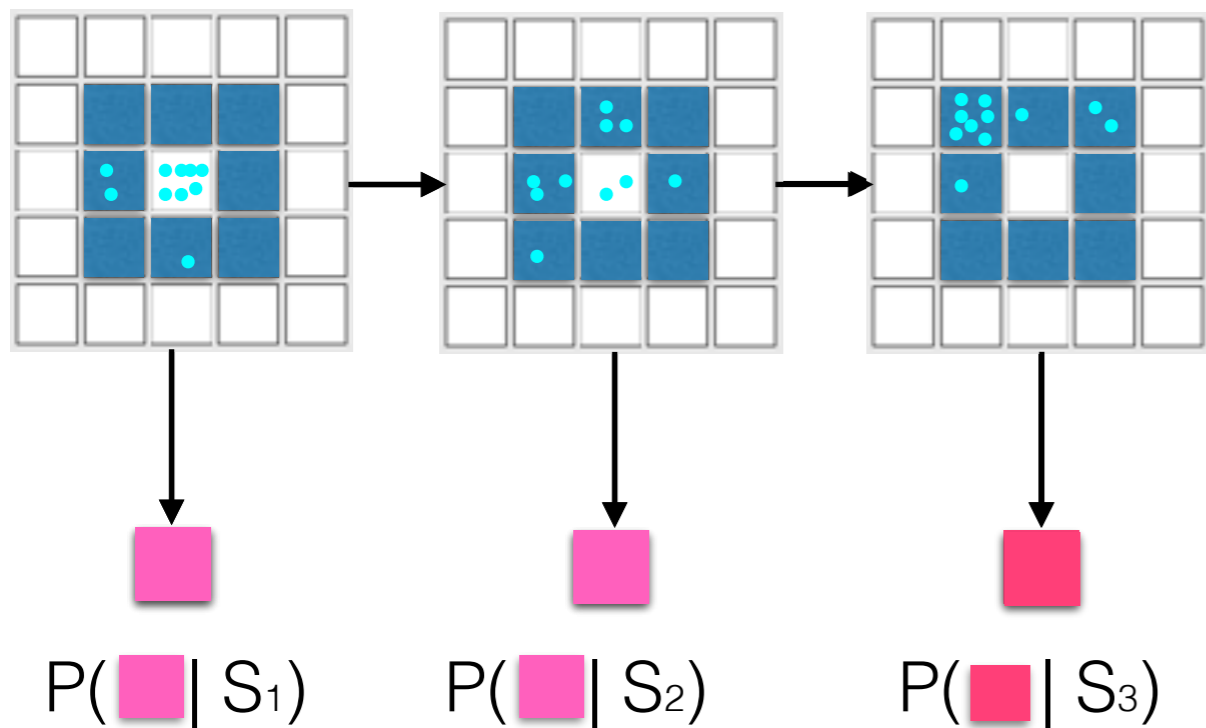
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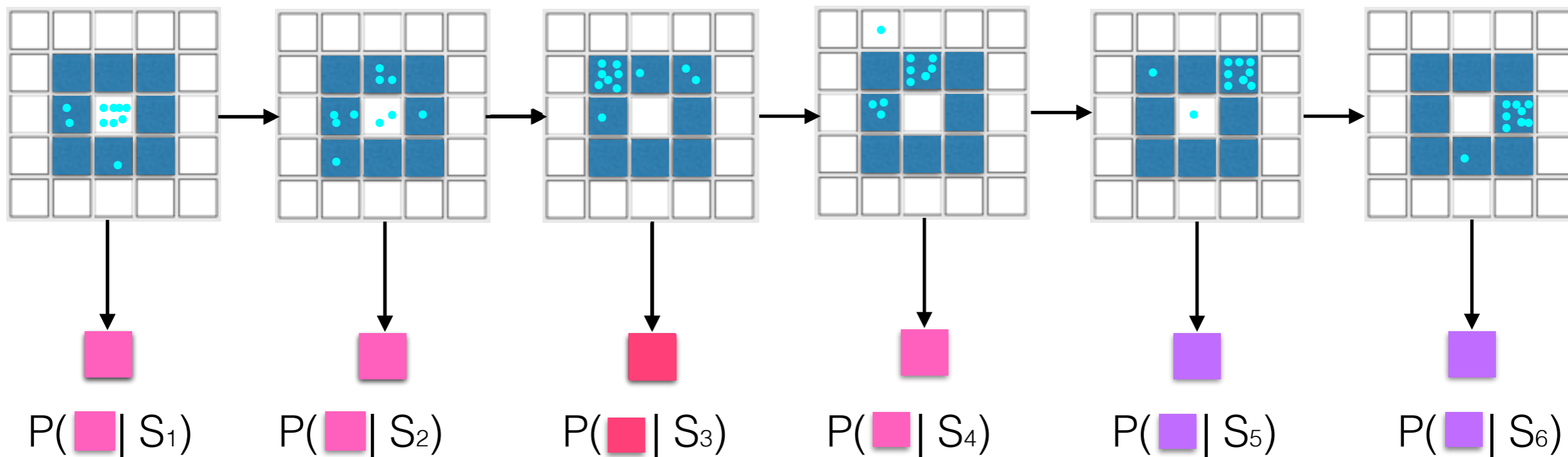


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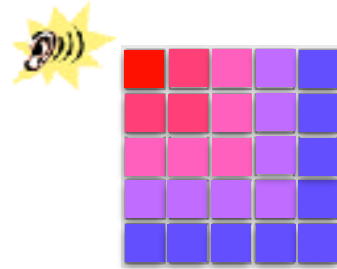


HMM PARTICLE FILTER

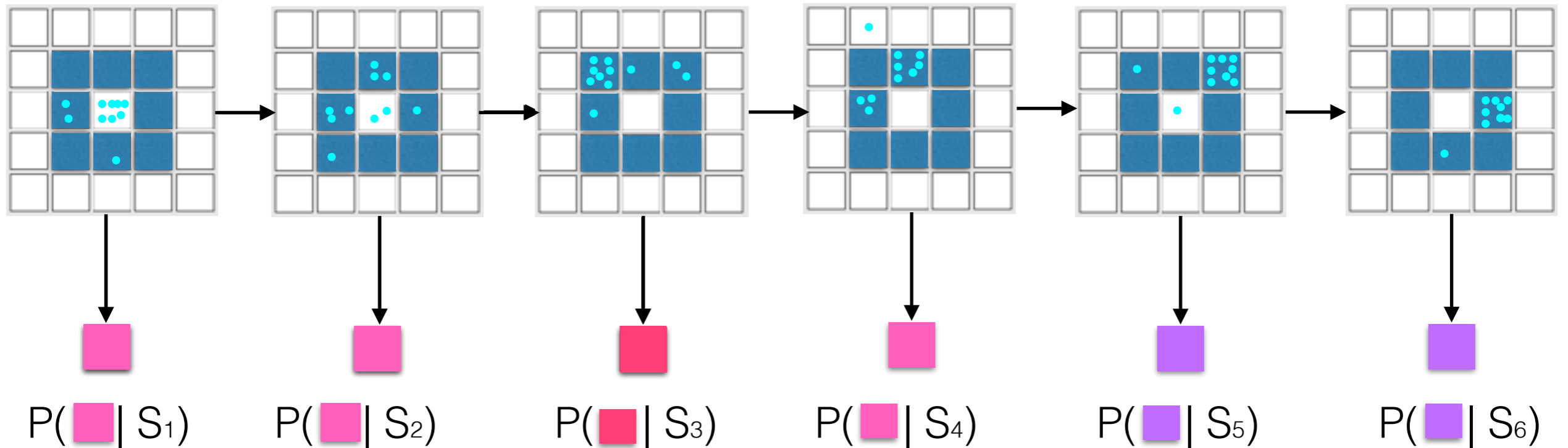
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HMM PARTICLE FILTER

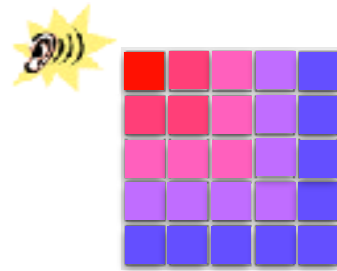


Instead of tracking each one, resample!

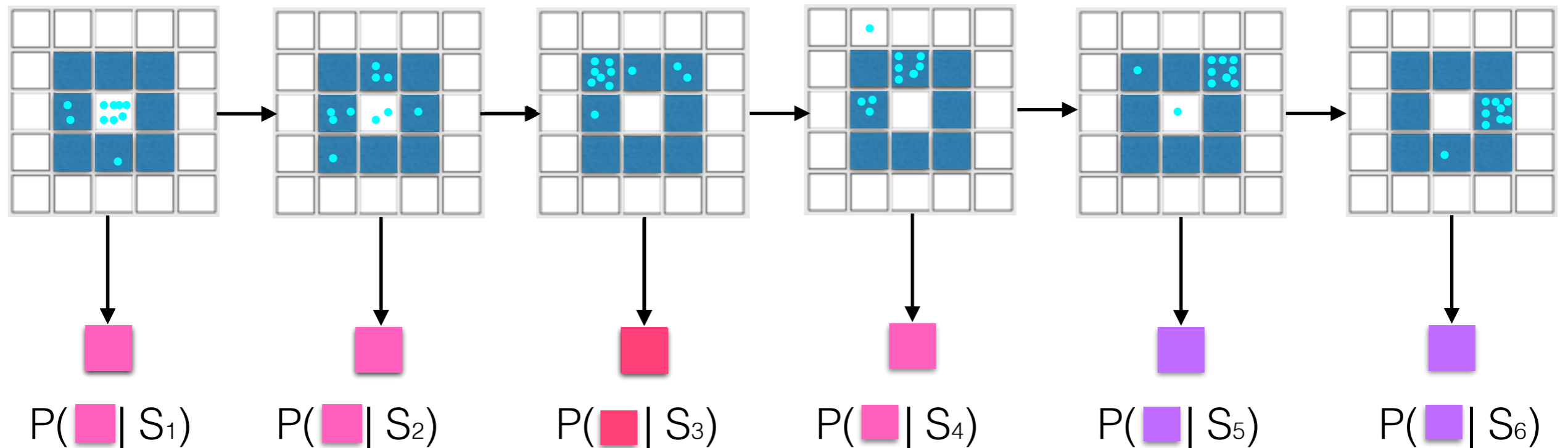


- On every round, transfer particles from previous states according to transition probability

HMM PARTICLE FILTER

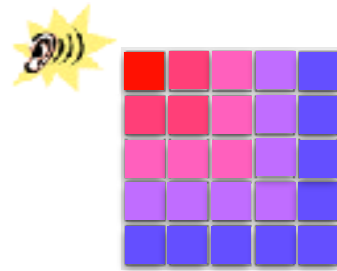


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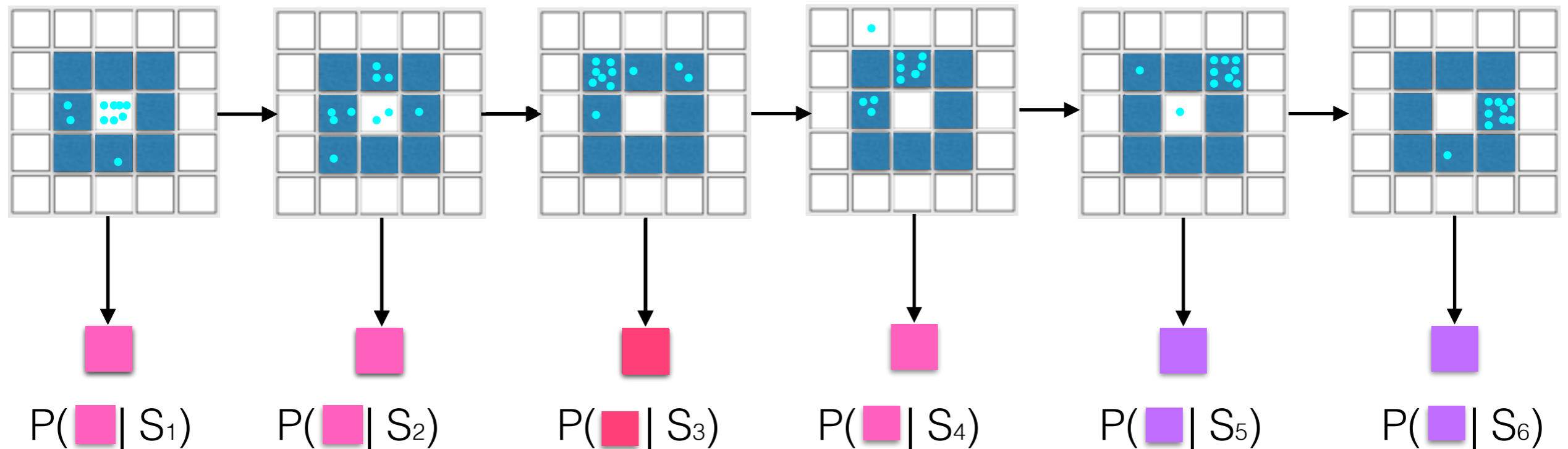


- On every round, transfer particles from previous states according to transition probability
- Resample particles according to $P(\text{observation}|\text{state})$

HMM PARTICLE FILTER



Instead of tracking each one, resample!



- On every round, transfer particles from previous states according to transition probability
- Resample particles according to $P(\text{observation}|\text{state})$
- Use new particles to proceed

Particle Filtering

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Particle Filtering

- Without resampling, we carry many particles with very small probabilities
 - too many samples needed for a good estimate
- By resampling, we got rid of samples with very small probabilities
 - Hence fewer samples suffice

HMM PARTICLE FILTER

- Inference time only depends on number of samples
- Of course more the samples the better accuracy
- Often we don't need too many samples. Why ?

Gibbs Sampling

Gibbs Sampling

- Repeat n times for, n samples,

Gibbs Sampling

- Repeat n times for, n samples,
 - Start with arbitrary value for variables

Gibbs Sampling

- Repeat n times for, n samples,
 - Start with arbitrary value for variables
 - Replace each variable by new sample from $P(\text{Variable} | \text{all other variables})$

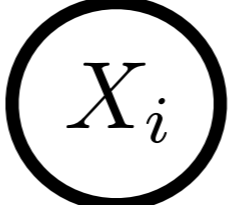
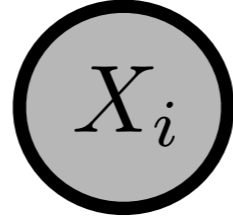
Gibbs Sampling

- Repeat n times for, n samples,
 - Start with arbitrary value for variables
 - Replace each variable by new sample from $P(\text{Variable} | \text{all other variables})$
 - Go over all variables multiple times

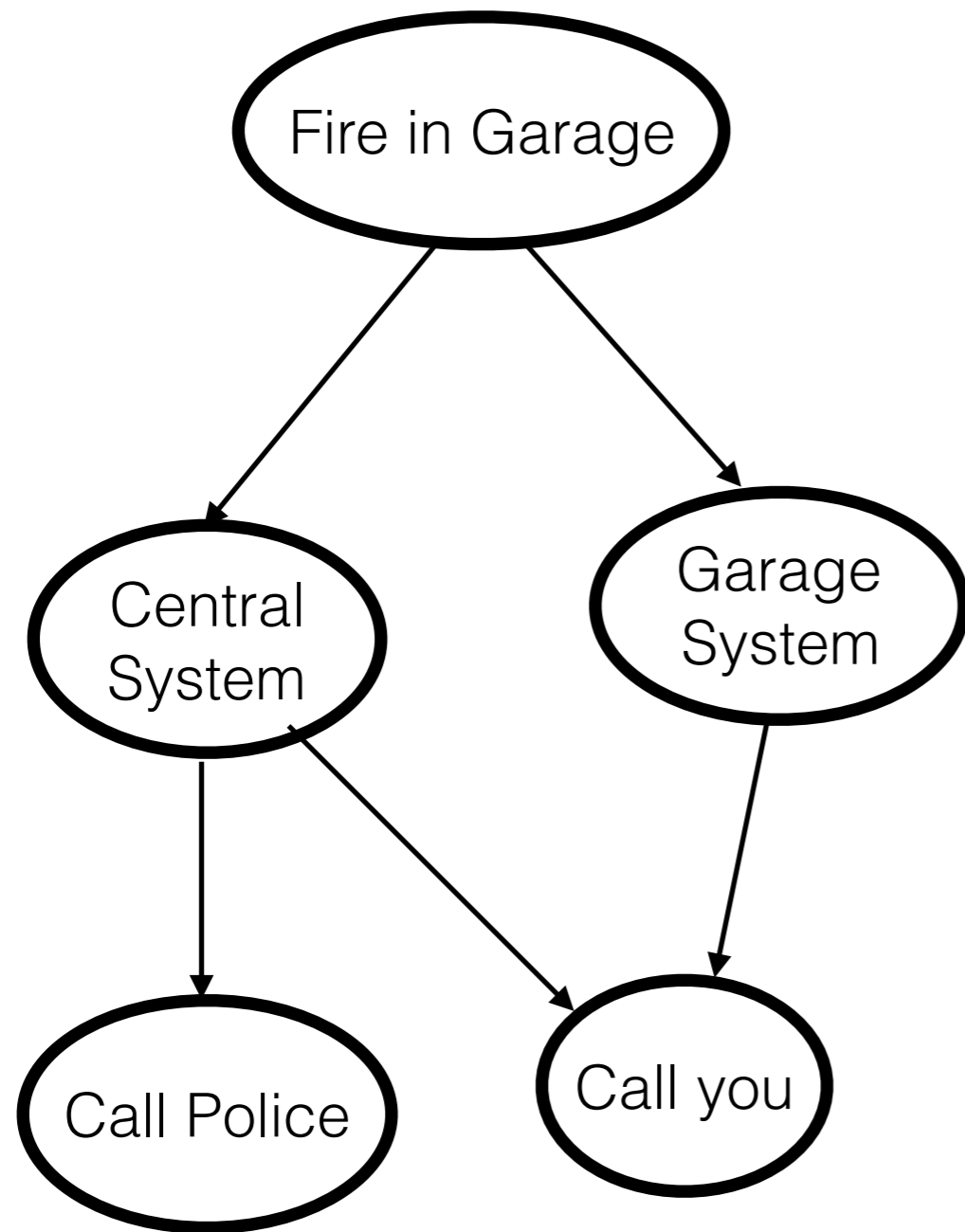
Gibbs Sampling

- Repeat n times for, n samples,
 - Start with arbitrary value for variables
 - Replace each variable by new sample from $P(\text{Variable} | \text{all other variables})$
 - Go over all variables multiple times
 - Return final sample of the N variables

GRAPHICAL MODELS

- Variables X_i s written as  if X_i is observed
- Variables X_i s written as  if X_i is latent
- Parameters are often left out (its understood and not explicitly written out). If present they don't have bounding objects
- A directed edge \longrightarrow is drawn connecting every parent to its child (from parent to child)

GRAPHICAL MODELS



BAYESIAN NETWORKS

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- Directed acyclic graph $G = (V, E)$ (**graph with no directed cycle**)
 - Edges going from parent nodes to child nodes
 - Direction indicates parent “generates” child

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Joint probability factorizes as:

$$P(X_1, \dots, X_N) = \prod_{i=1}^N P(X_i | \text{Parents}(X_i))$$

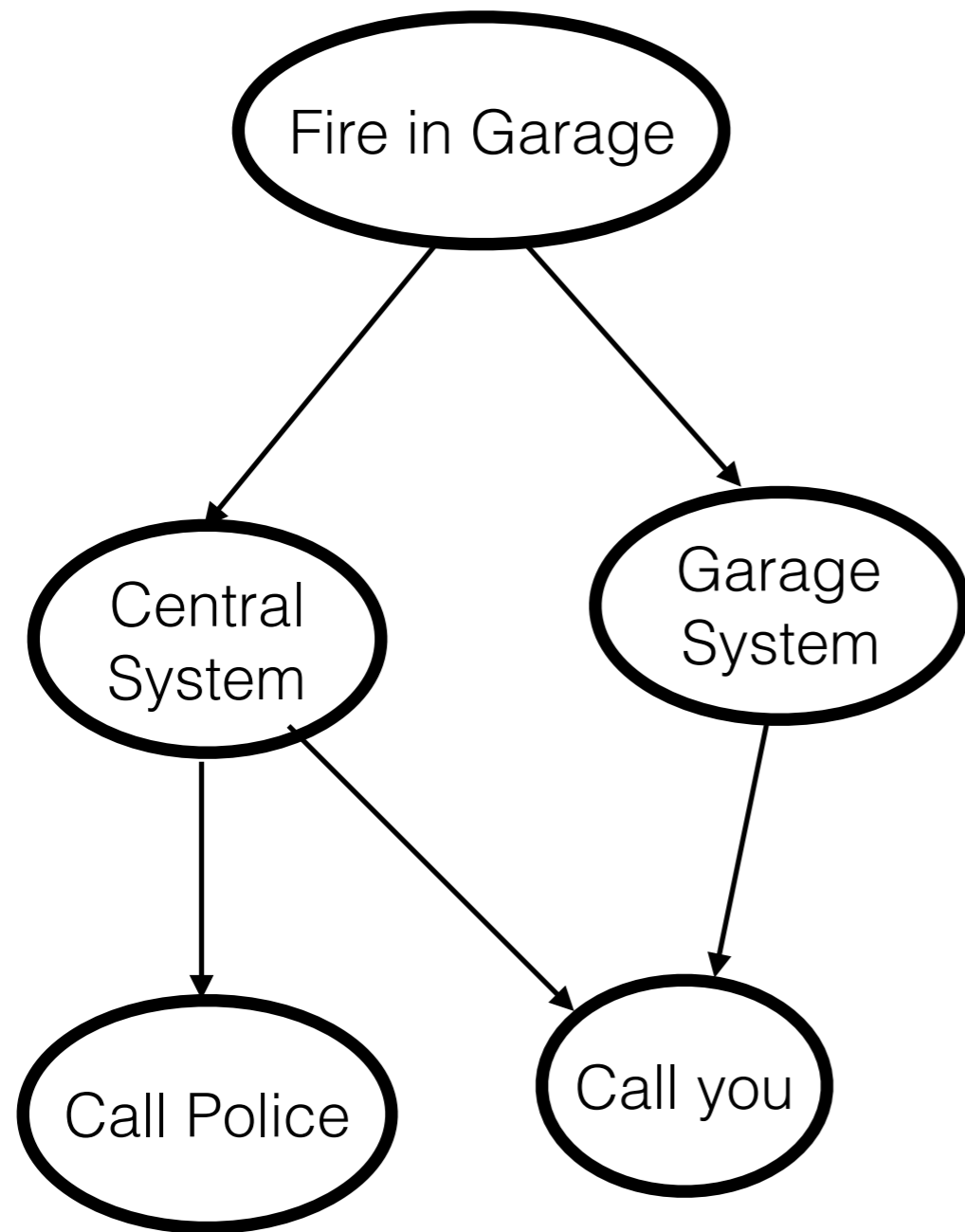
BAYESIAN NETWORKS

- Directed acyclic graph (DAG): $G = (V, E)$
- Joint distribution P_θ over X_1, \dots, X_n that factorizes over G :

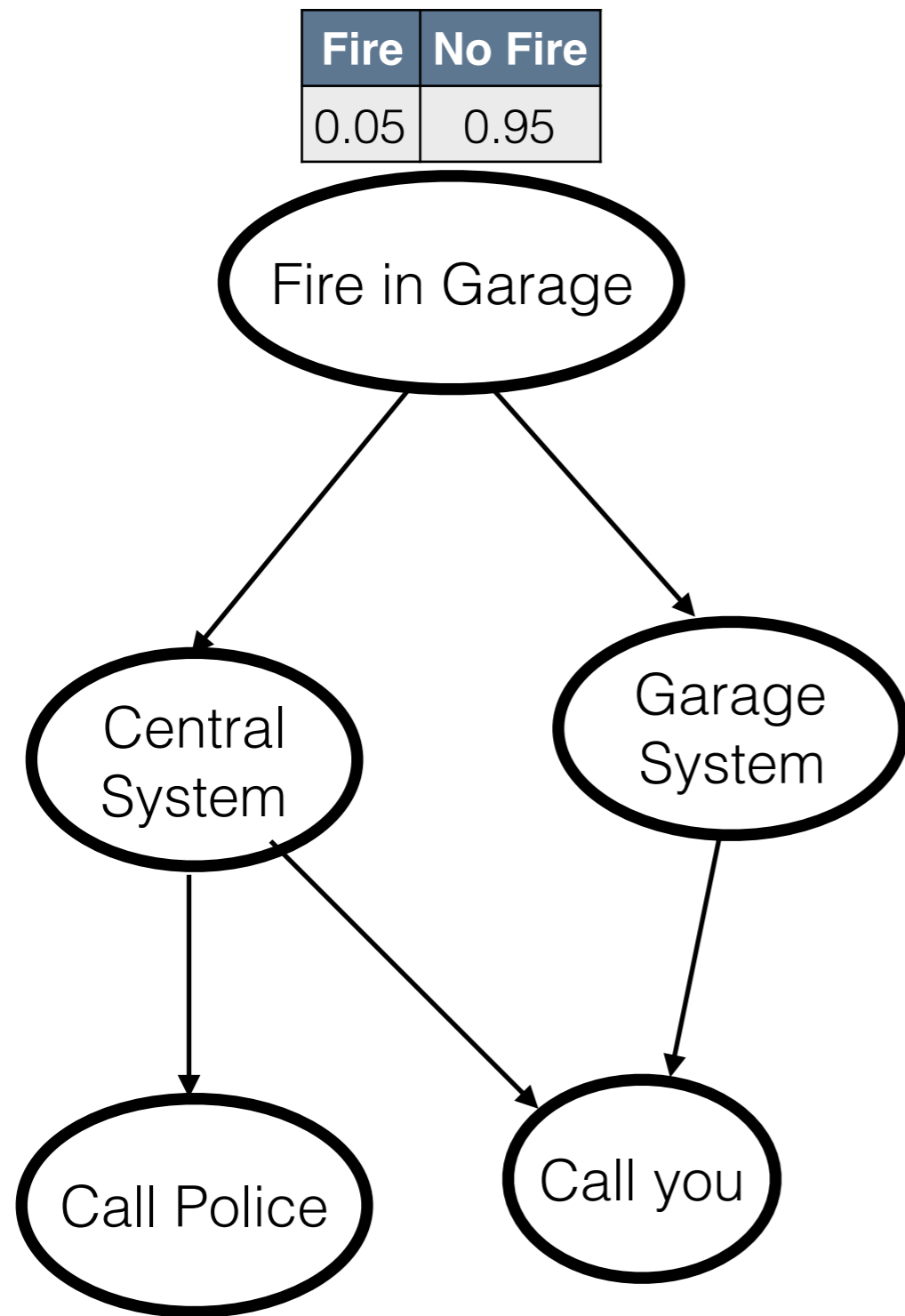
$$P_\theta(X_1, \dots, X_n) = \prod_{i=1}^n P_\theta(X_i | \text{Parent}(X_i))$$

- Hence Bayesian Networks are specified by G along with CPD's over the variables (given their parents)

GRAPHICAL MODELS



GRAPHICAL MODELS

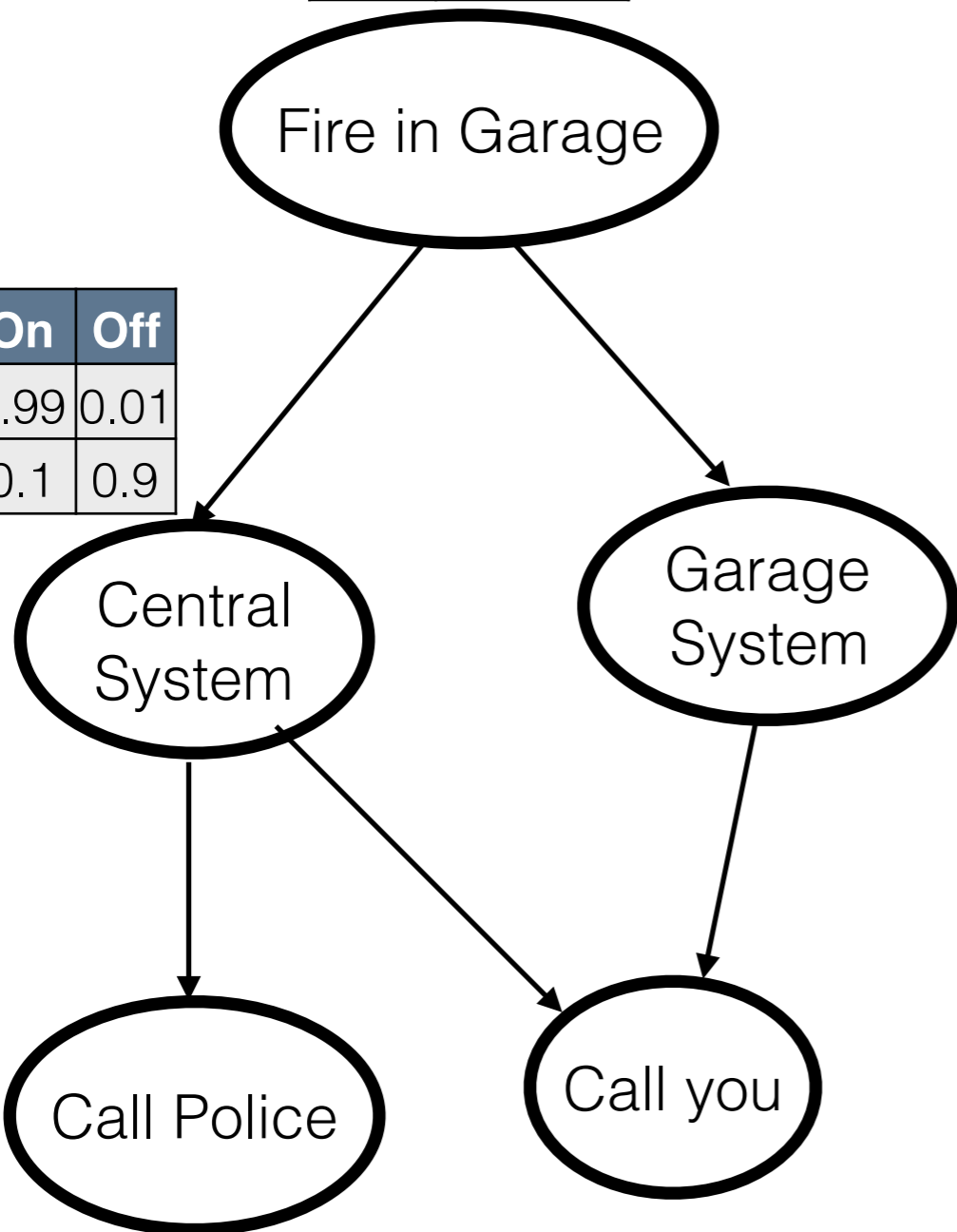
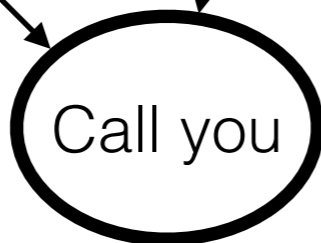
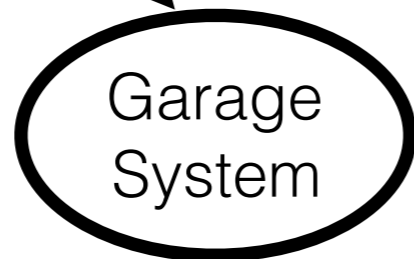
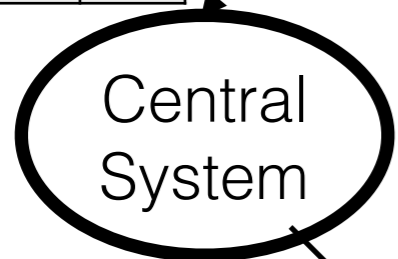


GRAPHICAL MODELS

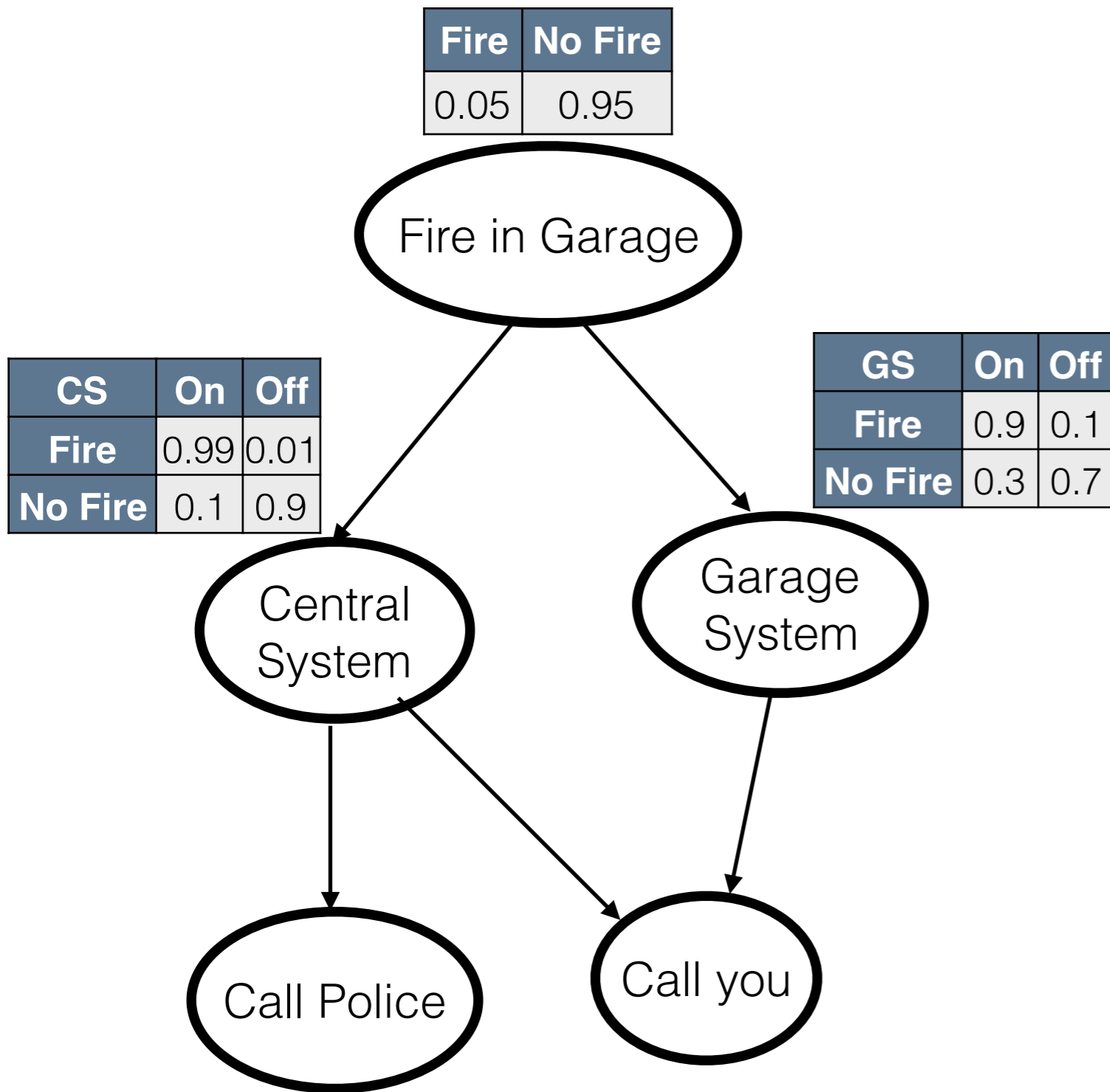
Fire	No Fire
0.05	0.95



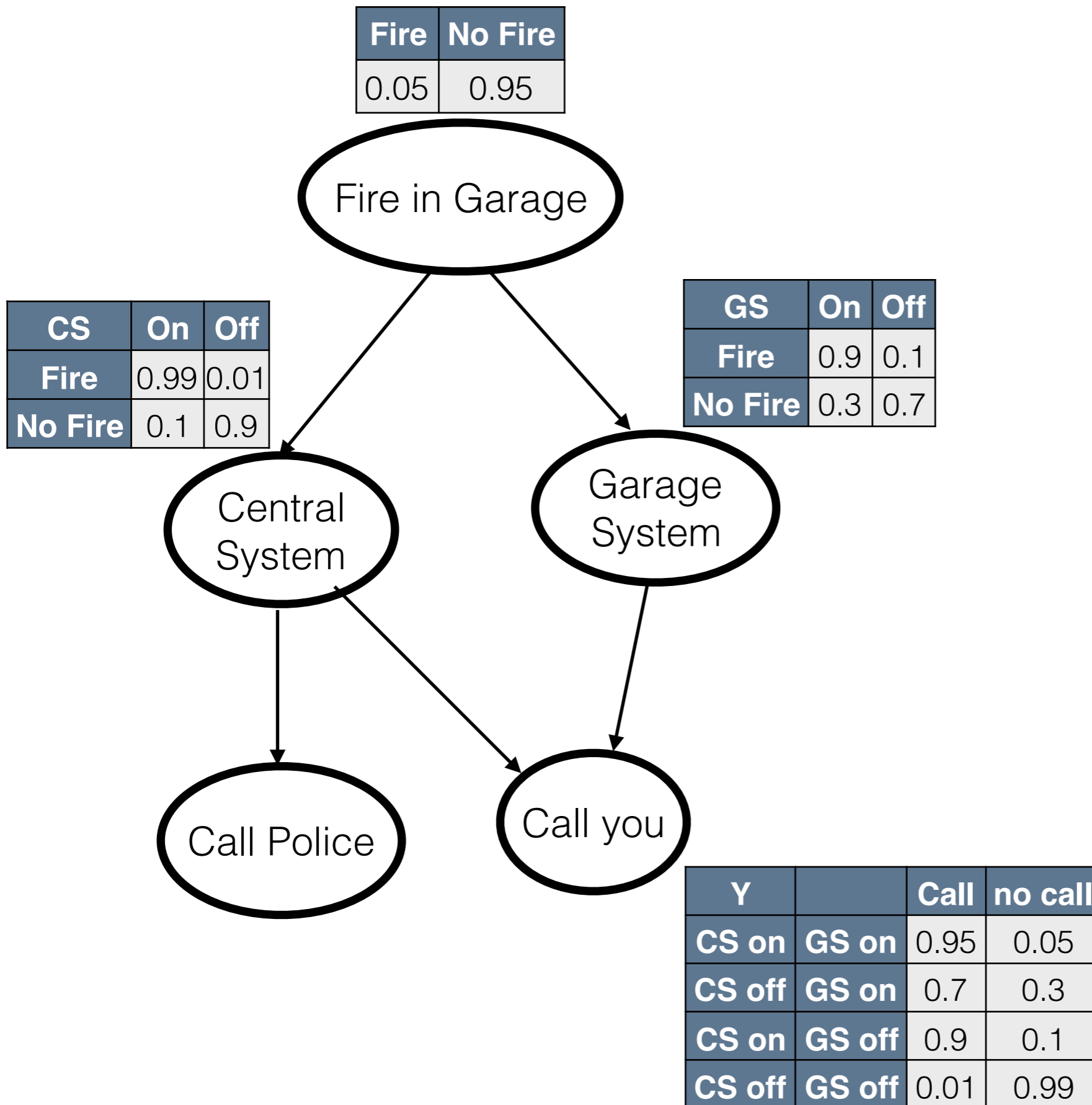
CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



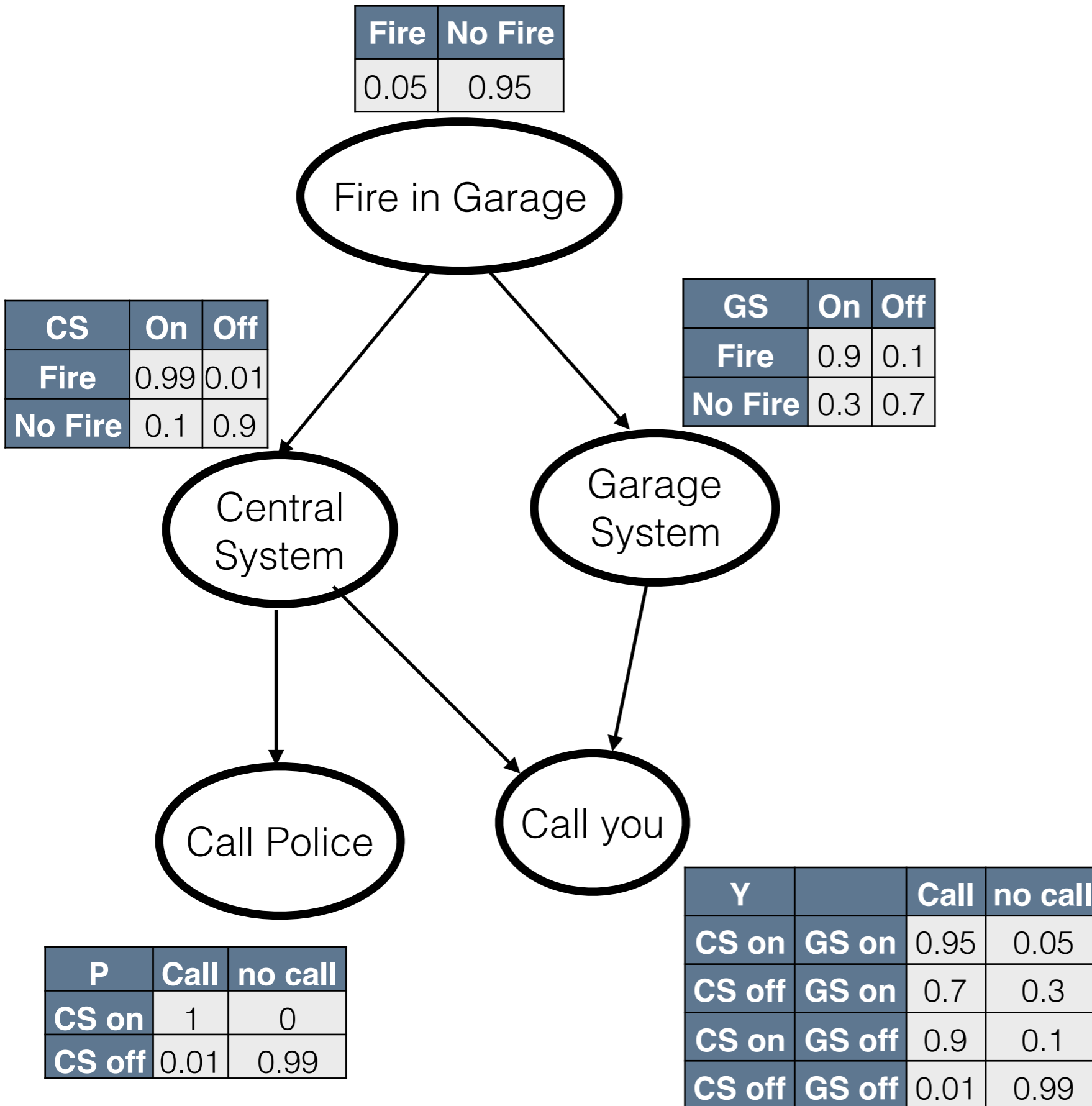
GRAPHICAL MODELS



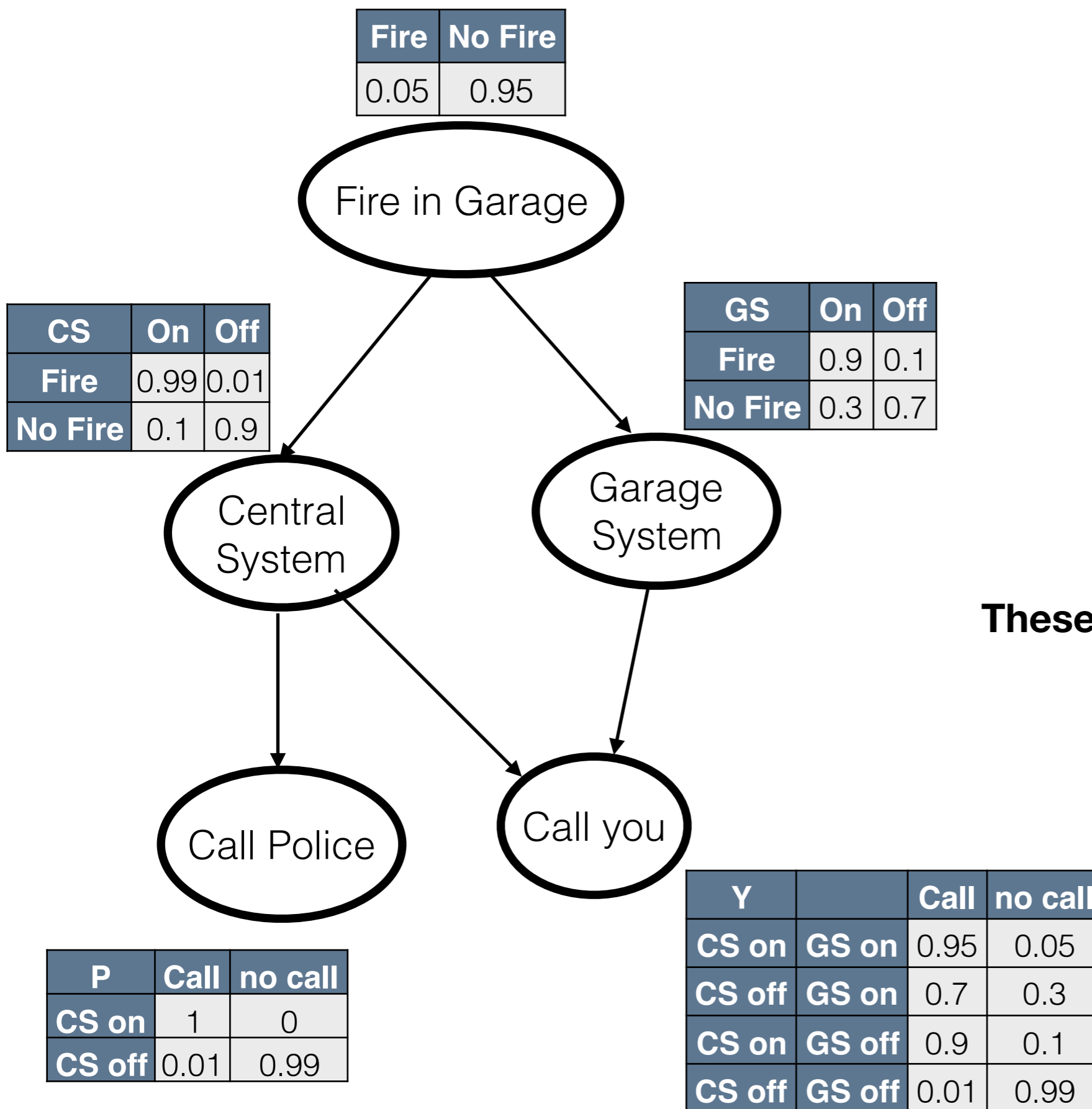
GRAPHICAL MODELS



GRAPHICAL MODELS



GRAPHICAL MODELS



These tables are the parameters

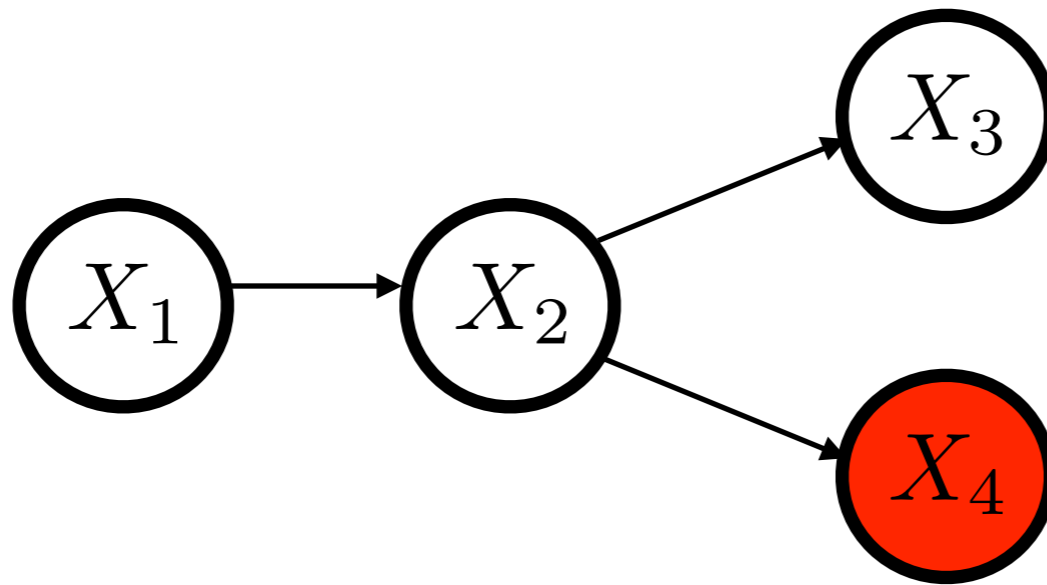
Inference in BN

VARIABLE ELIMINATION: EXAMPLES

- Marginals are enough:

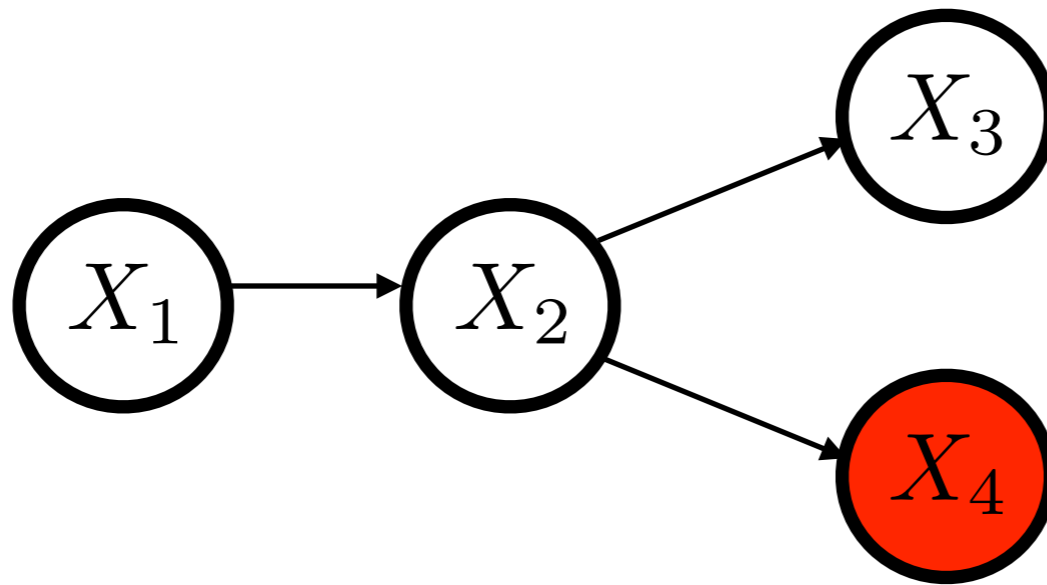
$$P(X_j = x_j, X_k = x_k | X_i = x_i, X_h = x_h) = \frac{P(X_j = x_j, X_k = x_k, X_i = x_i, X_h = x_h)}{P(X_i = x_i, X_h = x_h)}$$

VARIABLE ELIMINATION: EXAMPLES



$P(\text{Given variables}) = \text{Sum over all other variables } (P(\text{All variables}))$
 $= \text{Sum over all other variables } (\text{Product } P(X_i | \text{Parents}(X_i)))$

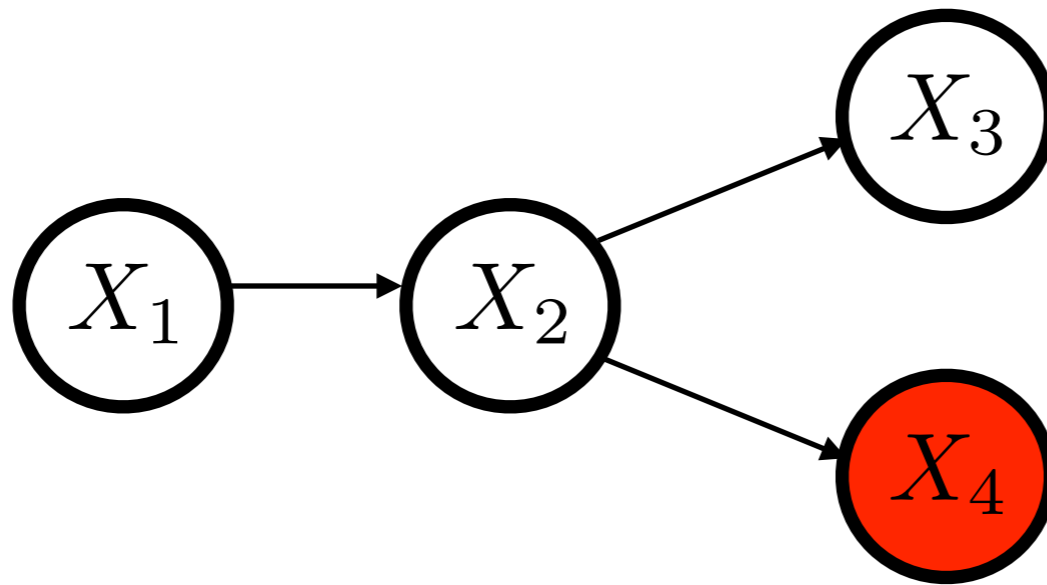
VARIABLE ELIMINATION: EXAMPLES



$P(\text{Given variables}) = \text{Sum over all other variables } (P(\text{All variables}))$
 $= \text{Sum over all other variables } (\text{Product } P(X_i | \text{Parents}(X_i)))$

$$P(X_4) = \sum_{x_1} \sum_{x_2} \sum_{x_3} P(X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4)$$

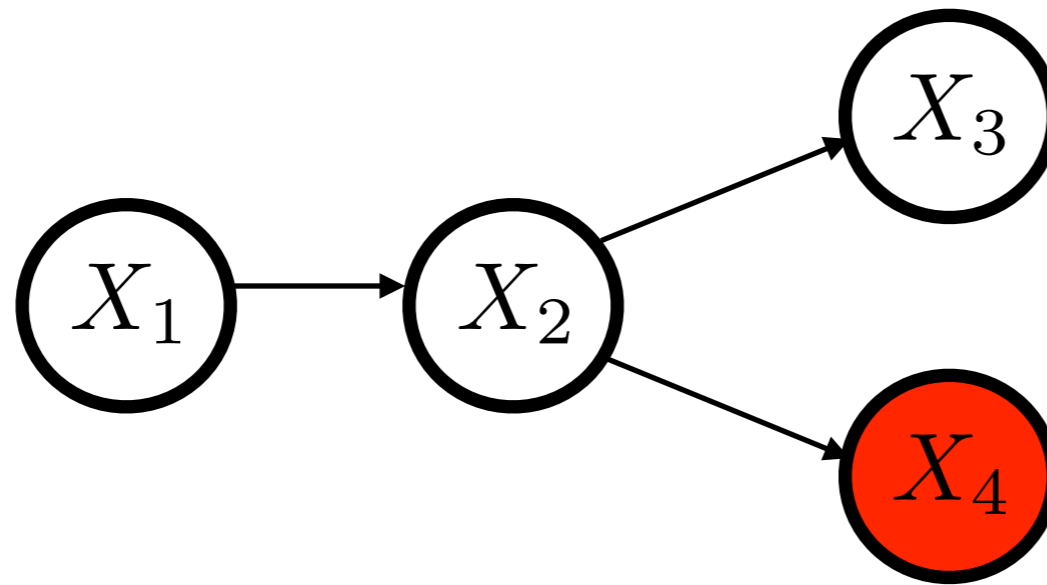
VARIABLE ELIMINATION: EXAMPLES



$P(\text{Given variables}) = \text{Sum over all other variables } (P(\text{All variables}))$
 $= \text{Sum over all other variables } (\text{Product } P(X_i | \text{Parents}(X_i)))$

$$\begin{aligned} P(X_4) &= \sum_{x_1} \sum_{x_2} \sum_{x_3} P(X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4) \\ &= \sum_{x_1} \sum_{x_2} \sum_{x_3} (P(X_1 = x_1) \cdot P(X_2 = x_2 | X_1 = x_1) \cdot P(X_3 = x_3 | X_2 = x_2) \cdot P(X_4 | X_2 = x_2)) \end{aligned}$$

VARIABLE ELIMINATION: EXAMPLES



$P(\text{Given variables}) = \text{Sum over all other variables } (P(\text{All variables}))$
 $= \text{Sum over all other variables } (\text{Product } P(X_i | \text{Parents}(X_i)))$

$$\begin{aligned} P(X_4) &= \sum_{x_1} \sum_{x_2} \sum_{x_3} P(X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4) \\ &= \sum_{x_1} \sum_{x_2} \sum_{x_3} (P(X_1 = x_1) \cdot P(X_2 = x_2 | X_1 = x_1) \cdot P(X_3 = x_3 | X_2 = x_2) \cdot P(X_4 | X_2 = x_2)) \\ &= \sum_{x_1} \left(P(X_1 = x_1) \sum_{x_2} \left(P(X_2 = x_2 | X_1 = x_1) P(X_4 | X_2 = x_2) \left(\sum_{x_3} P(X_3 = x_3 | X_2 = x_2) \right) \right) \right) \end{aligned}$$

VARIABLE ELIMINATION: BAYESIAN NETWORK

Initialize **List** with conditional probability distributions

Pick an order of elimination I for remaining variables

For each $X_i \in I$

 Find distributions in **List** containing variable X_i and remove them

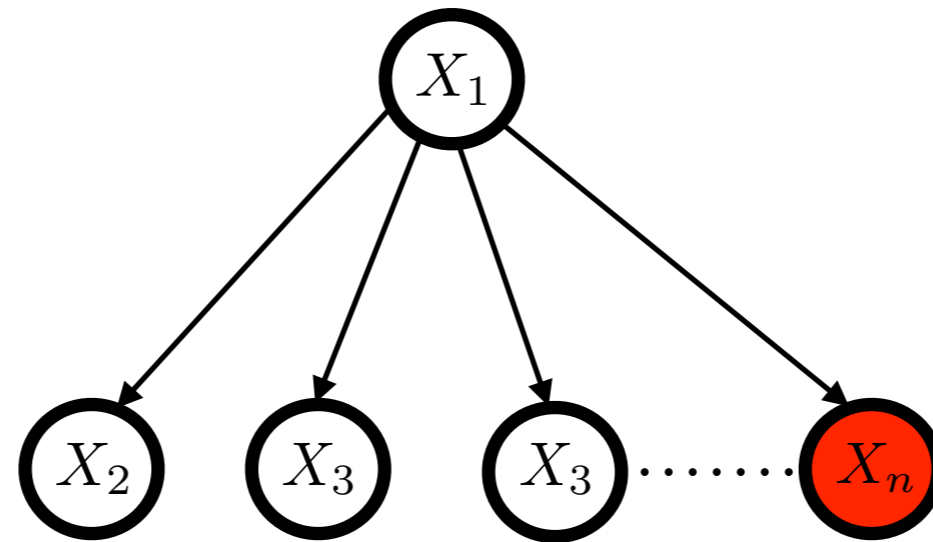
 Define new distribution as the sum (over values of X_i) of the product of these distributions

 Place the new distribution on **List**

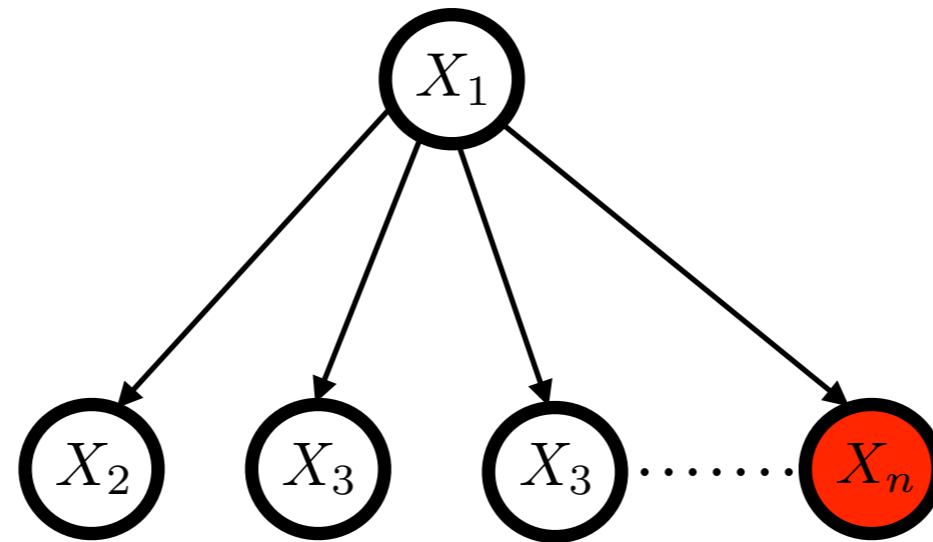
End

Return **List**

VARIABLE ELIMINATION: ORDER MATTERS

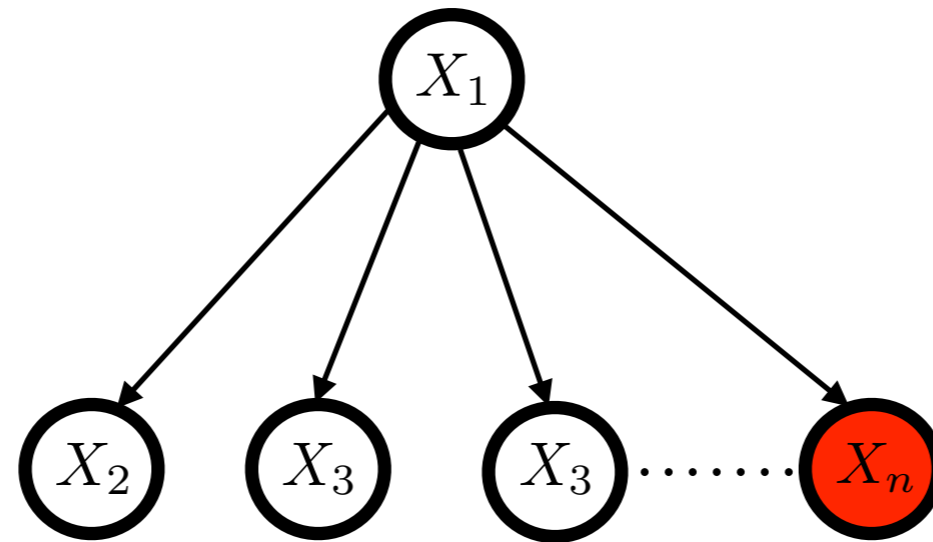


VARIABLE ELIMINATION: ORDER MATTERS



List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

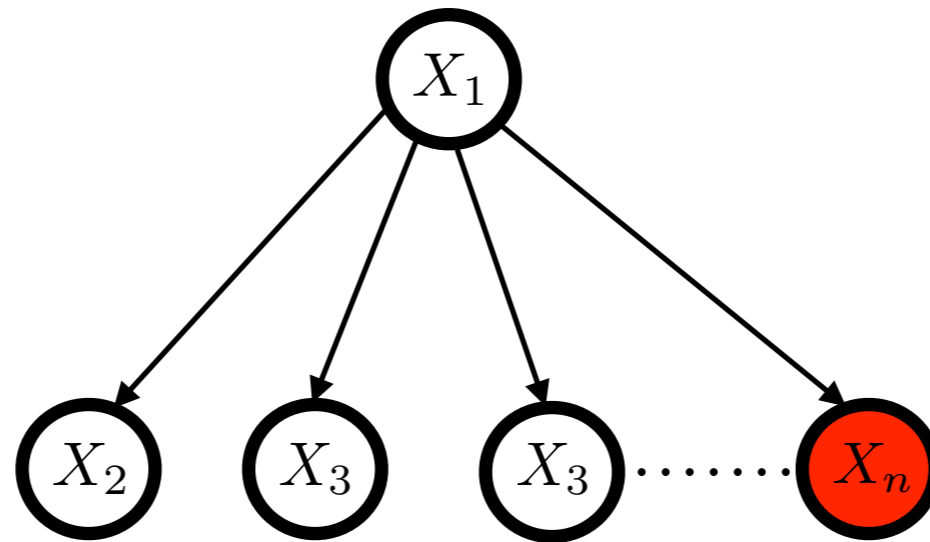
VARIABLE ELIMINATION: ORDER MATTERS



List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

Say $I = (1, 2, 3, \dots, n-1)$

VARIABLE ELIMINATION: ORDER MATTERS

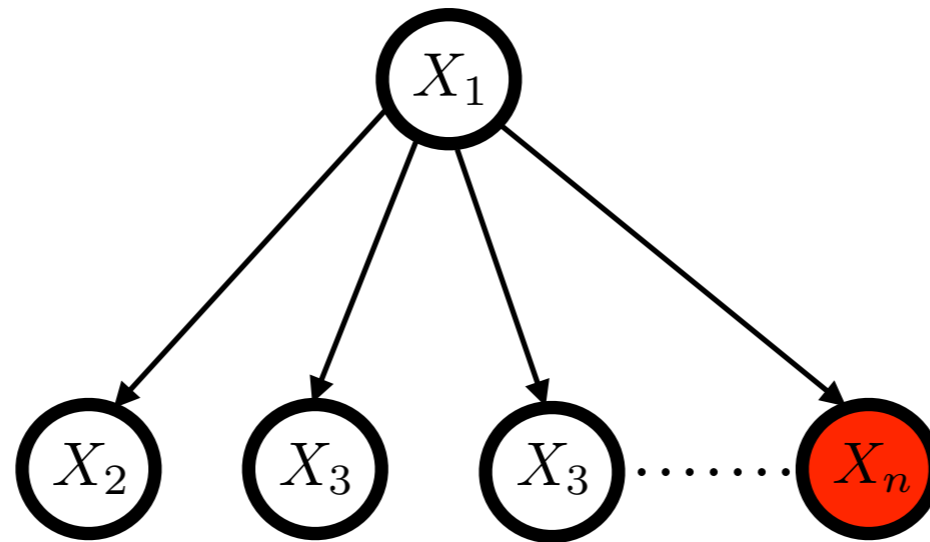


List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

Say $I = (1, 2, 3, \dots, n-1)$

Iteration 1: Eliminate X_1

VARIABLE ELIMINATION: ORDER MATTERS



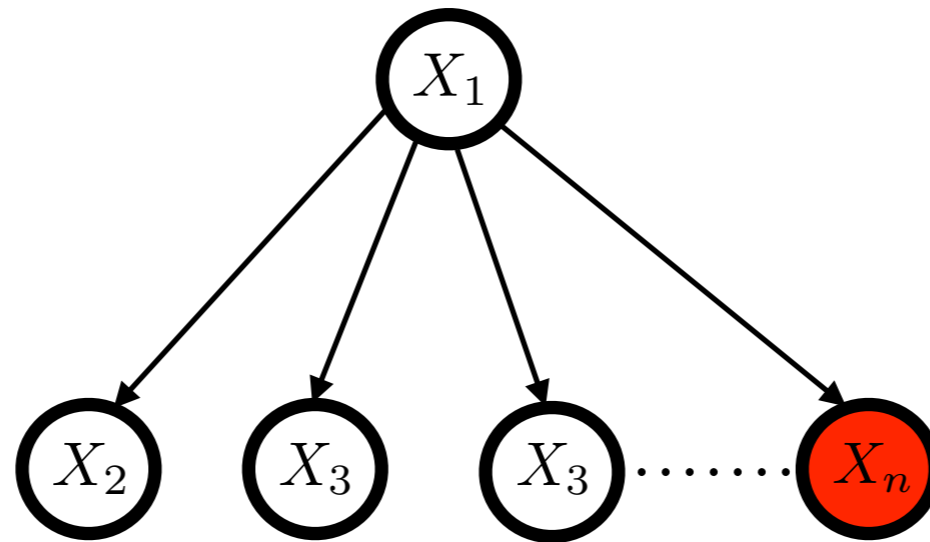
List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

Say $I = (1, 2, 3, \dots, n-1)$

Iteration 1: Eliminate X_1

All terms in list involve X_1 so remove all of them

VARIABLE ELIMINATION: ORDER MATTERS



List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

Say $I = (1, 2, 3, \dots, n-1)$

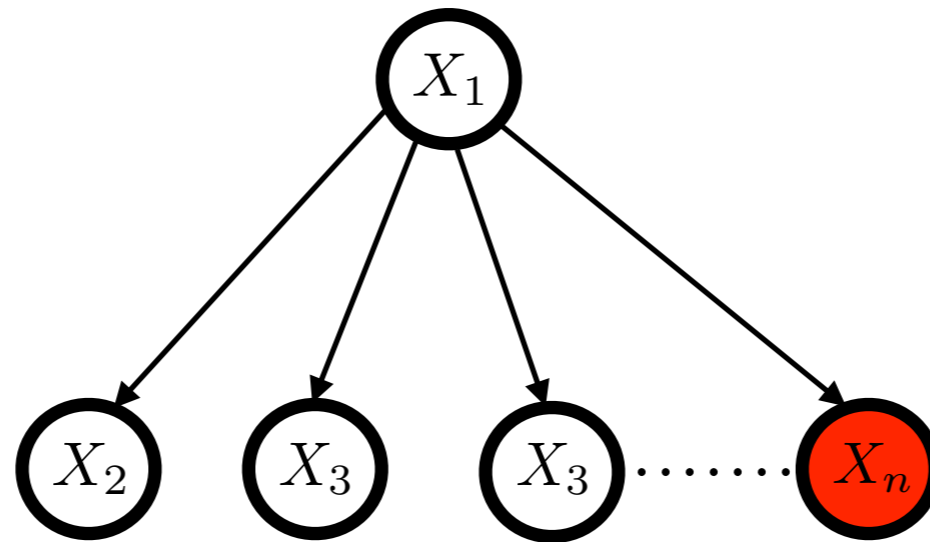
Iteration 1: Eliminate X_1

All terms in list involve X_1 so remove all of them

Replace them by table:

$$L_2(x_1, \dots, x_n) = \sum_{x_1} \left(P(X_1 = x_1) \prod_{t=2}^n P(X_t = x_t | X_1 = x_1) \right)$$

VARIABLE ELIMINATION: ORDER MATTERS



List initialized to $\{P(Y_1), P(Y_2|Y_1), P(Y_3|Y_1), \dots, P(Y_n|X_1)\}$

Say

What is the size of table L1?

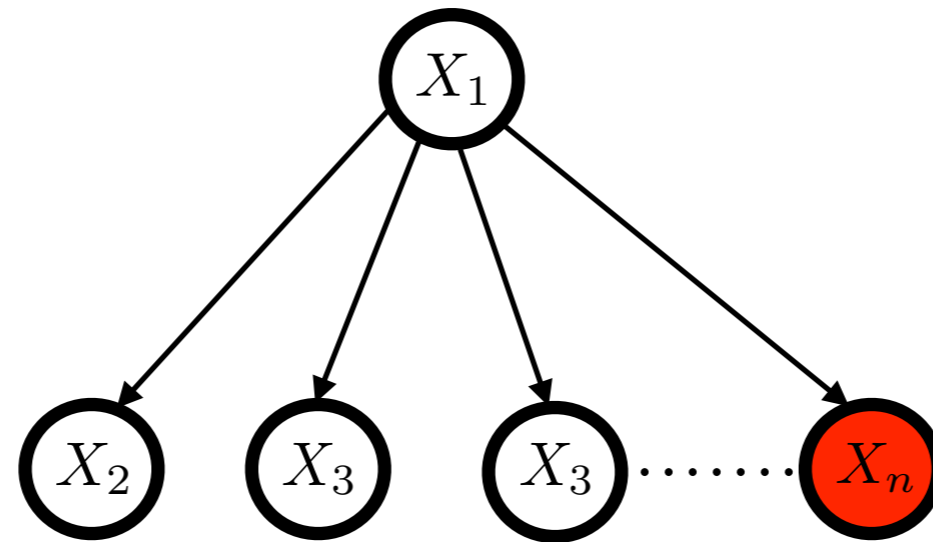
Iteration 1: Eliminate X_1

All terms in list involve X_1 so remove all of them

Replace them by table:

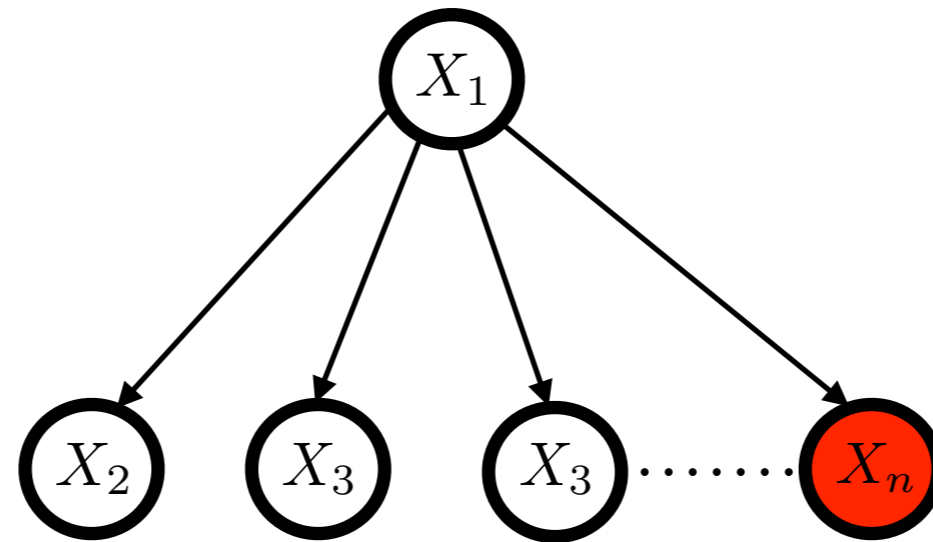
$$L_2(x_1, \dots, x_n) = \sum_{x_1} \left(P(X_1 = x_1) \prod_{t=2}^n P(X_t = x_t | X_1 = x_1) \right)$$

VARIABLE ELIMINATION: ORDER MATTERS



List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

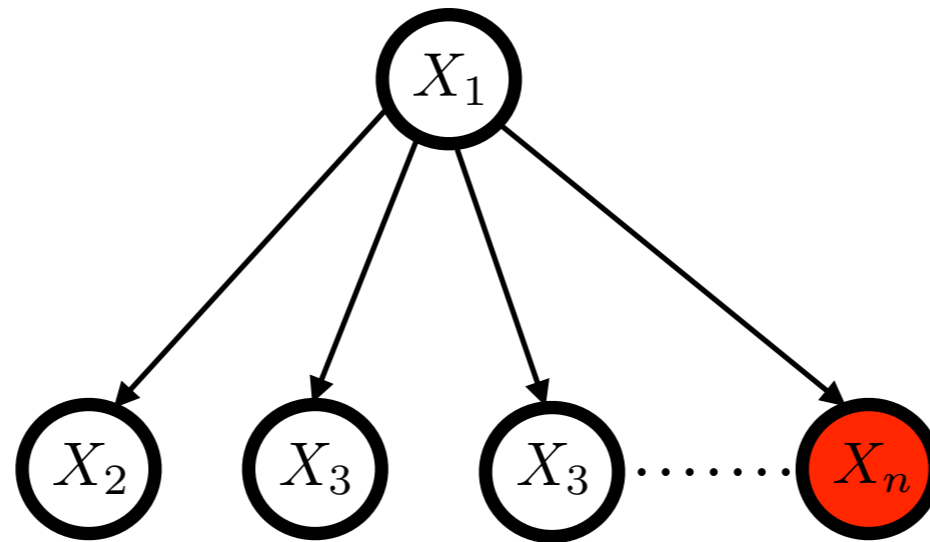
VARIABLE ELIMINATION: ORDER MATTERS



List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

Say $I = (n-1, n-2, \dots, 1)$

VARIABLE ELIMINATION: ORDER MATTERS

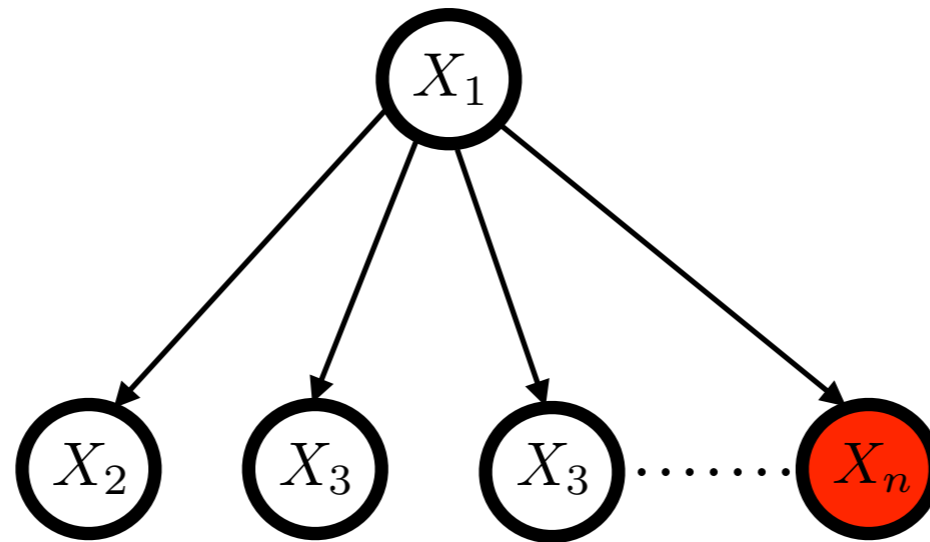


List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

Say $I = (n-1, n-2, \dots, 1)$

Iteration 1: Eliminate X_{n-1}

VARIABLE ELIMINATION: ORDER MATTERS



List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

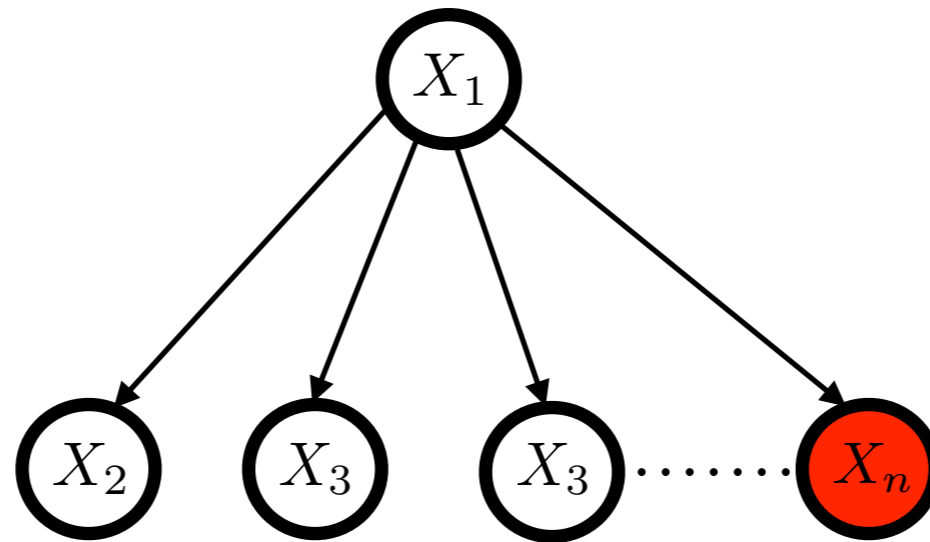
Say $I = (n-1, n-2, \dots, 1)$

Iteration 1: Eliminate X_{n-1}

Remove $P(X_n|X_1)$ from List and replace by

$$L_{n-1}(x_1) = \sum_{x_{n-1}} P(X_{n-1}|X_1 = x_1) = 1$$

VARIABLE ELIMINATION: ORDER MATTERS



List initialized to : $\{P(X_1), P(X_2|X_1), P(X_3|X_1), \dots, P(X_n|X_1)\}$

Say $I = (n-1, n-2, \dots, 1)$

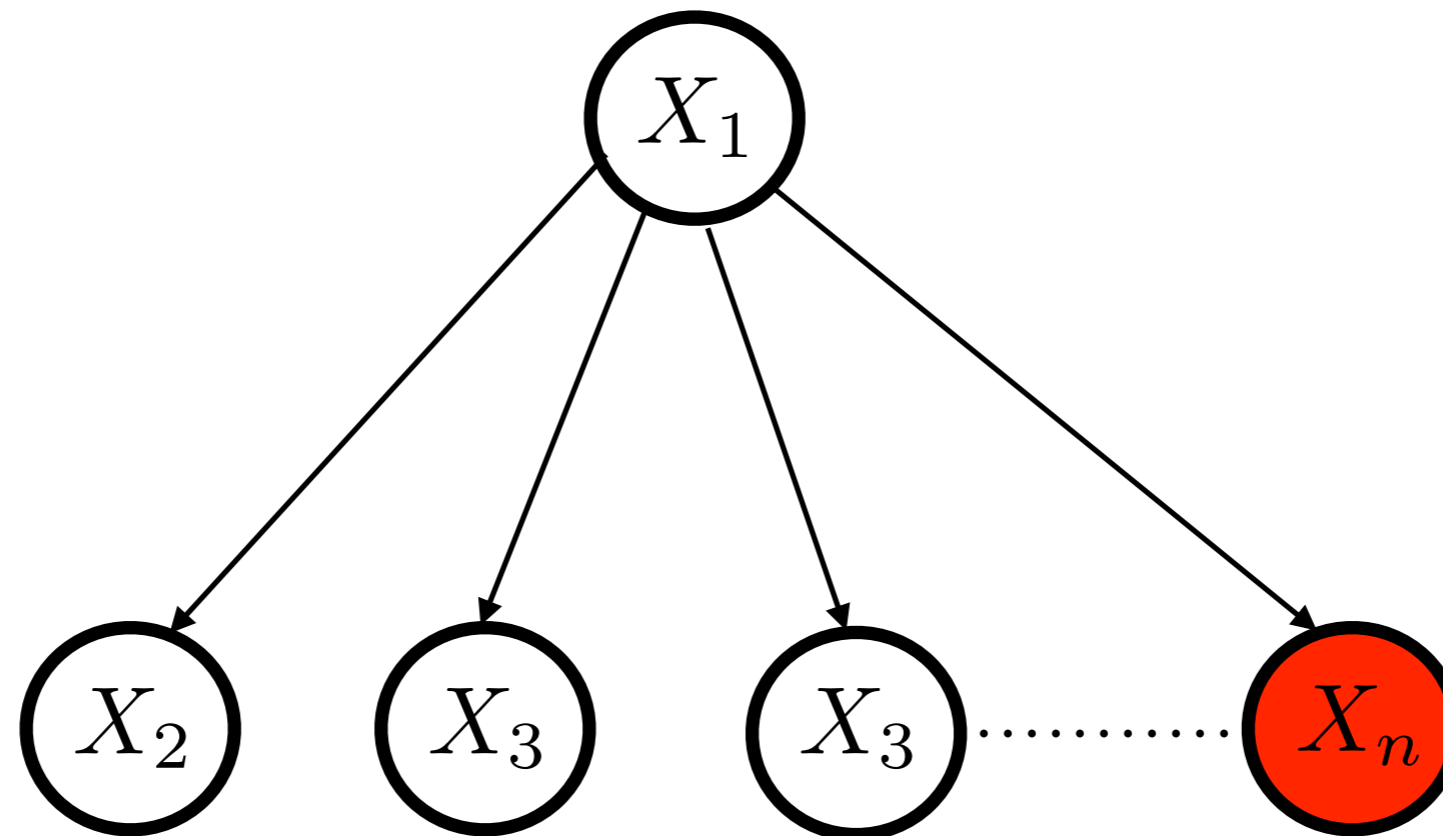
Iteration 1: Eliminate X_{n-1}

Remove $P(X_n|X_1)$ from List and replace by

$$L_{n-1}(x_1) = \sum_{x_{n-1}} P(X_{n-1}|X_1 = x_1) = 1$$

All the way up to X_2 we replace by all ones message
In then end we only have $P(X_1), P(X_n|X_1)$

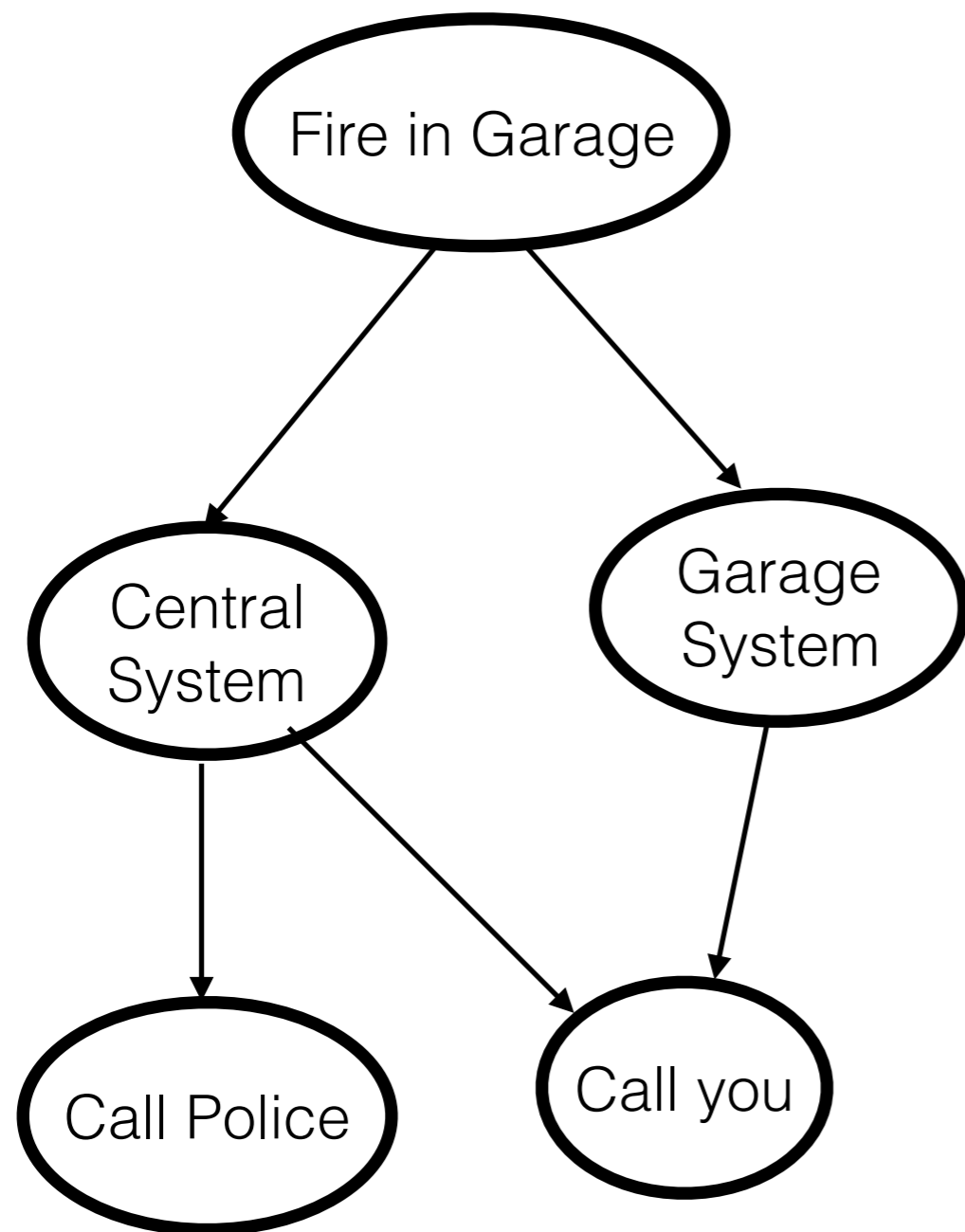
VARIABLE ELIMINATION: ORDER MATTERS



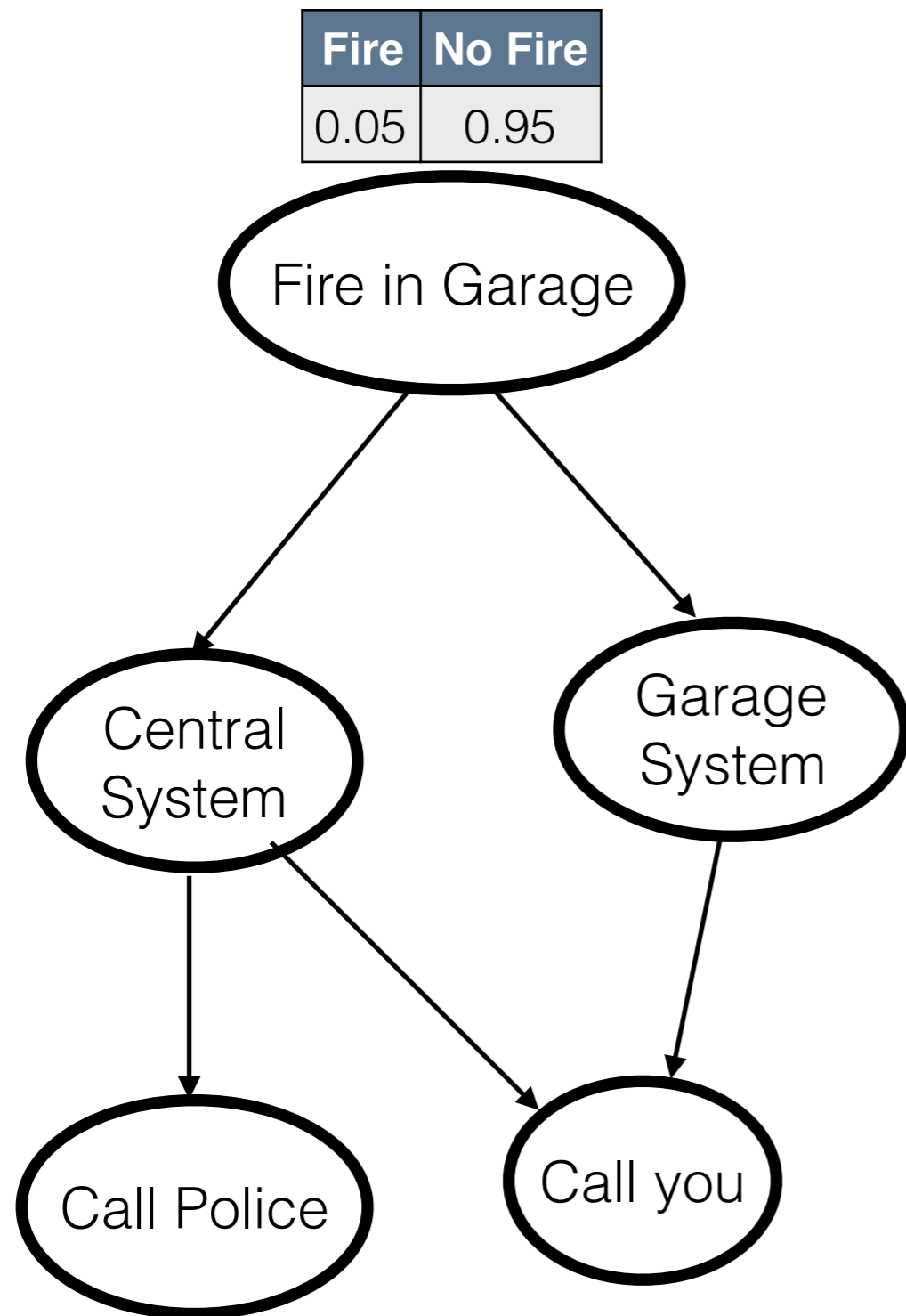
Right order: $O(n)$

Wrong order: $O(2^n)$

REJECTION SAMPLING



REJECTION SAMPLING



REJECTION SAMPLING

Fire	No Fire
0.05	0.95

Fire in Garage

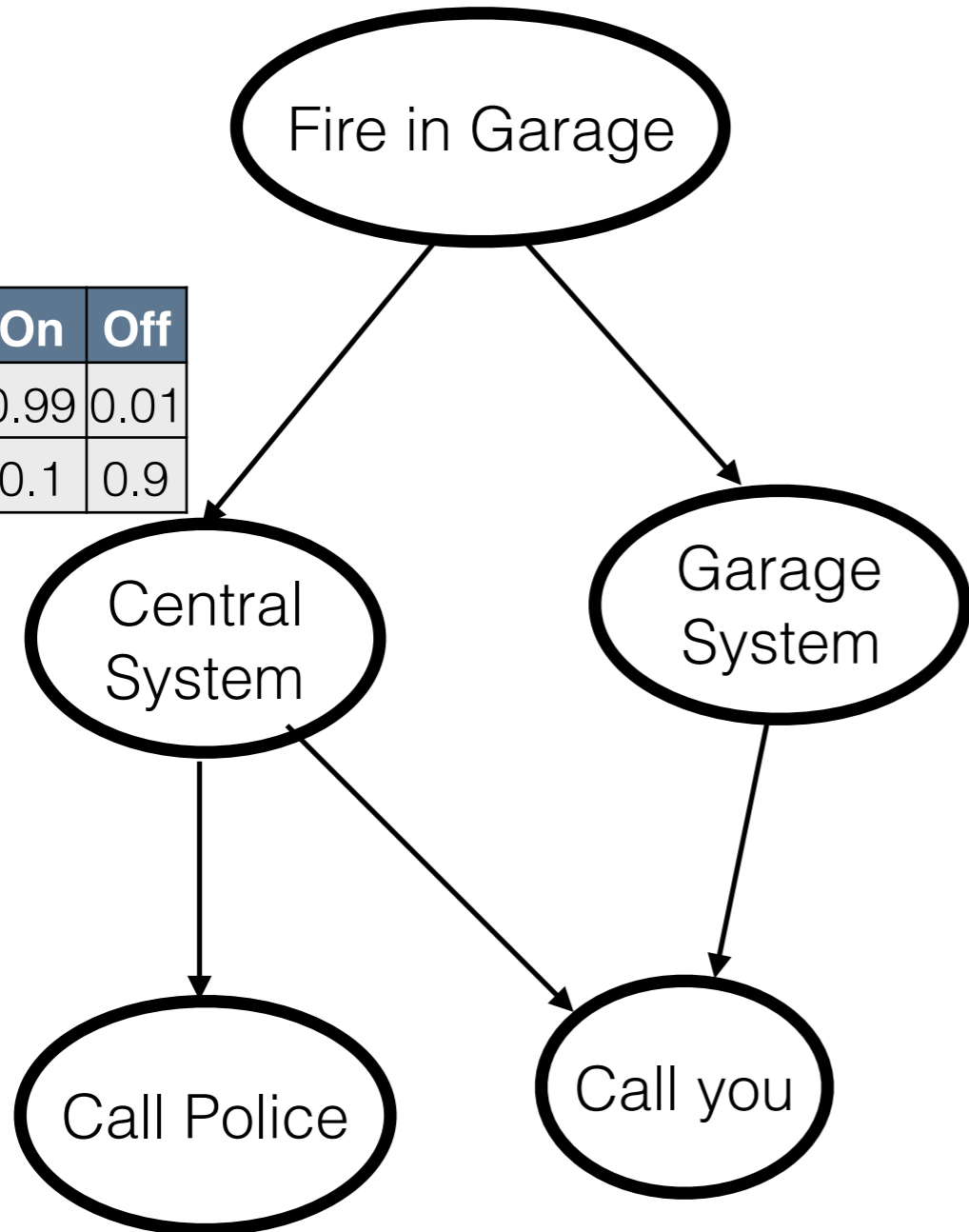
CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9

Central System

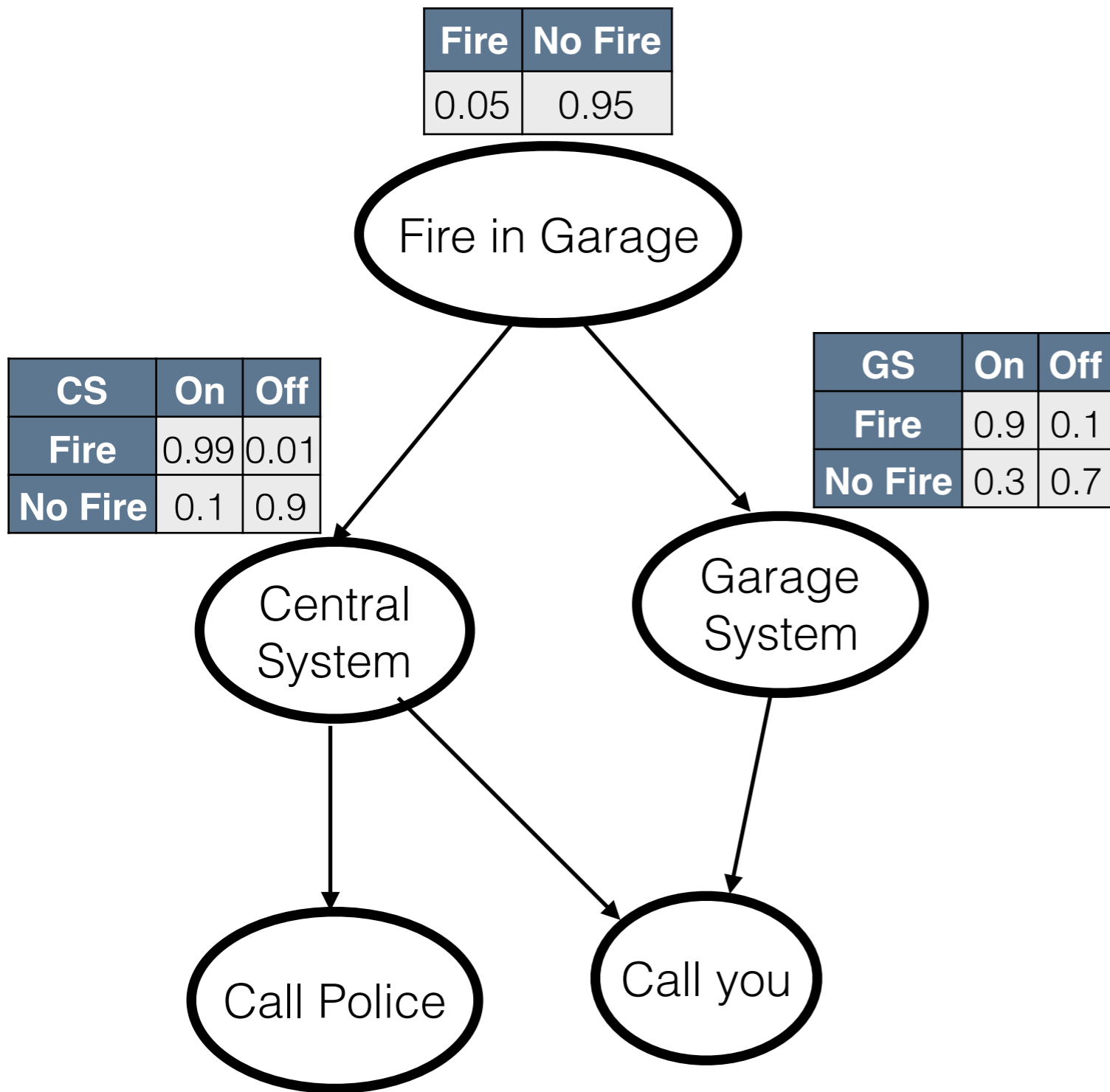
Garage System

Call Police

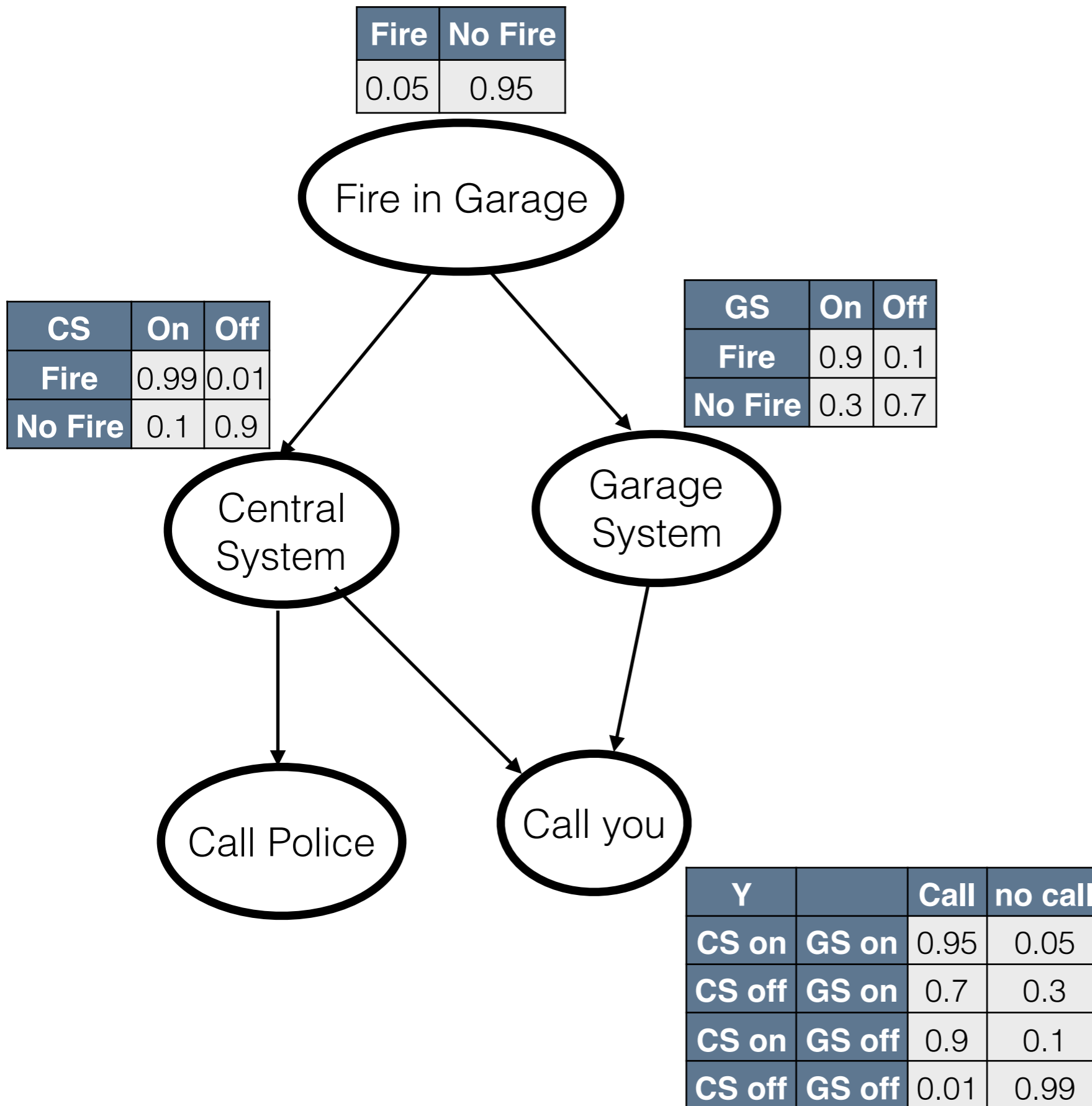
Call you



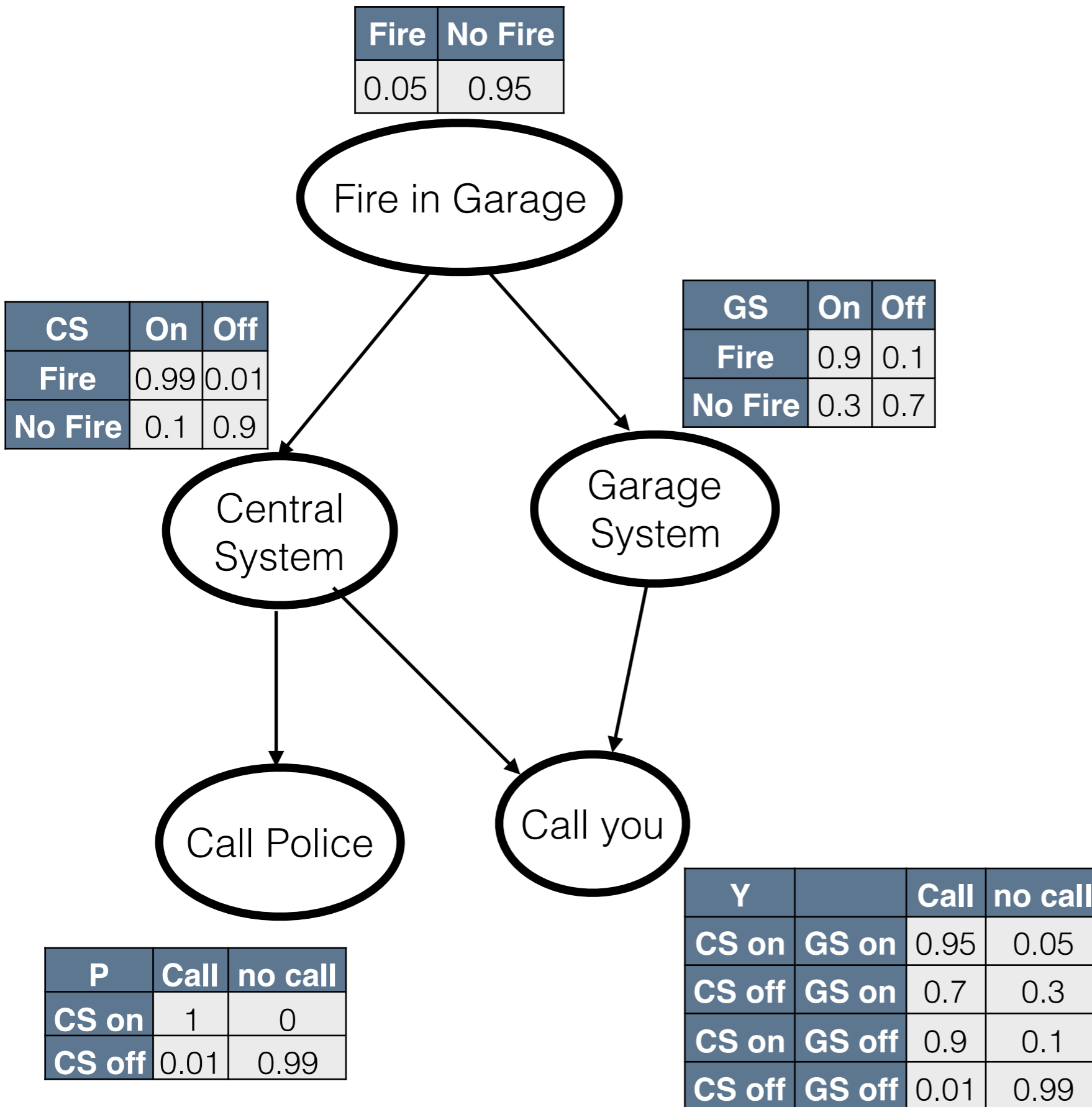
REJECTION SAMPLING



REJECTION SAMPLING



REJECTION SAMPLING



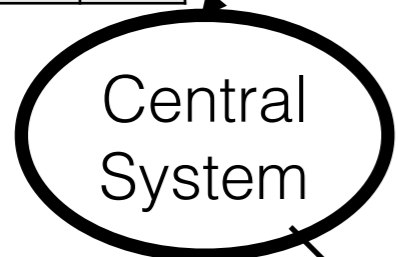
REJECTION SAMPLING

	F	CS	GS	P	Y
1					

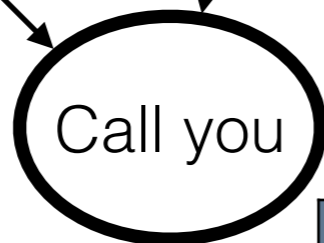
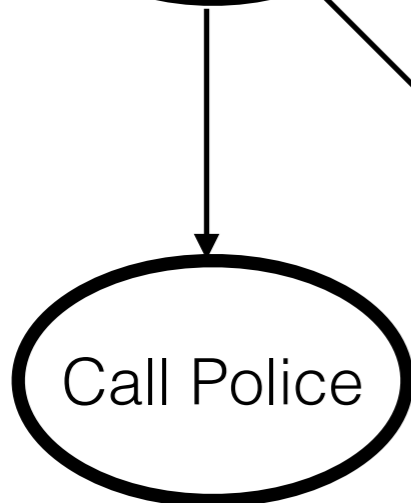
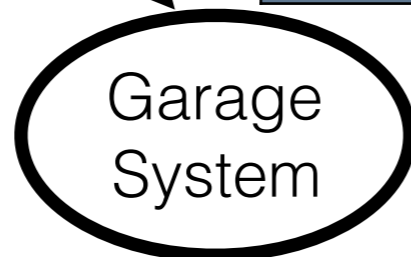
Fire	No Fire
0.05	0.95



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



P	Call	no call
CS on	1	0
CS off	0.01	0.99

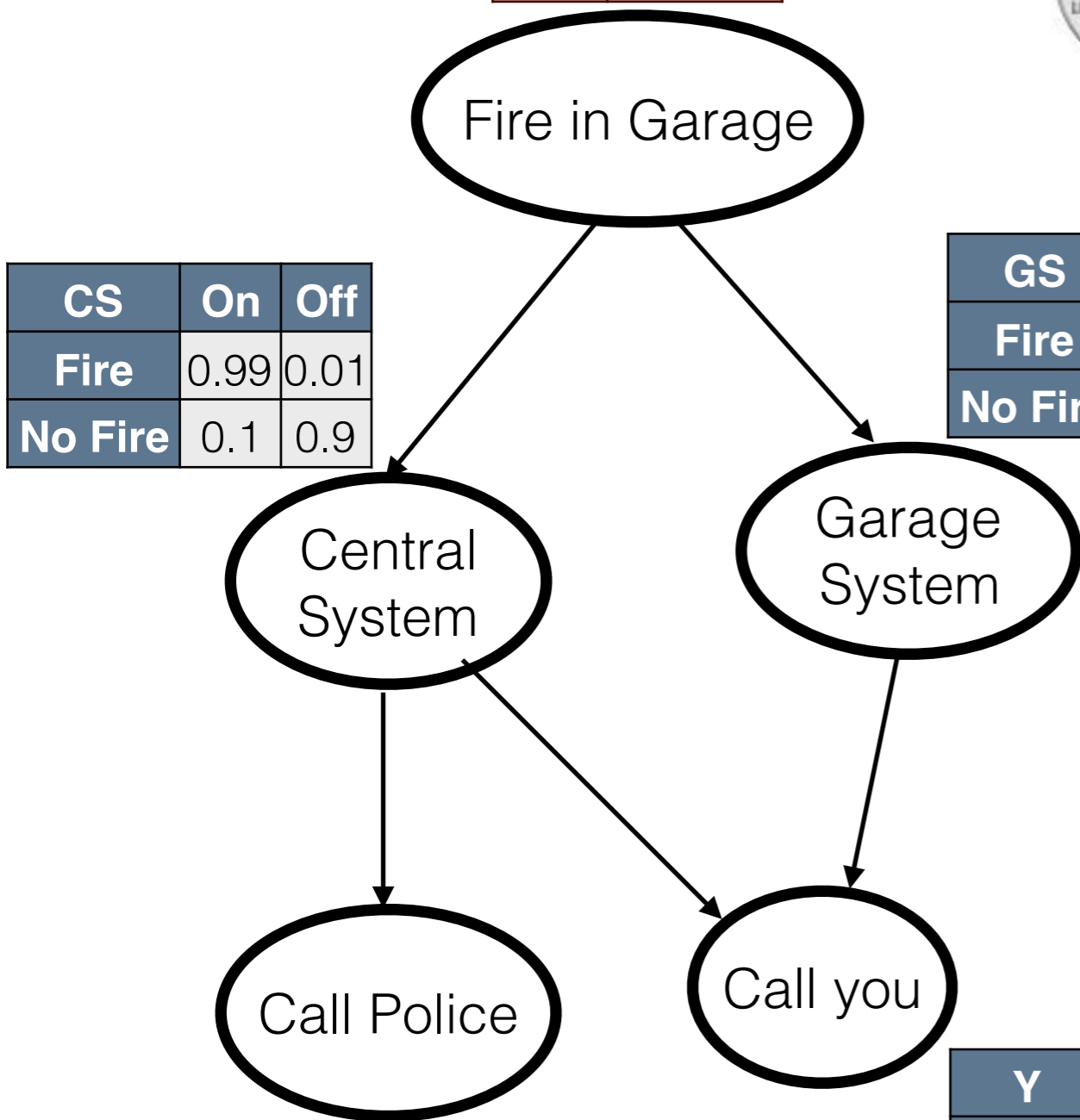
Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

REJECTION SAMPLING



Fire	No Fire
0.05	0.95

	F	CS	GS	P	Y
1	0				



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9

GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7

P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

REJECTION SAMPLING

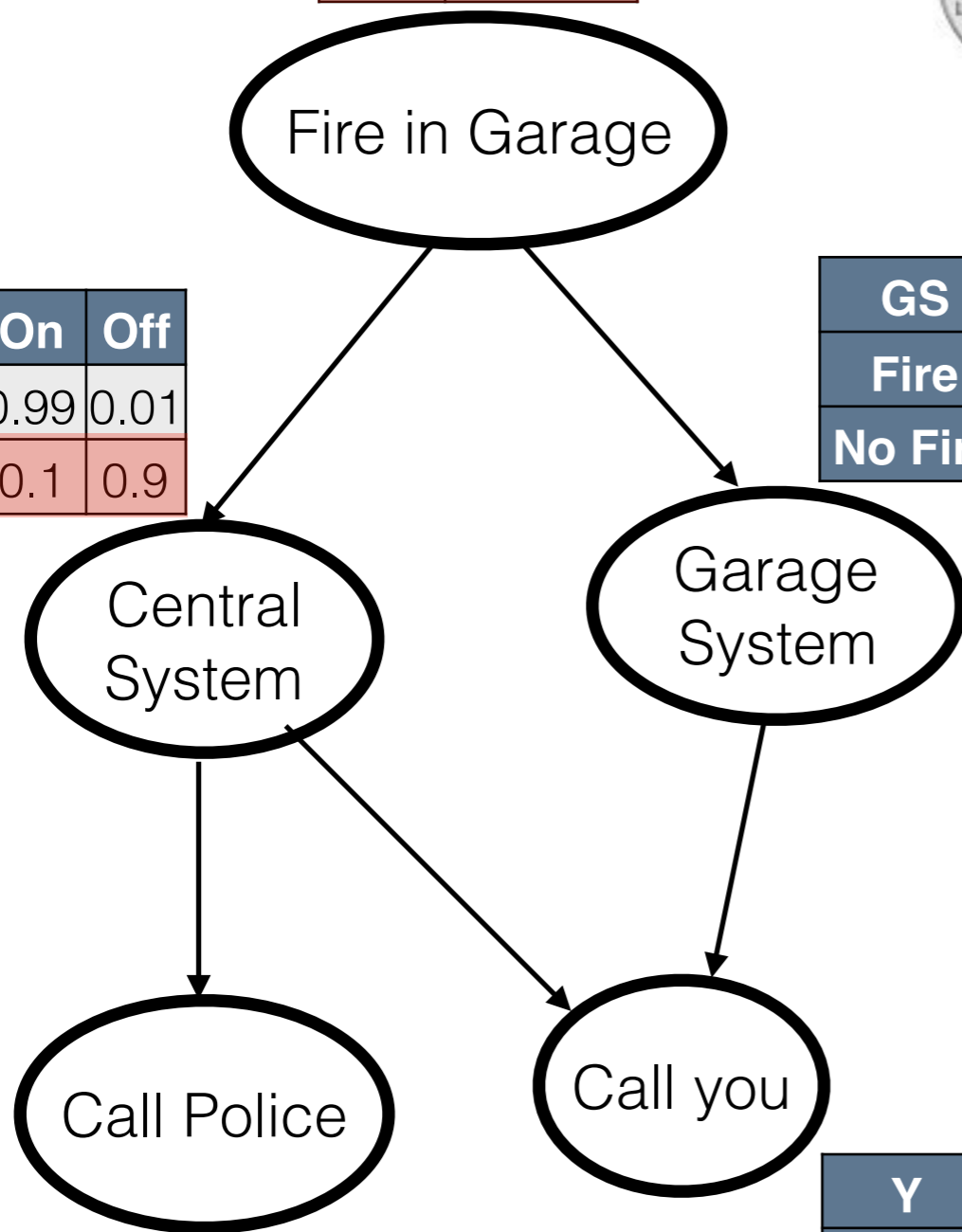


	F	CS	GS	P	Y
1	0				

Fire	No Fire
0.05	0.95

CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9

GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

REJECTION SAMPLING

Fire	No Fire
0.05	0.95

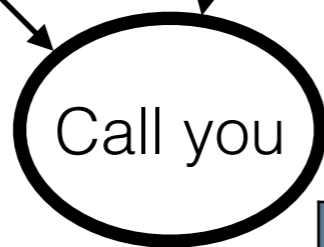
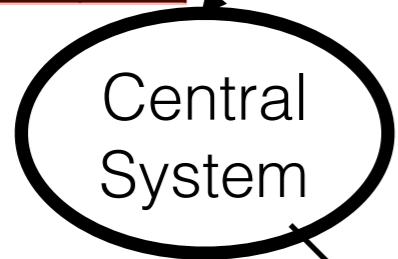


	F	CS	GS	P	Y
1	0	0			



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9

GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

REJECTION SAMPLING

Fire	No Fire
0.05	0.95

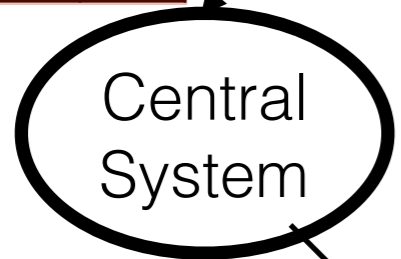


	F	CS	GS	P	Y
1	0	0			



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9

GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

REJECTION SAMPLING

	F	CS	GS	P	Y
1	0	0	1		

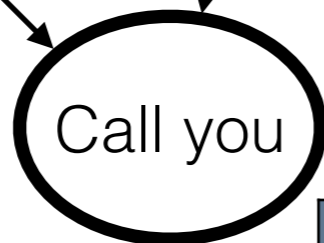
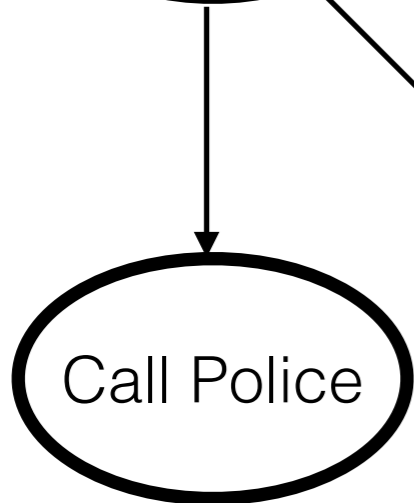
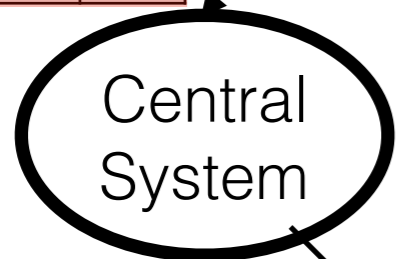
Fire	No Fire
0.05	0.95



GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

REJECTION SAMPLING

	F	CS	GS	P	Y
1	0	0	1		

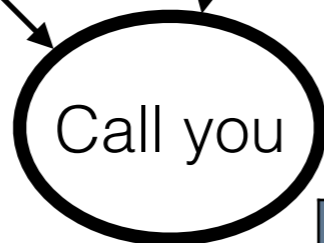
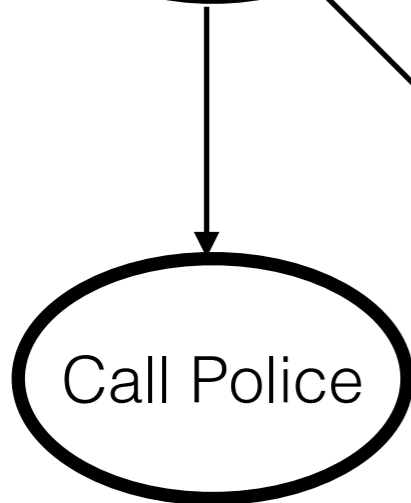
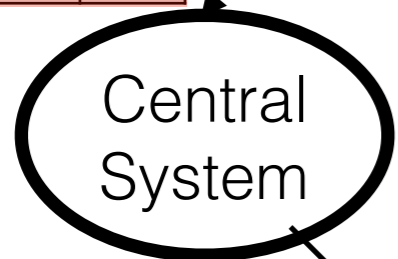
Fire	No Fire
0.05	0.95



GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

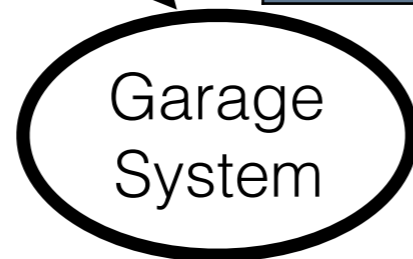
REJECTION SAMPLING

	F	CS	GS	P	Y
1	0	0	1	0	

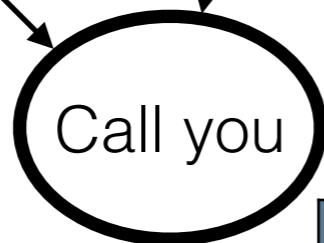
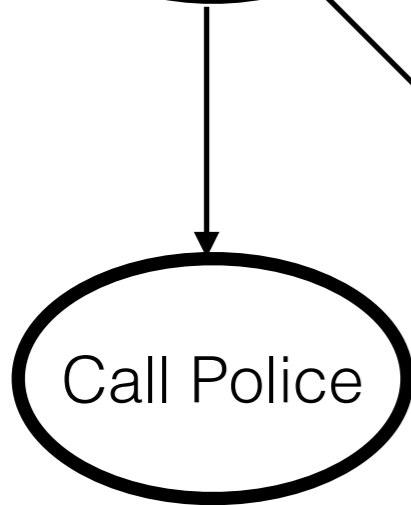
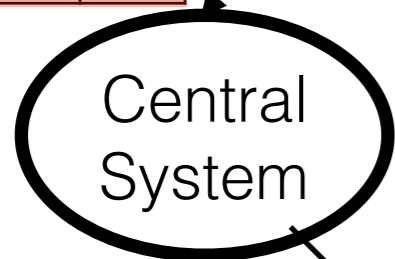
Fire	No Fire
0.05	0.95



GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

REJECTION SAMPLING

	F	CS	GS	P	Y
1	0	0	1	0	

Fire	No Fire
0.05	0.95



Fire in Garage

GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7

Garage System



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9

Central System



Call Police



Call you

P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

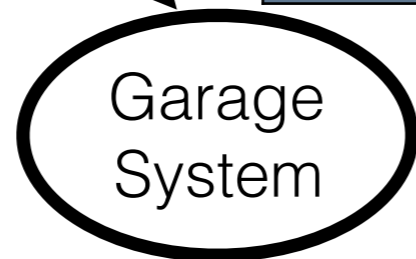
REJECTION SAMPLING

	F	CS	GS	P	Y
1	0	0	1	0	1

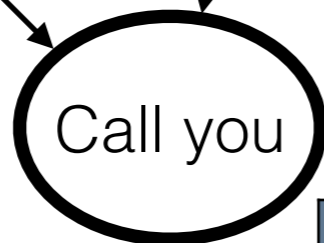
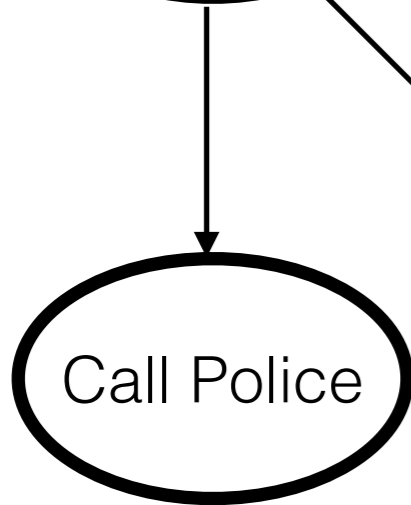
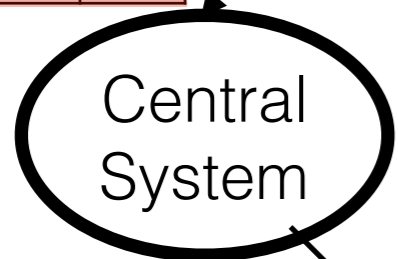
Fire	No Fire
0.05	0.95



GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9

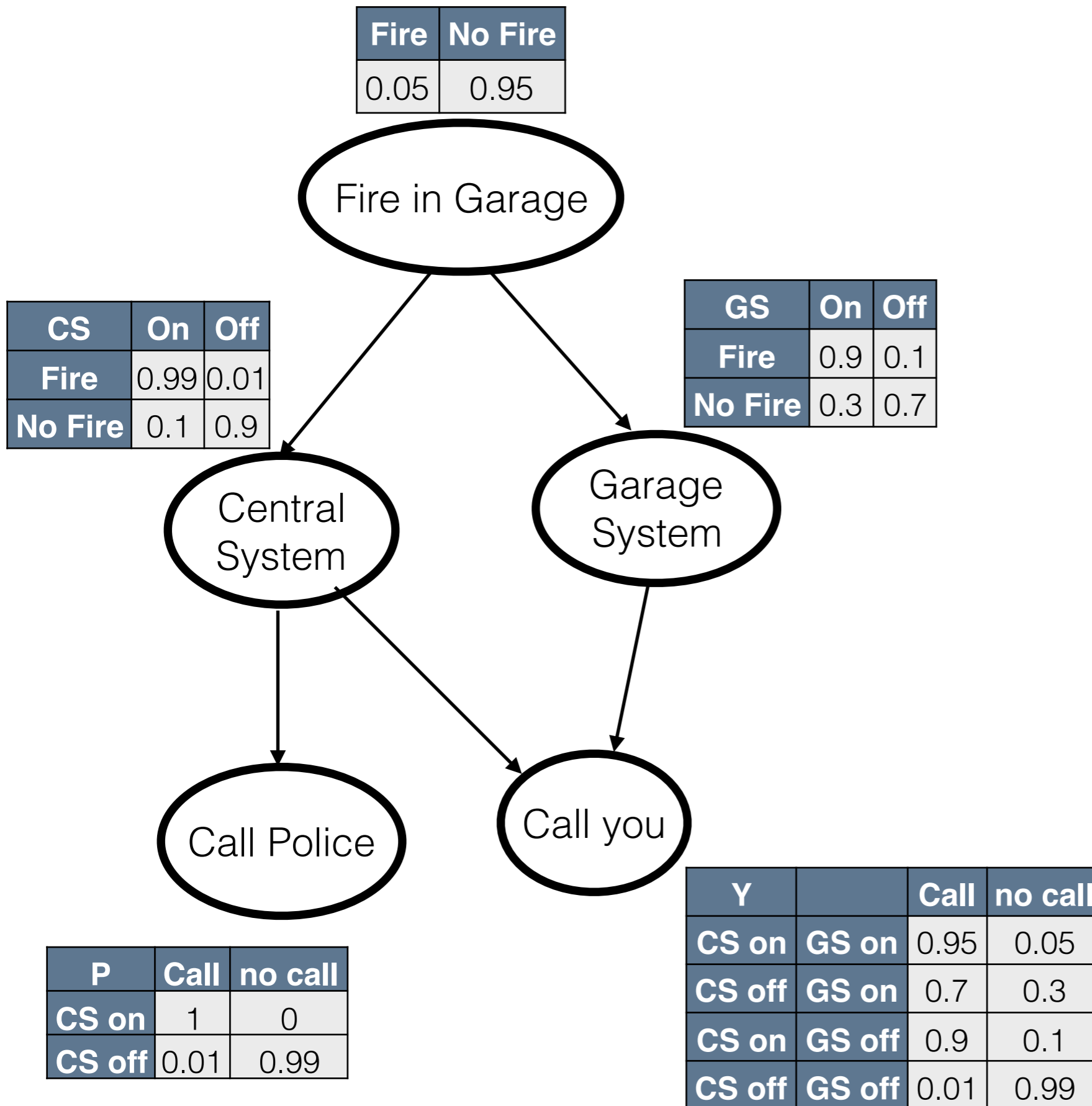


P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

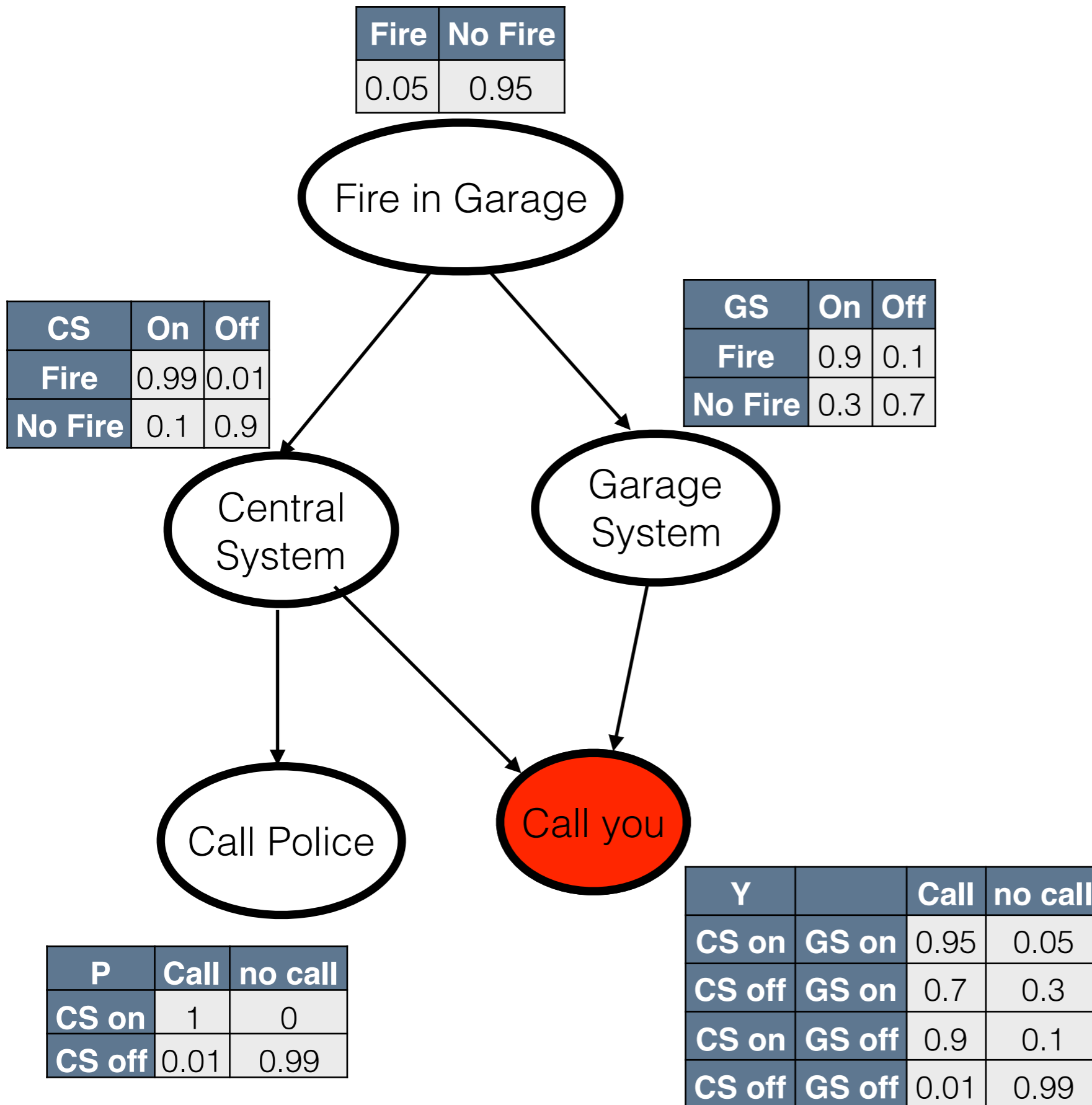


REJECTION SAMPLING



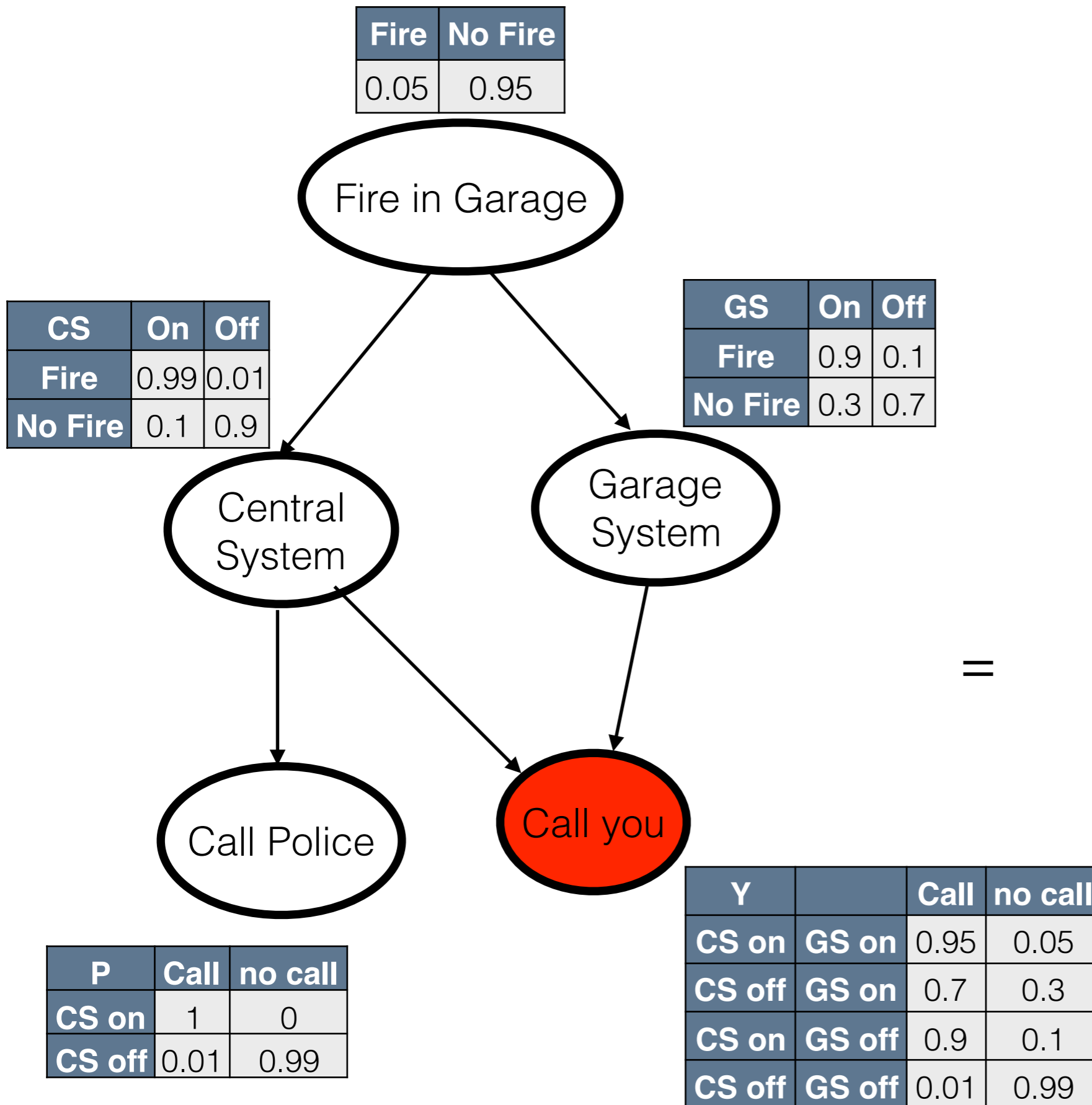
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

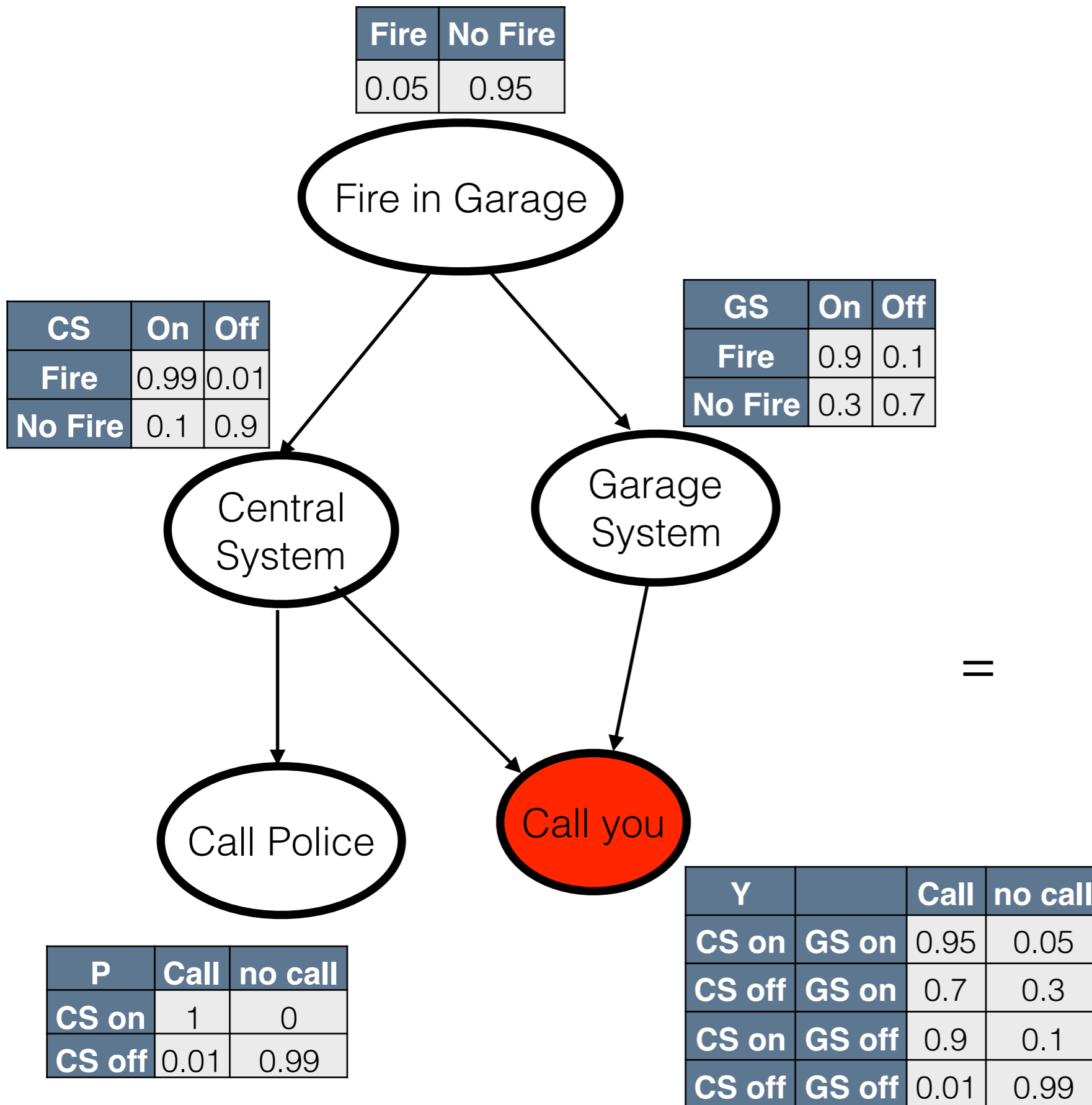
REJECTION SAMPLING



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	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

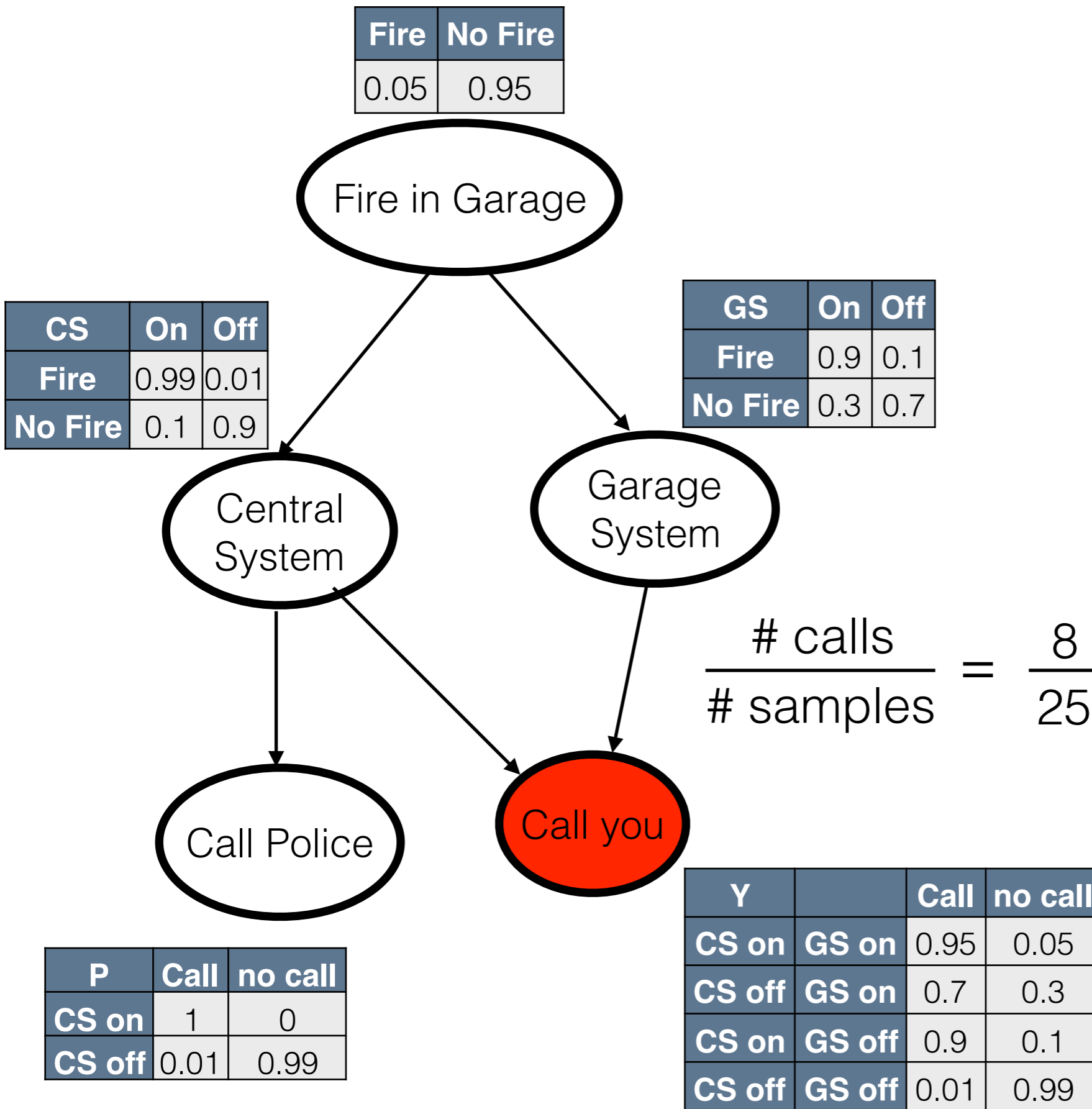
REJECTION SAMPLING



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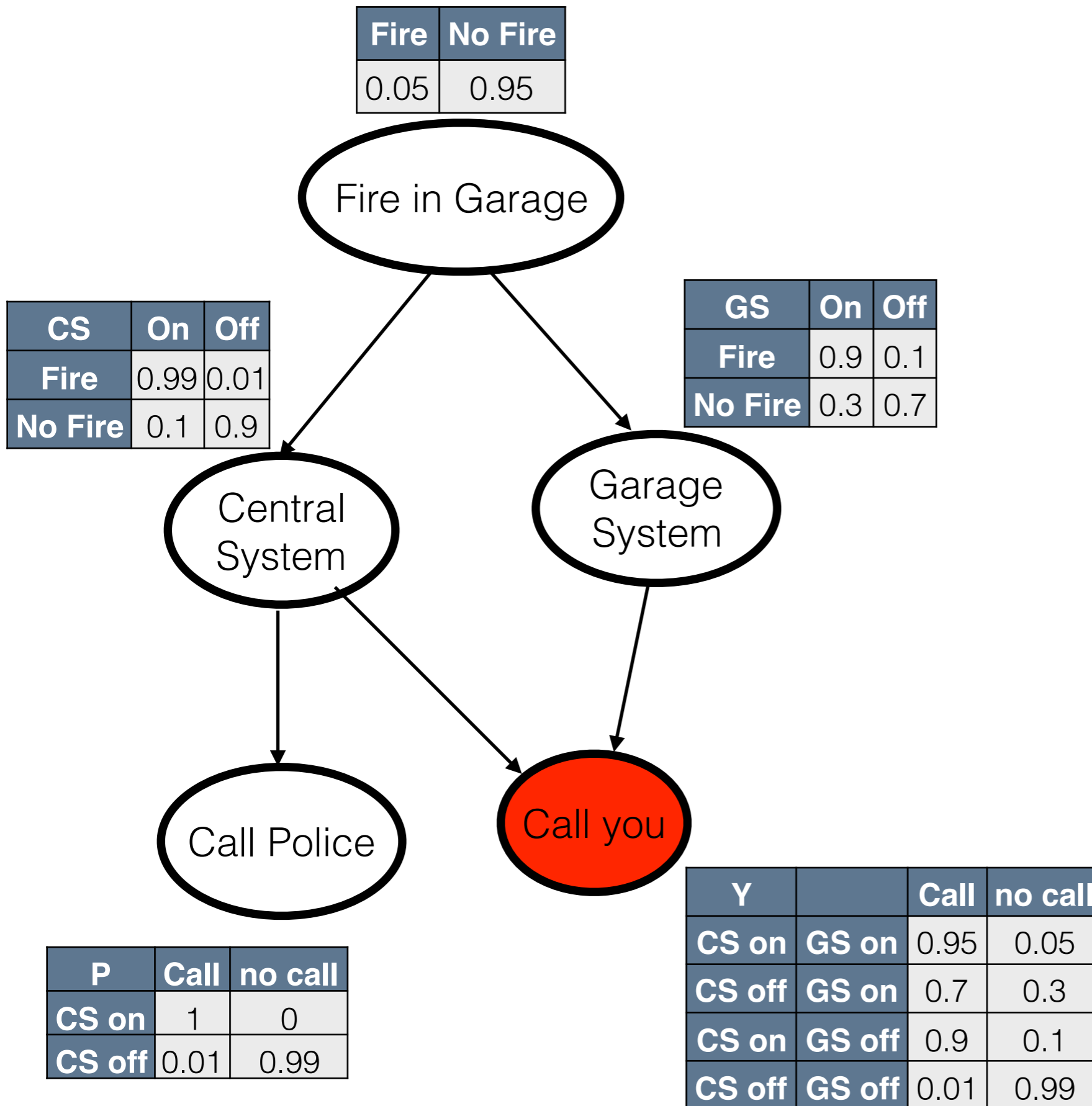
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



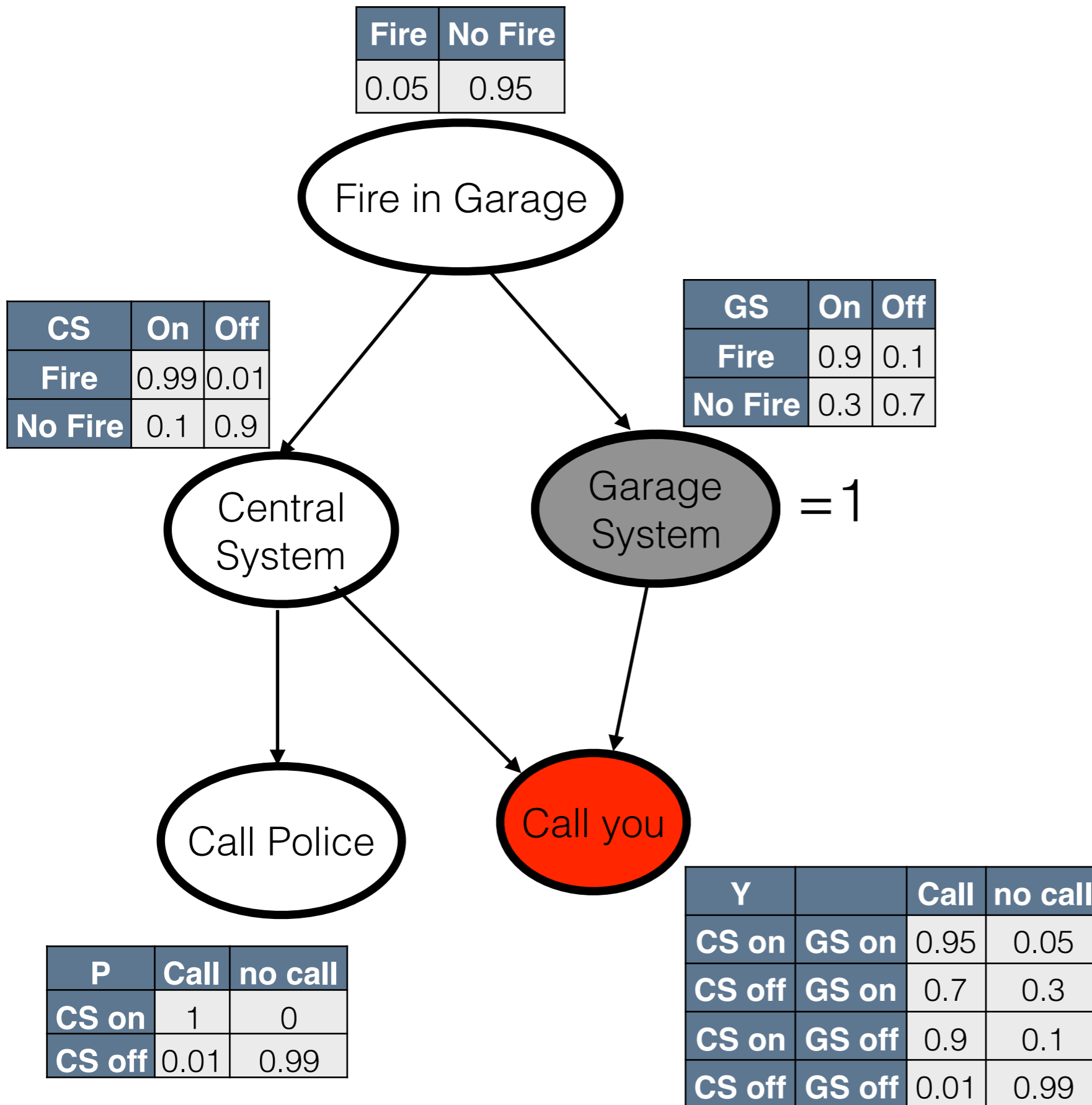
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



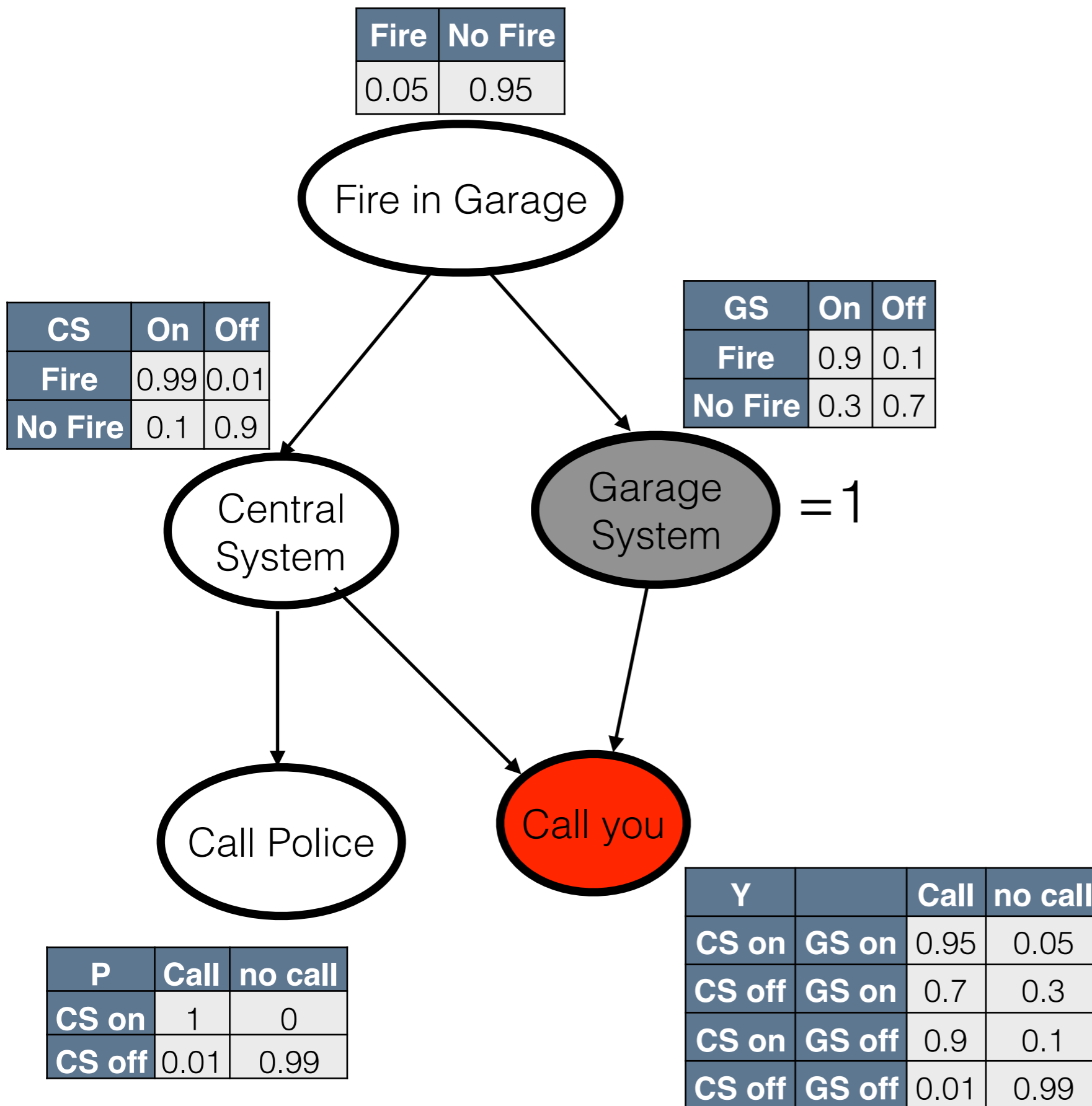
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



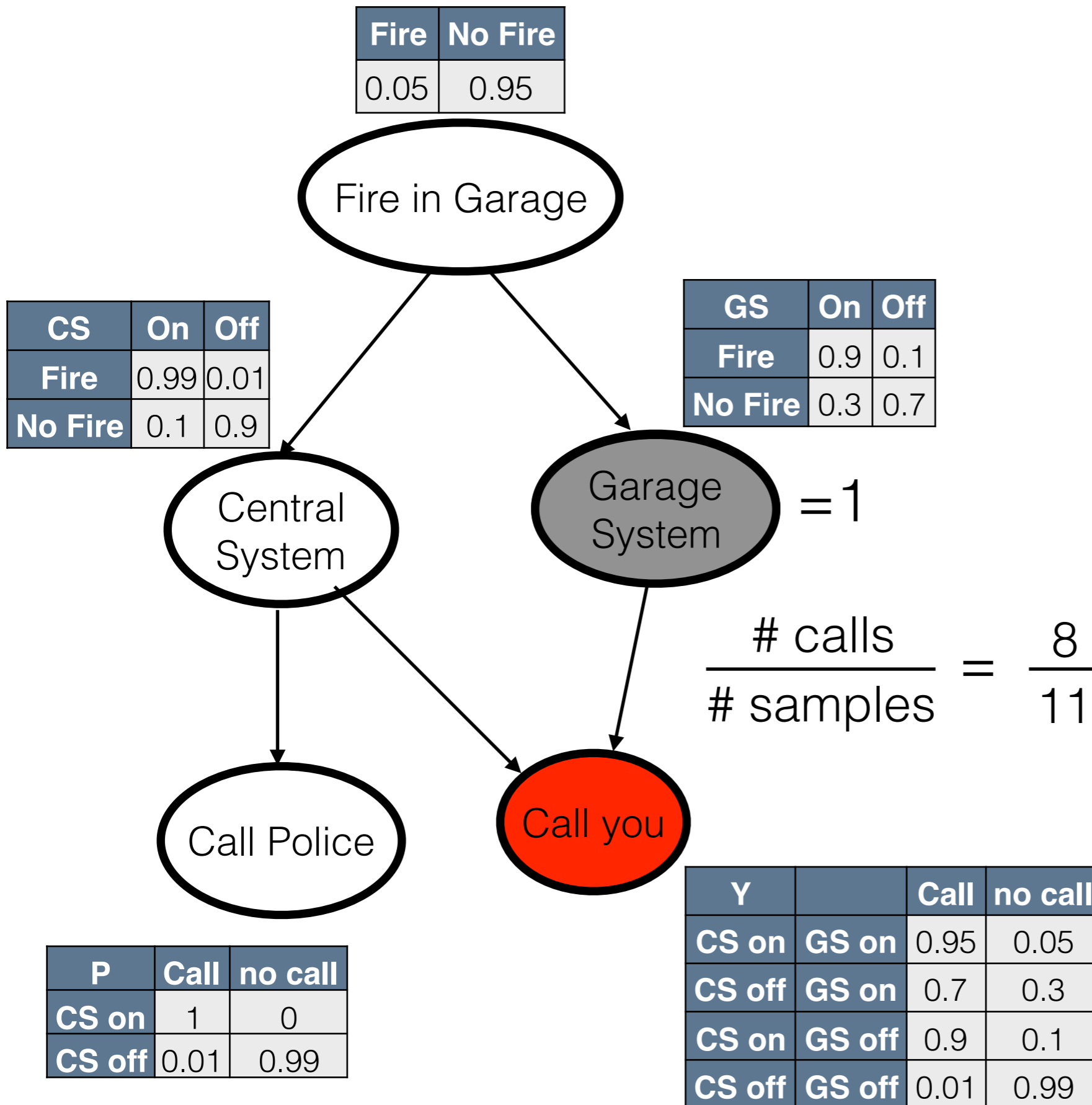
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



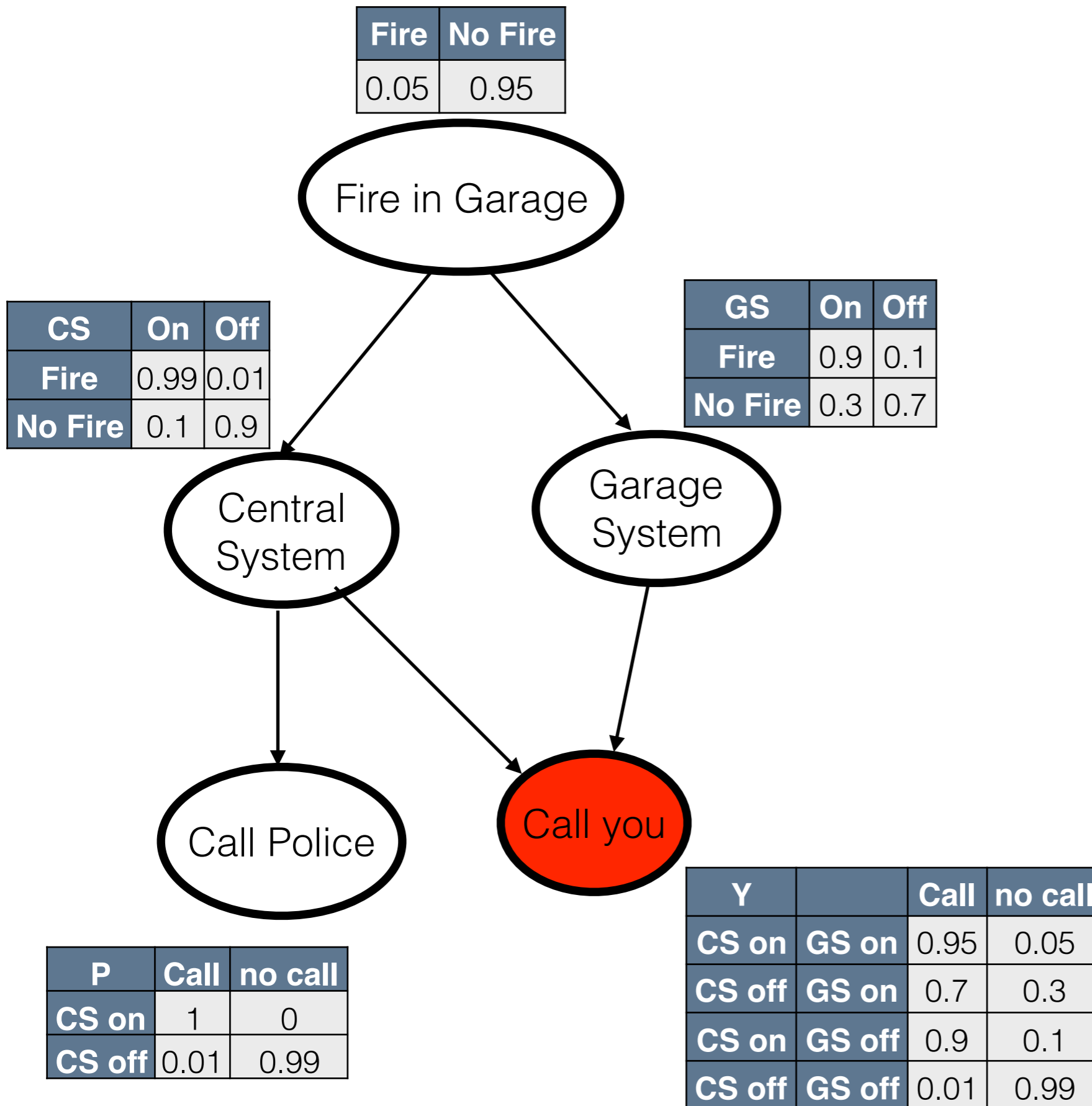
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



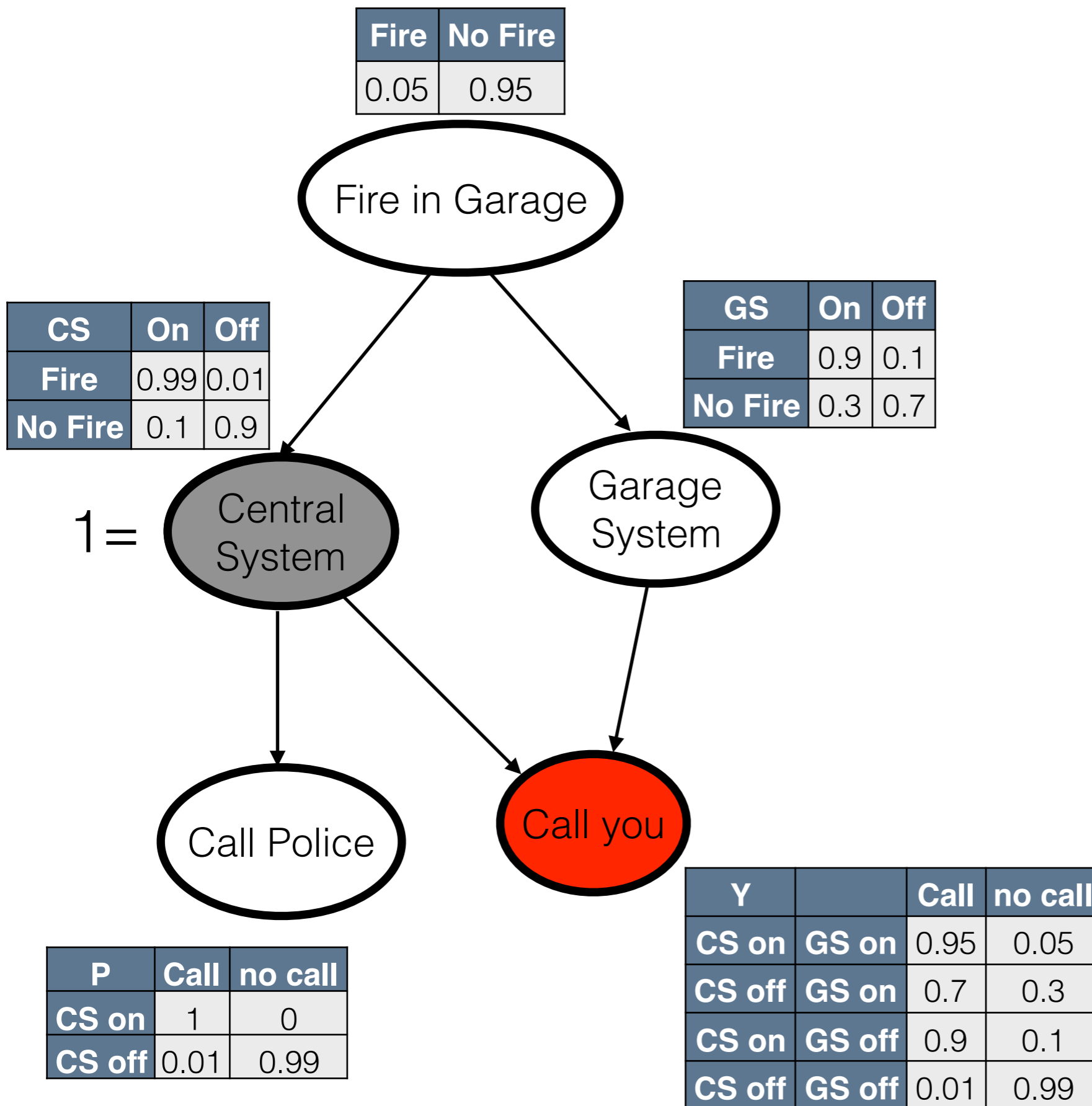
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



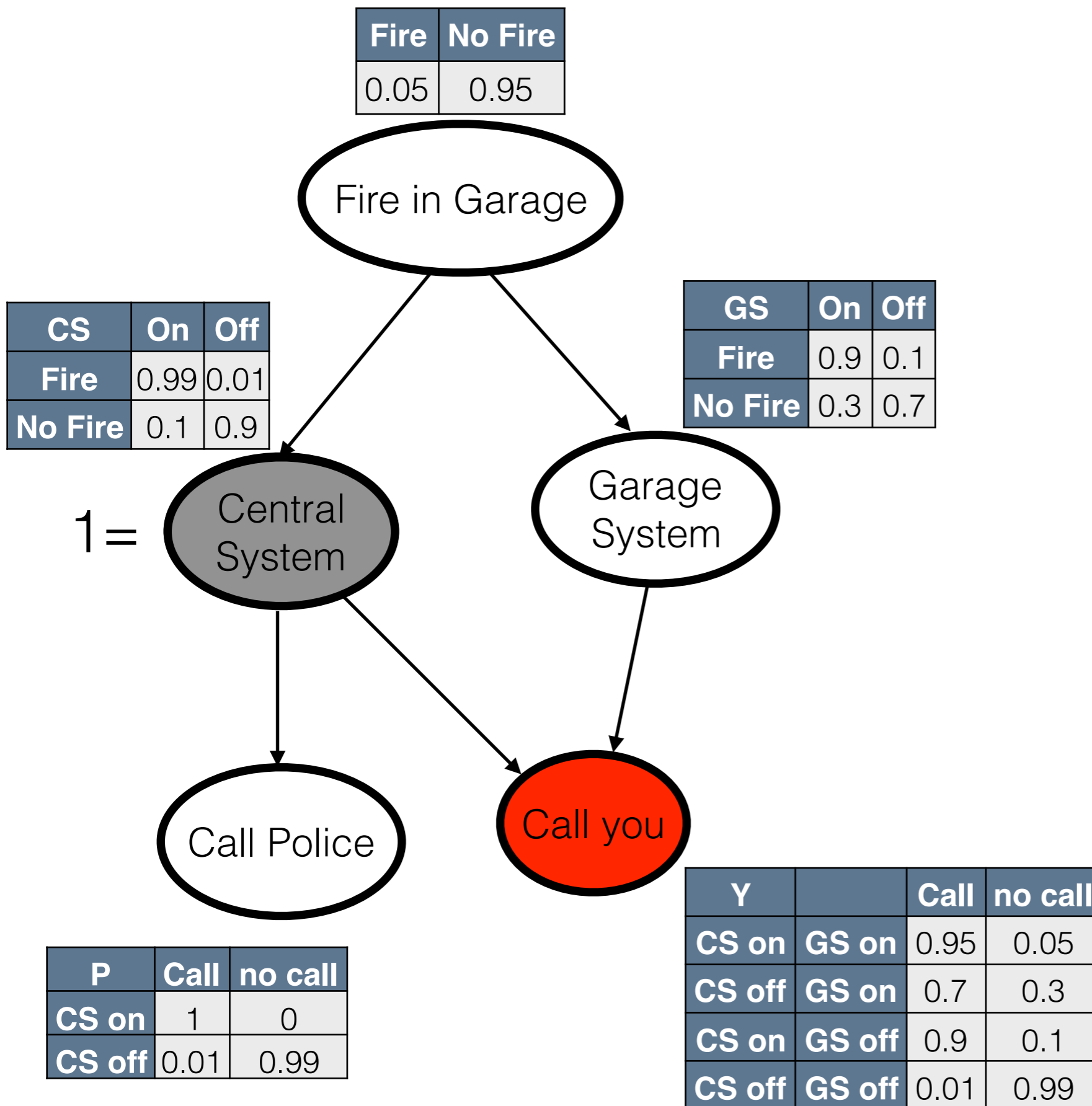
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



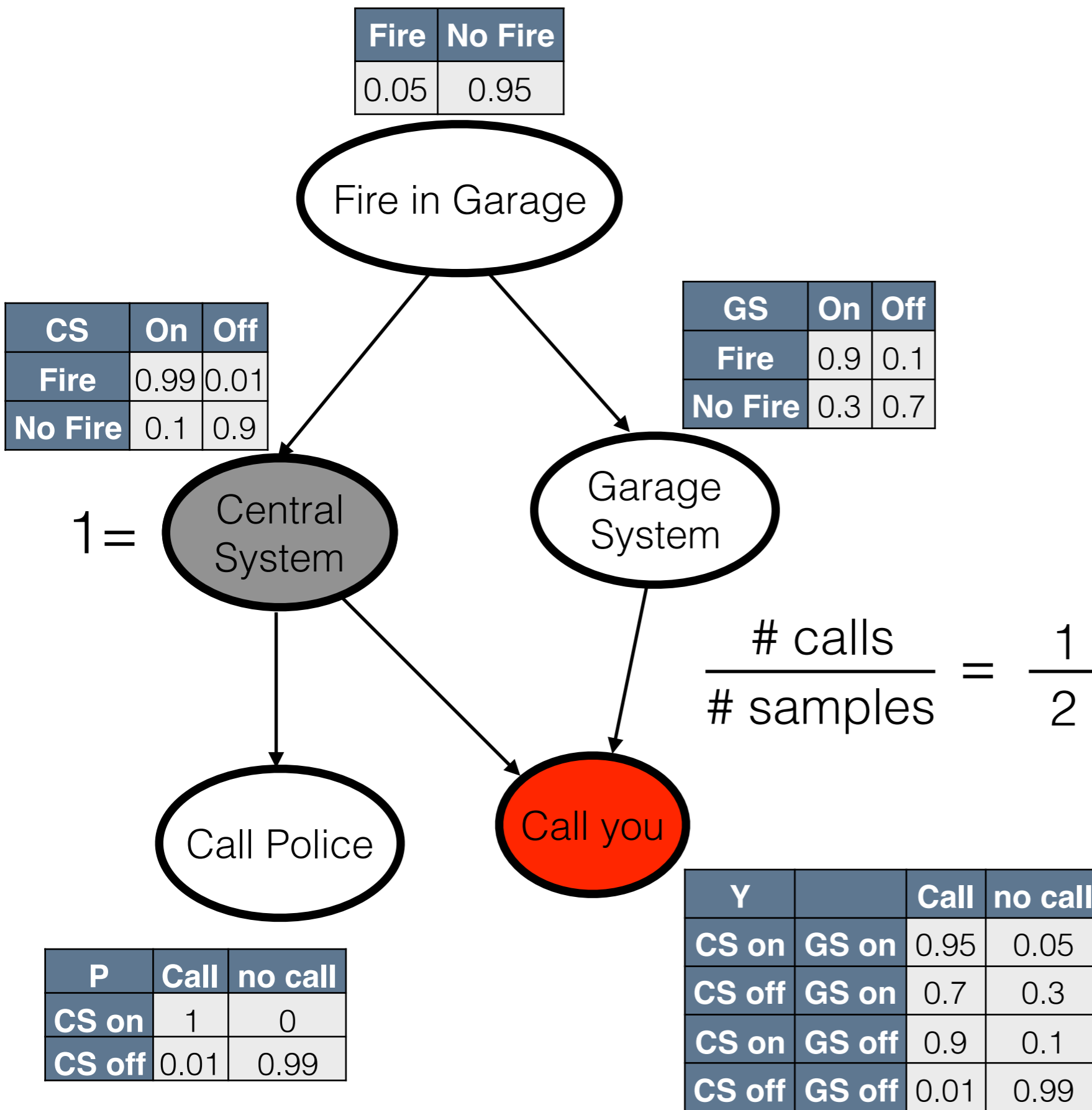
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



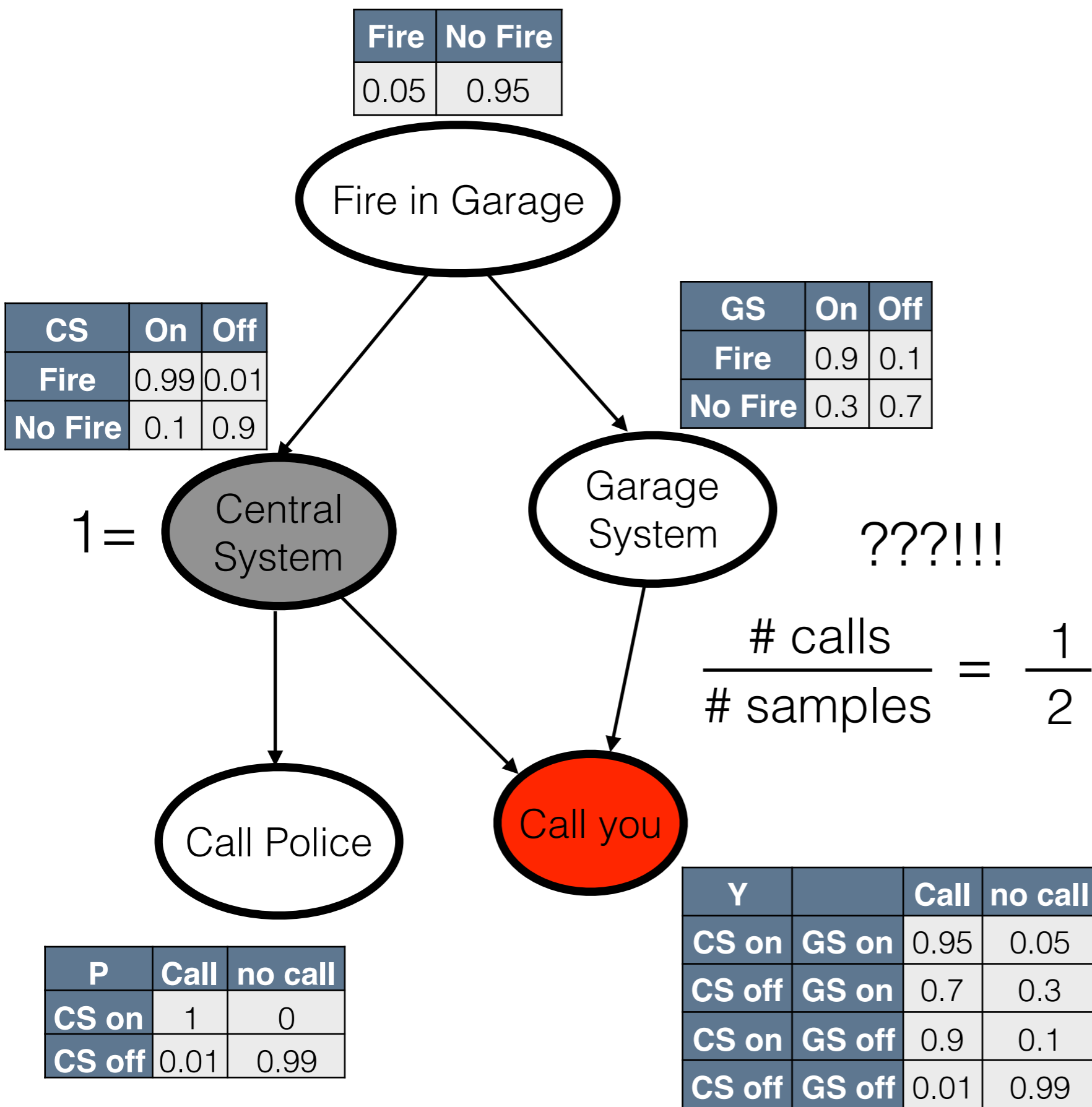
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



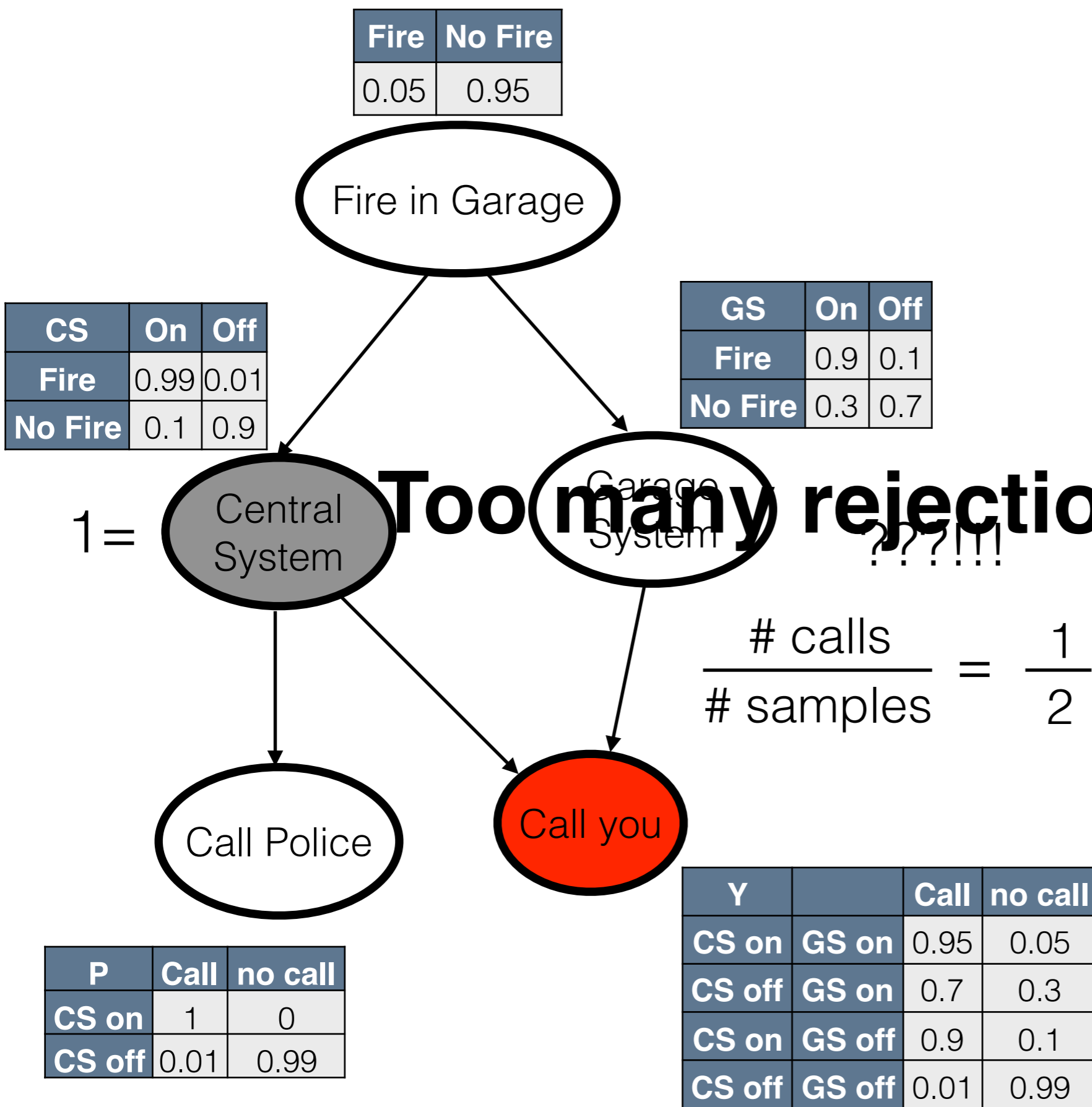
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



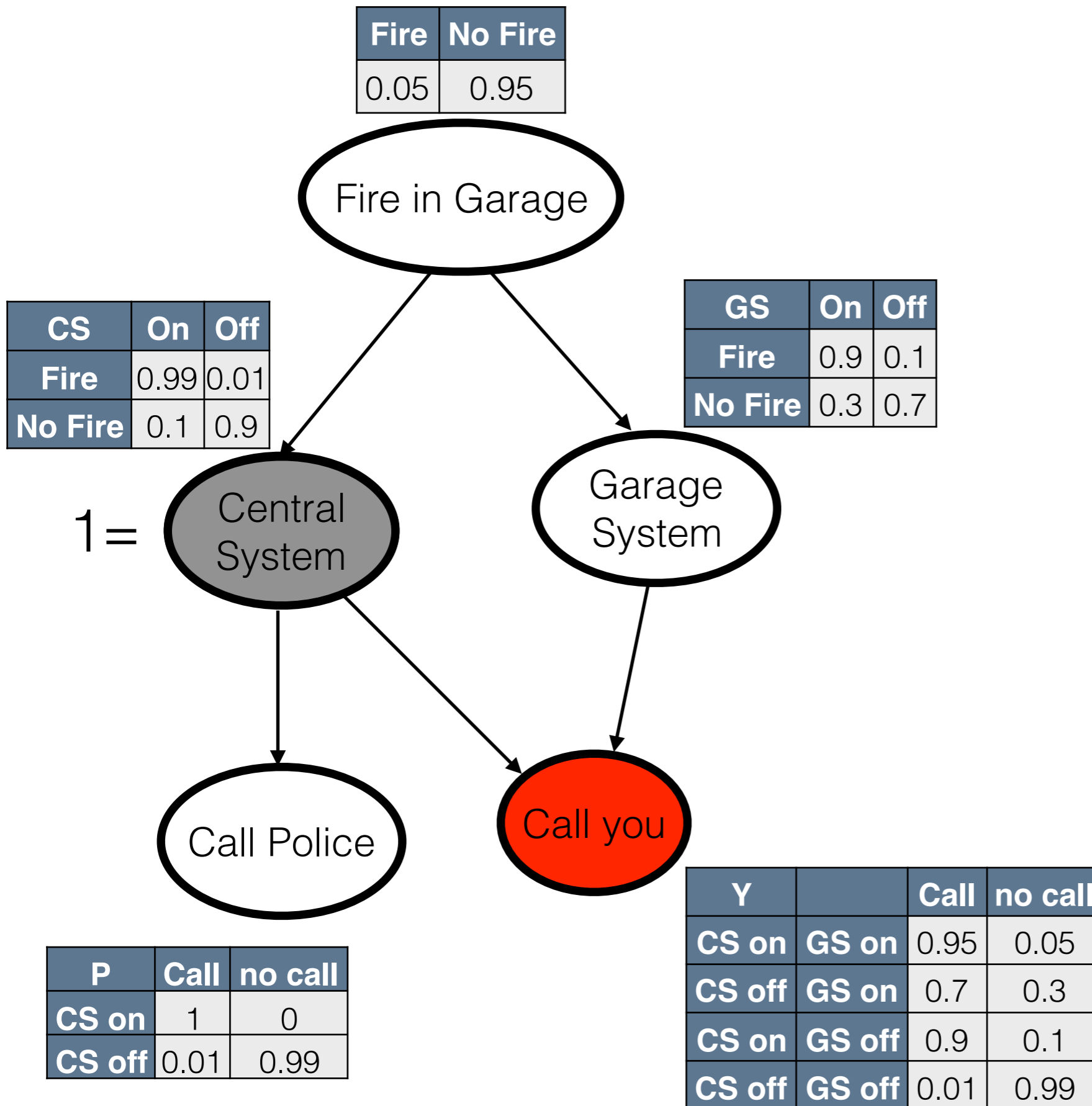
	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

REJECTION SAMPLING



	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

IMPORTANCE SAMPLING



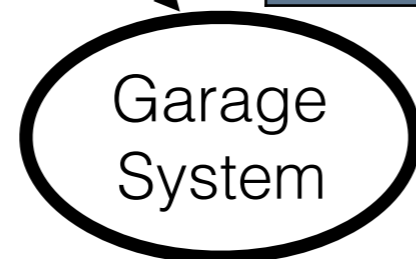
IMPORTANCE SAMPLING

If we could sample directly from distribution conditioned on $CS = 1$ it would be great!!!

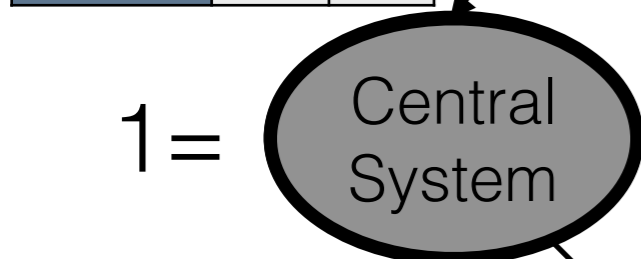
Fire	No Fire
0.05	0.95



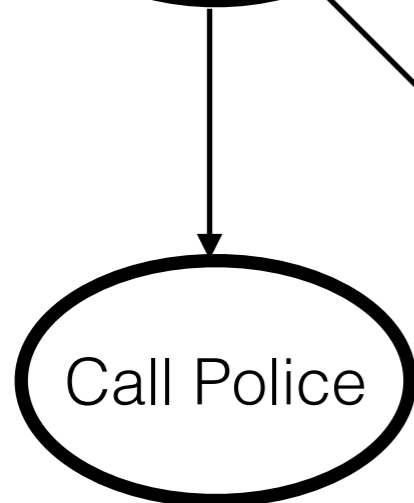
GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



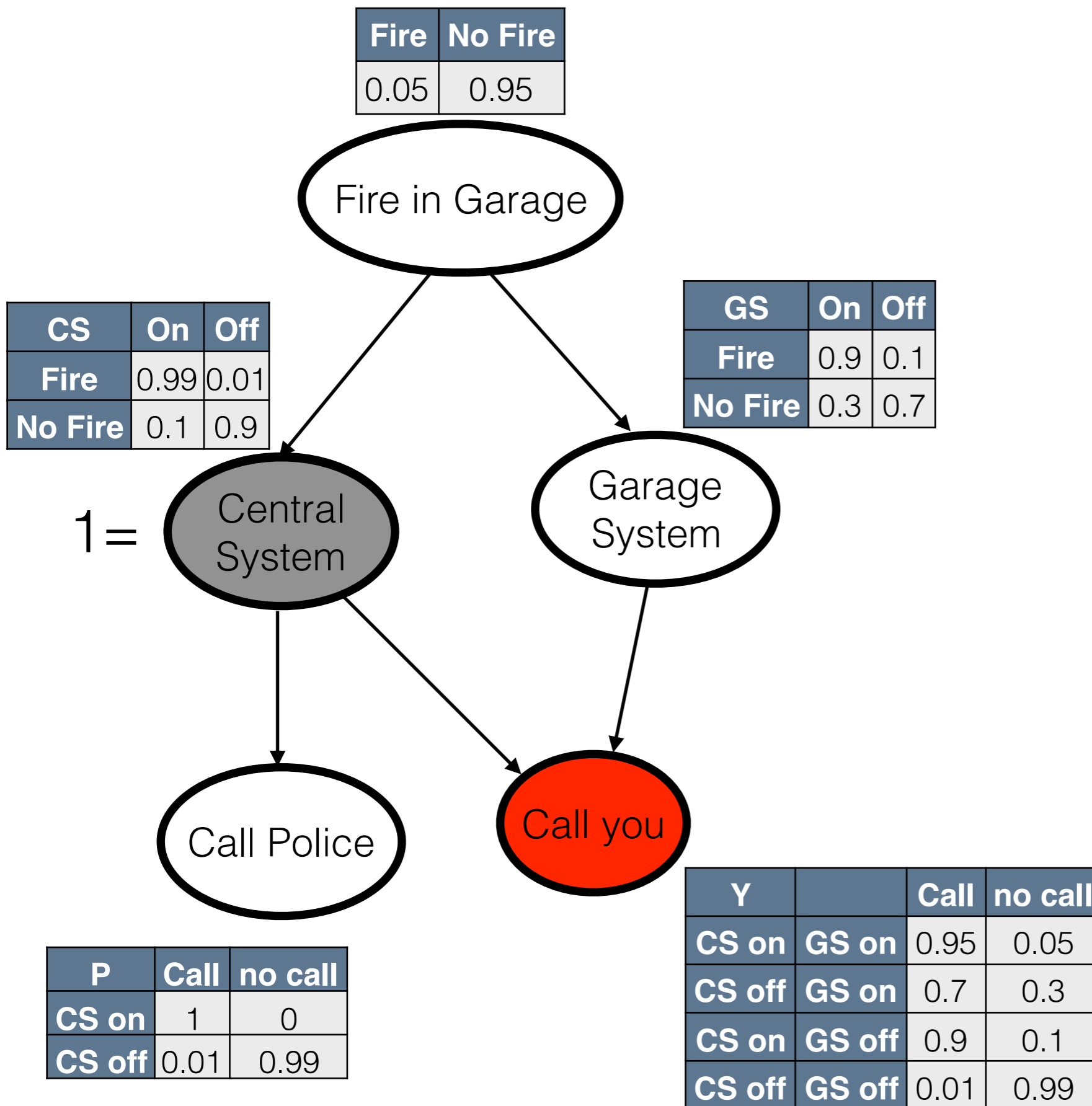
1 =



P	Call	no call
CS on	1	0
CS off	0.01	0.99

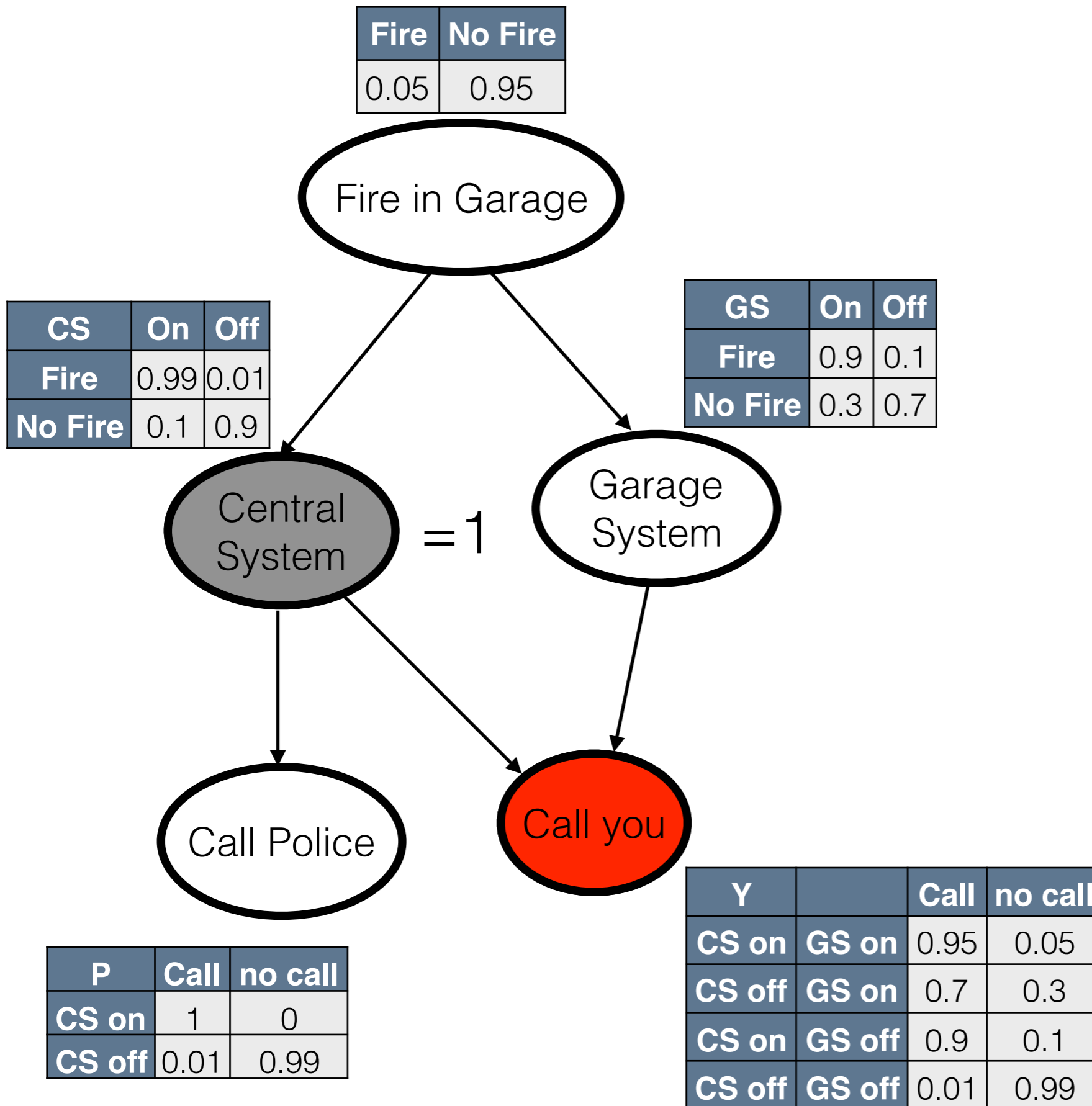
Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

IMPORTANCE SAMPLING



	F	CS	GS	P	Y
1	0	0	1	0	1
2	0	1	0	1	0
3	1	1	1	1	1
4	0	0	0	0	0
5	0	0	1	0	0
6	0	0	1	0	1
7	0	0	0	0	0
8	0	0	1	0	1
9	0	0	0	0	0
10	0	0	1	0	1
11	0	0	1	0	0
12	0	0	1	0	1
13	0	0	1	0	1
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	1	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	0	0	0	0	0
22	0	0	0	0	0
23	0	0	0	0	0
24	0	0	0	0	0
25	0	0	1	0	1

IMPORTANCE SAMPLING



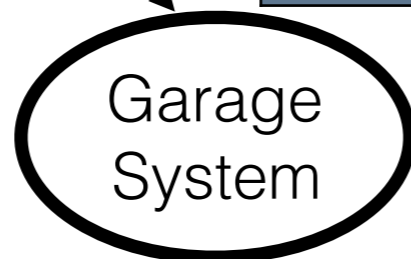
IMPORTANCE SAMPLING

	F	CS	GS	P	Y
1					

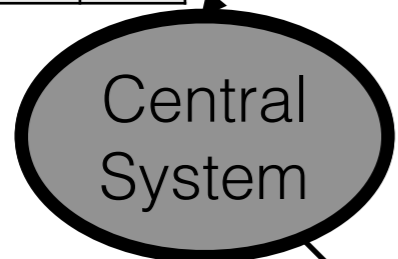
Fire	No Fire
0.05	0.95



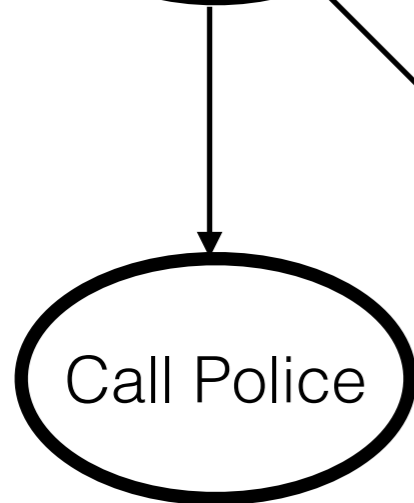
GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



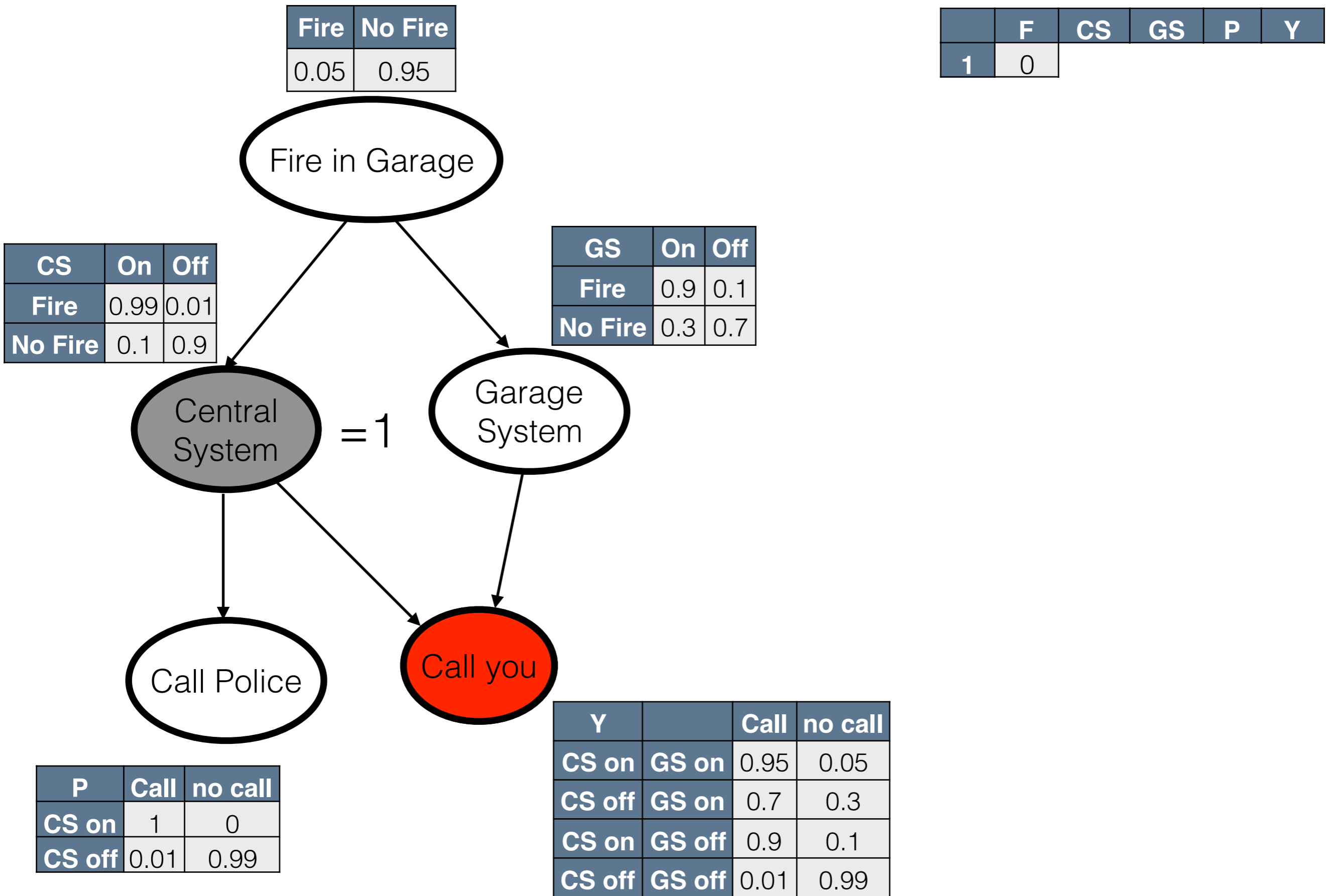
= 1



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

IMPORTANCE SAMPLING



IMPORTANCE SAMPLING

	F	CS	GS	P	Y
1	0	1			

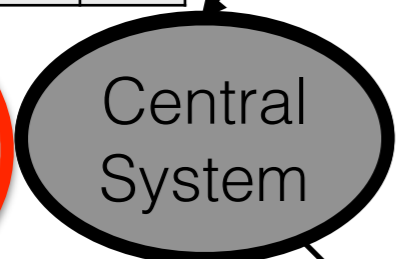
Fire	No Fire
0.05	0.95



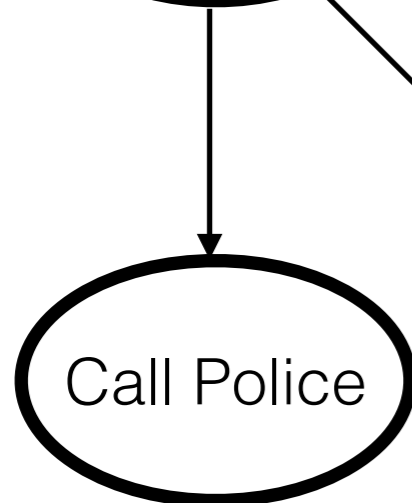
GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



= 1



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

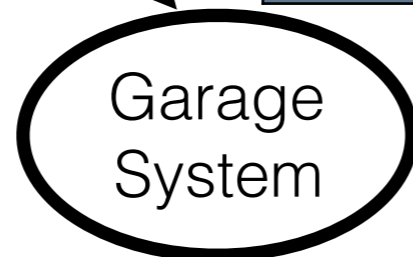
IMPORTANCE SAMPLING

	F	CS	GS	P	Y
1	0	1	0		

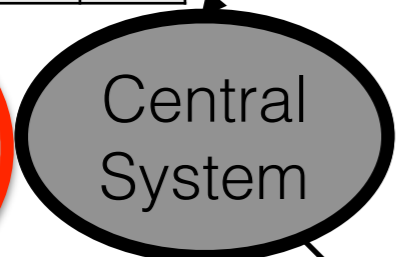
Fire	No Fire
0.05	0.95



GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



= 1



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

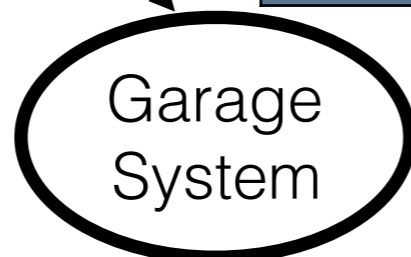
IMPORTANCE SAMPLING

	F	CS	GS	P	Y
1	0	1	0	1	

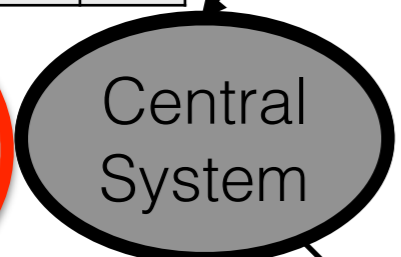
Fire	No Fire
0.05	0.95



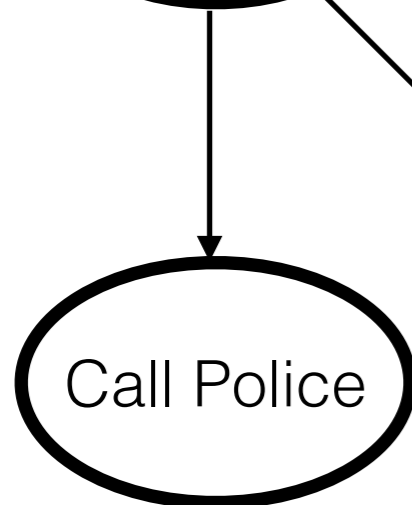
GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



= 1



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

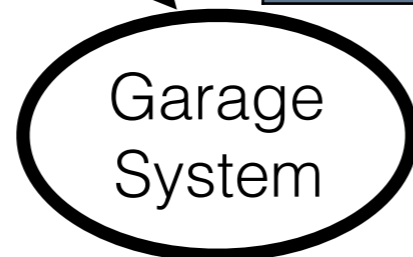
IMPORTANCE SAMPLING

	F	CS	GS	P	Y
1	0	1	0	1	0

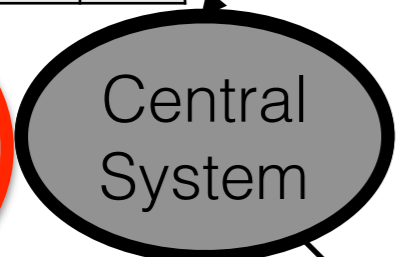
Fire	No Fire
0.05	0.95



GS	On	Off
Fire	0.9	0.1
No Fire	0.3	0.7



CS	On	Off
Fire	0.99	0.01
No Fire	0.1	0.9



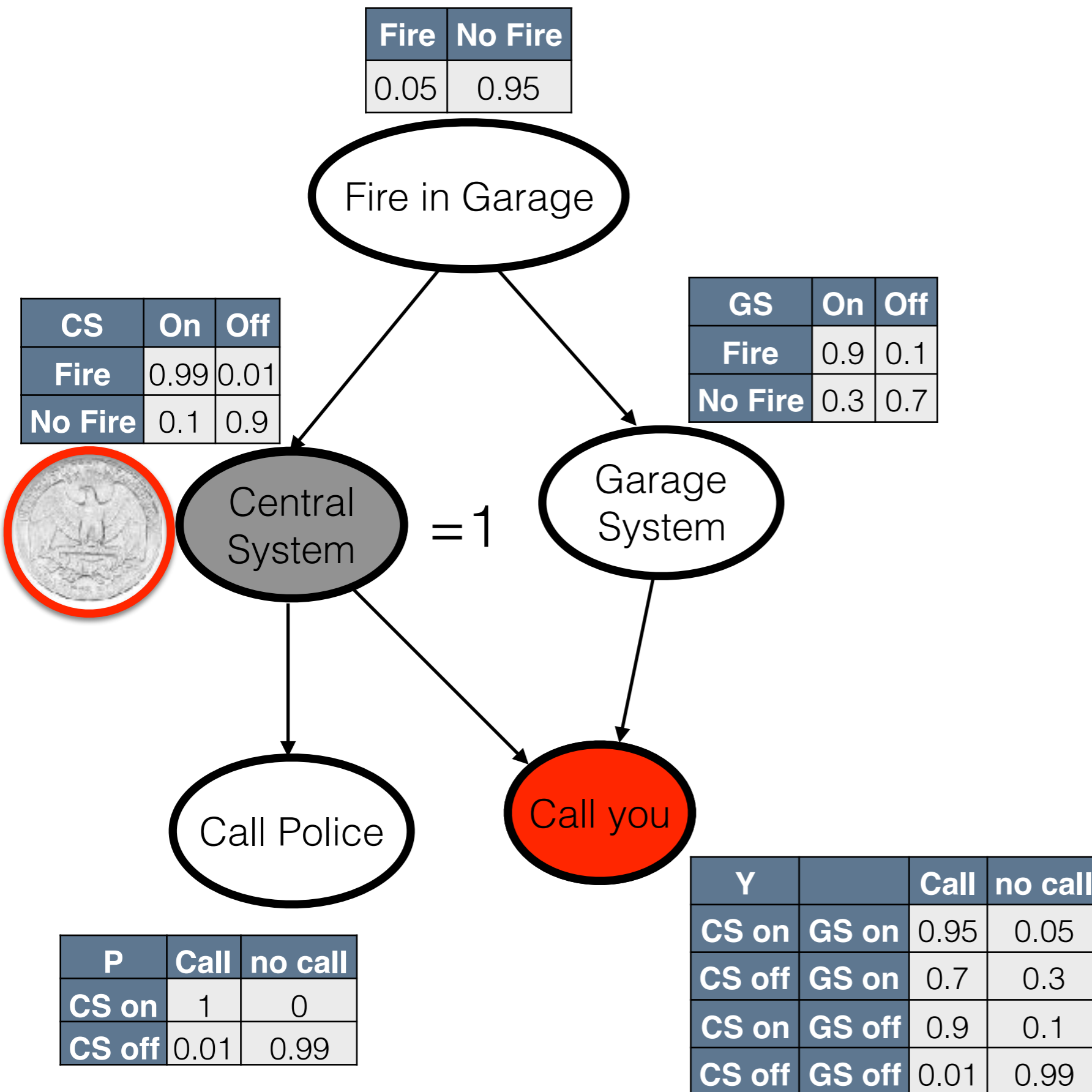
= 1



P	Call	no call
CS on	1	0
CS off	0.01	0.99

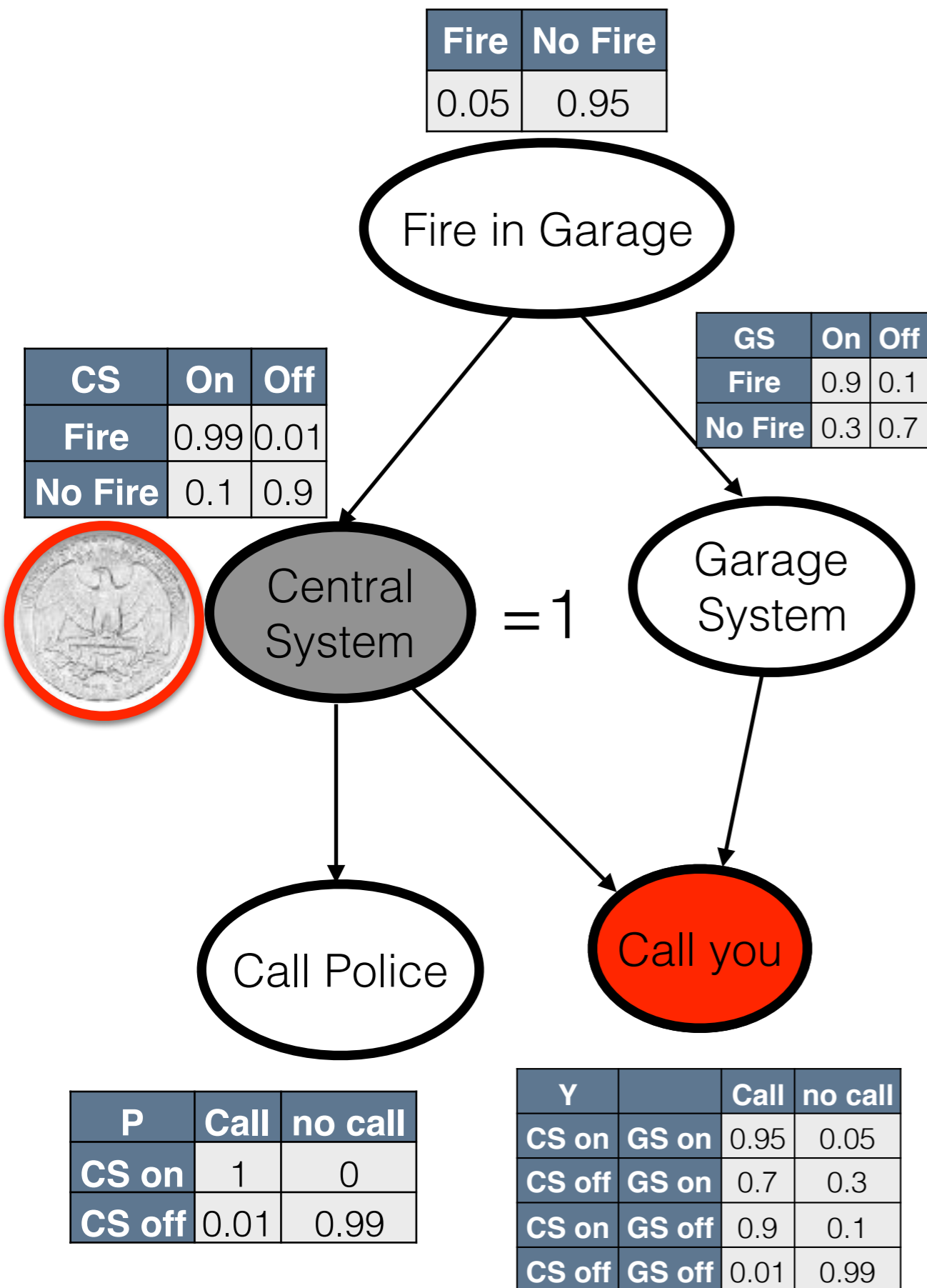
Y		Call	no call
CS on	GS on	0.95	0.05
CS off	GS on	0.7	0.3
CS on	GS off	0.9	0.1
CS off	GS off	0.01	0.99

IMPORTANCE SAMPLING



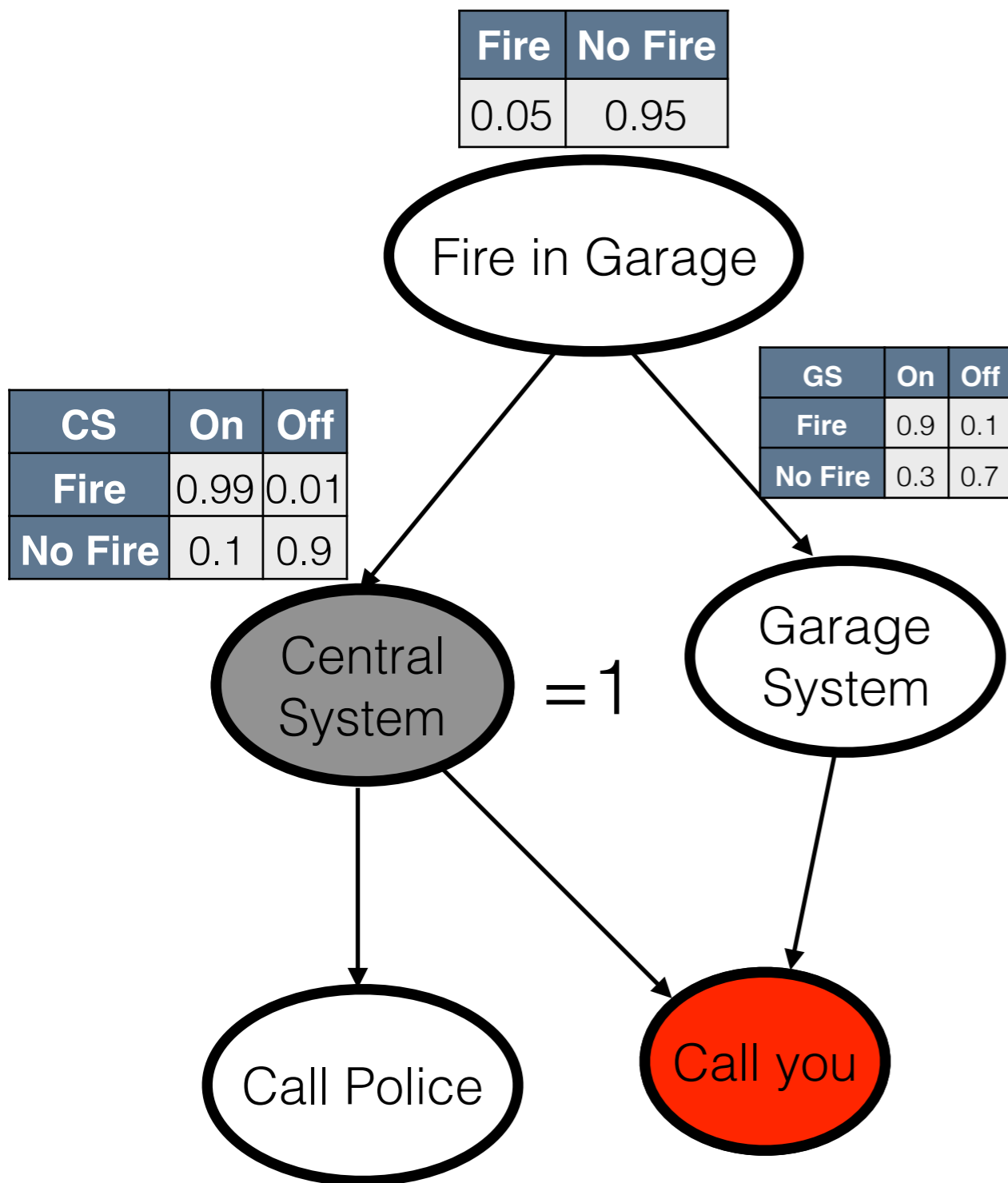
	F	CS	GS	P	Y
1	0	1	0	1	0
2	0	1	1	1	1
3	1	1	1	1	1
4	0	1	0	1	1
5	0	1	0	1	1
6	0	1	0	1	1
7	0	1	1	1	1
8	0	1	0	1	0
9	0	1	0	1	1
10	0	1	1	1	1
11	0	1	0	1	1
12	0	1	1	1	0
13	0	1	0	1	1
14	0	1	1	1	1
15	0	1	0	1	1
16	0	1	0	1	1
17	0	1	0	1	1
18	0	1	0	1	1
19	0	1	1	1	1
20	0	1	0	1	1
21	0	1	0	1	1
22	0	1	0	1	1
23	0	1	1	1	1
24	0	1	0	1	1
25	0	1	1	1	1

IMPORTANCE SAMPLING



	F	CS	GS	P	Y
1	0	1	0	1	0
2	0	1	1	1	1
3	1	1	1	1	1
4	0	1	0	1	1
5	0	1	0	1	1
6	0	1	0	1	1
7	0	1	1	1	1
8	0	1	0	1	0
9	0	1	0	1	1
10	0	1	1	1	1
11	0	1	0	1	1
12	0	1	1	1	0
13	0	1	0	1	1
14	0	1	1	1	1
15	0	1	0	1	1
16	0	1	0	1	1
17	0	1	0	1	1
18	0	1	0	1	1
19	0	1	1	1	1
20	0	1	0	1	1
21	0	1	0	1	1
22	0	1	0	1	1
23	0	1	1	1	1
24	0	1	0	1	1
25	0	1	1	1	1

IMPORTANCE SAMPLING

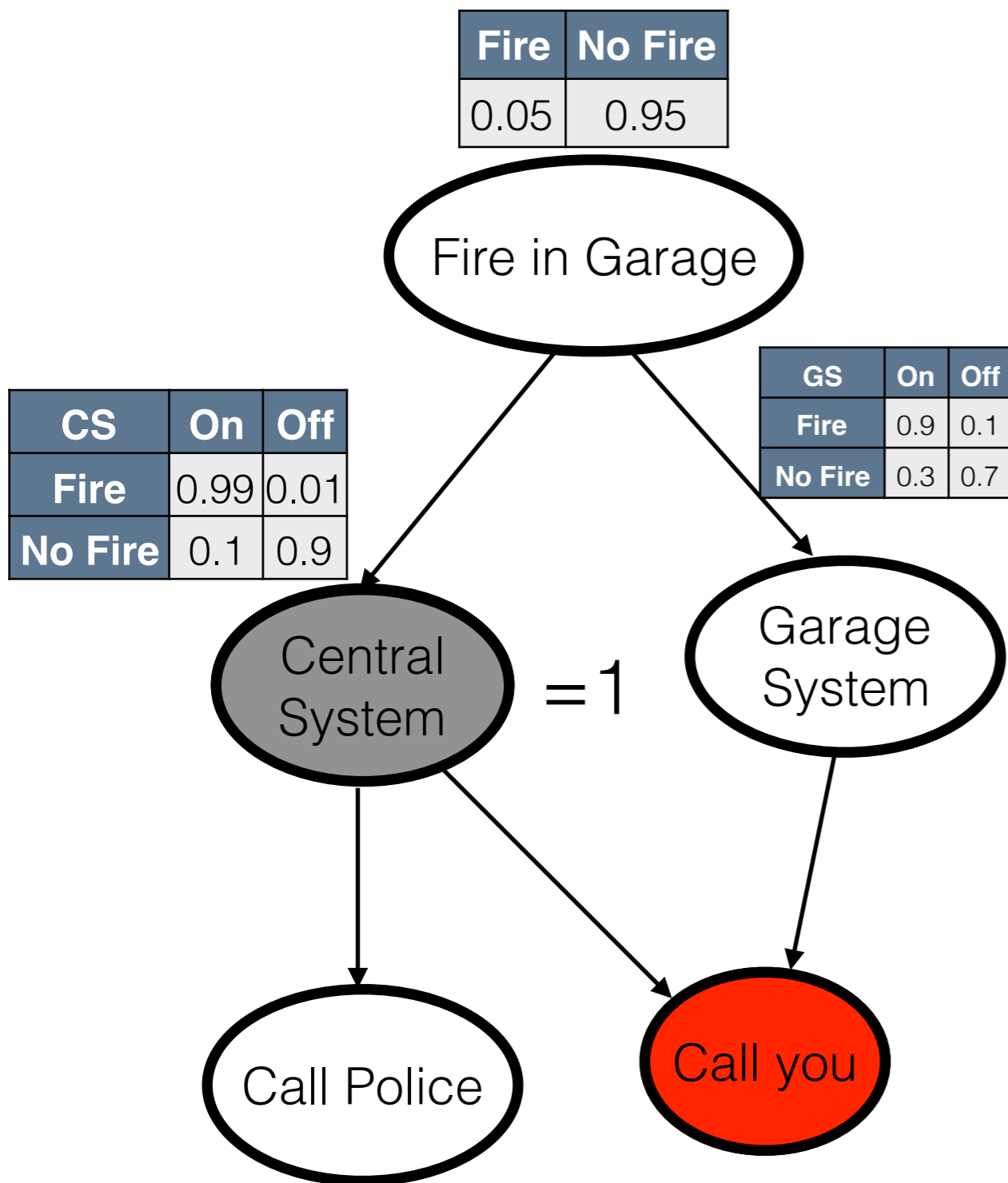


P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no
CS	GS	0.95	0.05
CS	GS	0.7	0.3
CS	GS	0.9	0.1
CS	GS	0.01	0.99

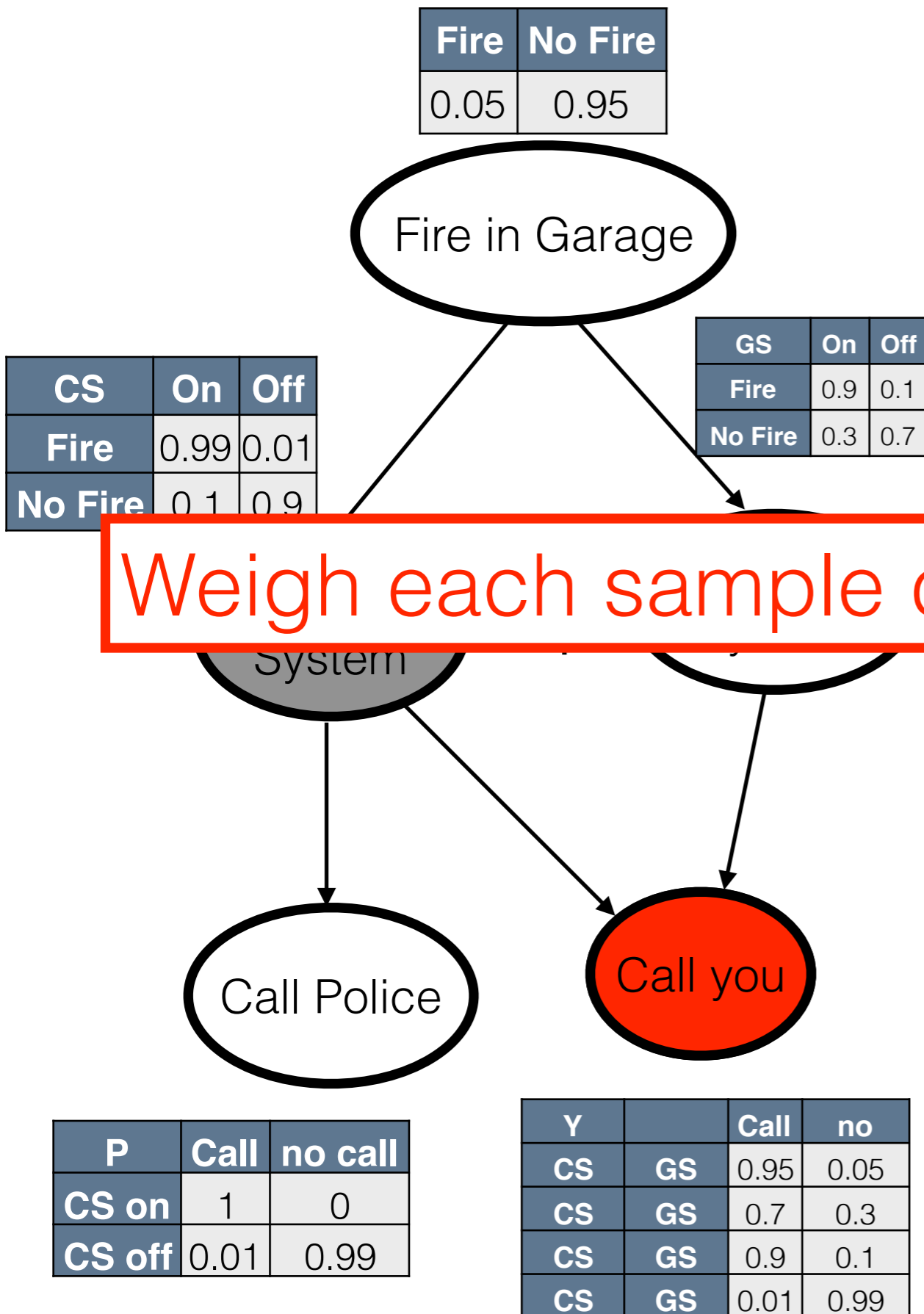
	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	
2	0	1	1	1	1	
3	1	1	1	1	1	
4	0	1	0	1	1	
5	0	1	0	1	1	
6	0	1	0	1	1	
7	0	1	1	1	1	
8	0	1	0	1	0	
9	0	1	0	1	1	
10	0	1	1	1	1	
11	0	1	0	1	1	
12	0	1	1	1	0	
13	0	1	0	1	1	
14	0	1	1	1	1	
15	0	1	0	1	1	
16	0	1	0	1	1	
17	0	1	0	1	1	
18	0	1	0	1	1	
19	0	1	1	1	1	
20	0	1	0	1	1	
21	0	1	0	1	1	
22	0	1	0	1	1	
23	0	1	1	1	1	
24	0	1	0	1	1	
25	0	1	1	1	1	

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	
2	0	1	1	1	1	
3	1	1	1	1	1	
4	0	1	0	1	1	
5	0	1	0	1	1	
6	0	1	0	1	1	
7	0	1	1	1	1	
8	0	1	0	1	0	
9	0	1	0	1	1	
10	0	1	1	1	1	
11	0	1	0	1	1	
12	0	1	1	1	0	
13	0	1	0	1	1	
14	0	1	1	1	1	
15	0	1	0	1	1	
16	0	1	0	1	1	
17	0	1	0	1	1	
18	0	1	0	1	1	
19	0	1	1	1	1	
20	0	1	0	1	1	
21	0	1	0	1	1	
22	0	1	0	1	1	
23	0	1	1	1	1	
24	0	1	0	1	1	
25	0	1	1	1	1	

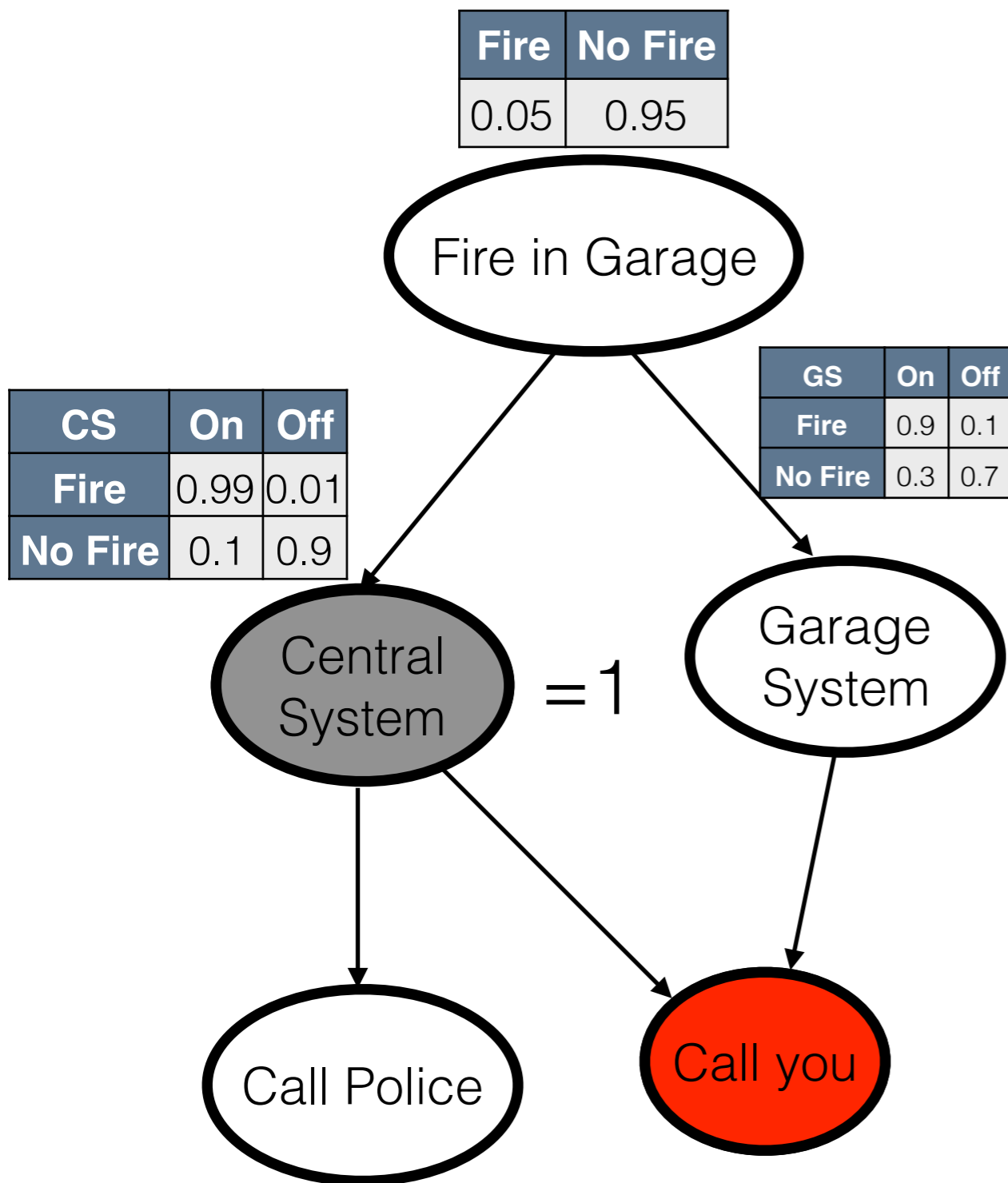
IMPORTANCE SAMPLING



Weigh each sample differently!

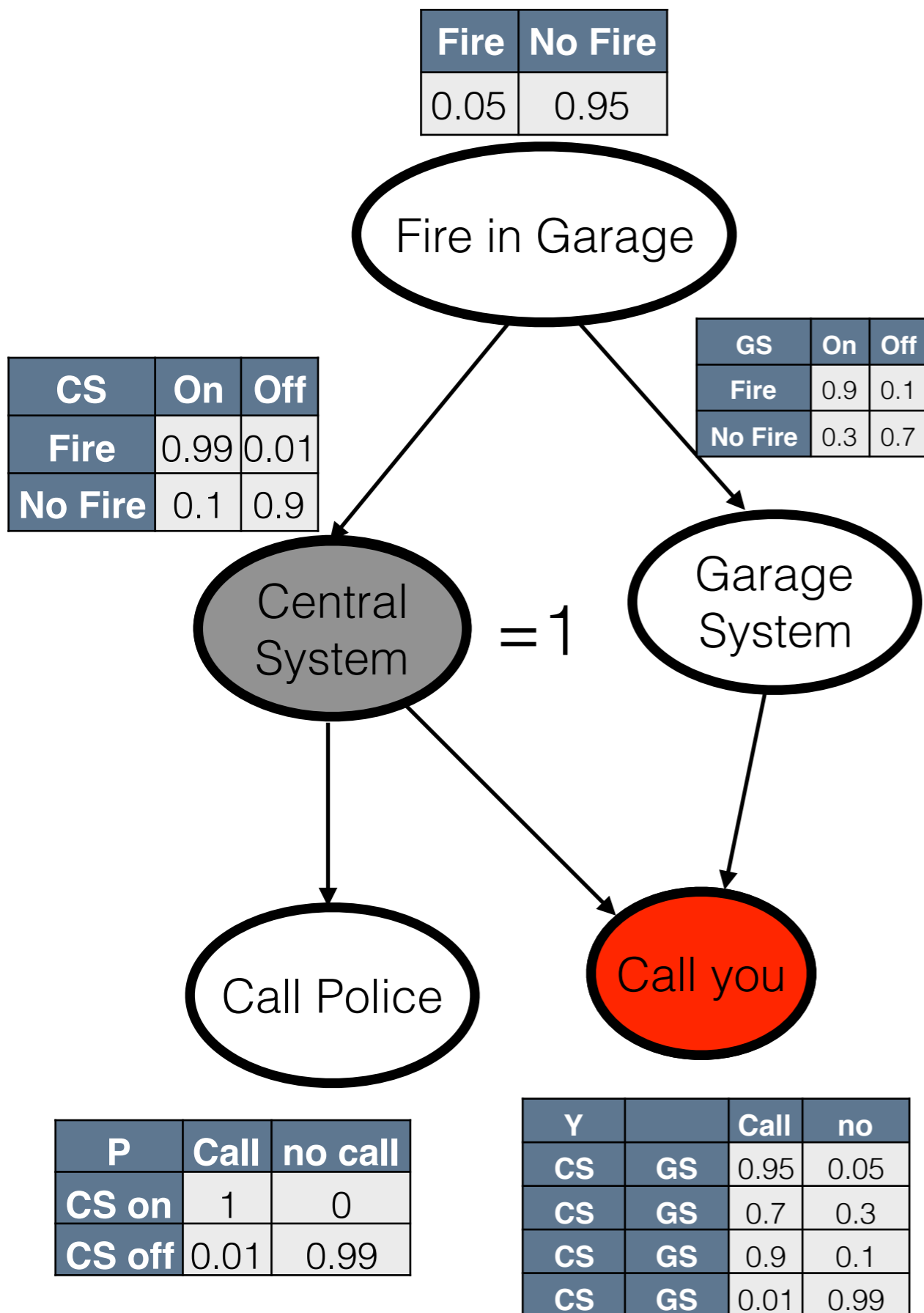
	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	
2	0	1	1	1	1	
3	1	1	1	1	1	
4	0	1	0	1	1	
5	0	1	0	1	1	
6	0	1	0	1	1	
7	0	1	1	1	1	
8	0	1	0	1	0	
9	0	1	0	1	1	
10	0	1	1	1	1	
11	0	1	0	1	1	
12	0	1	1	1	0	
13	0	1	0	1	1	
14	0	1	1	1	1	
15	0	1	0	1	1	
16	0	1	0	1	1	
17	0	1	0	1	1	
18	0	1	0	1	1	
19	0	1	1	1	1	
20	0	1	0	1	1	
21	0	1	0	1	1	
22	0	1	0	1	1	
23	0	1	1	1	1	
24	0	1	0	1	1	
25	0	1	1	1	1	

IMPORTANCE SAMPLING



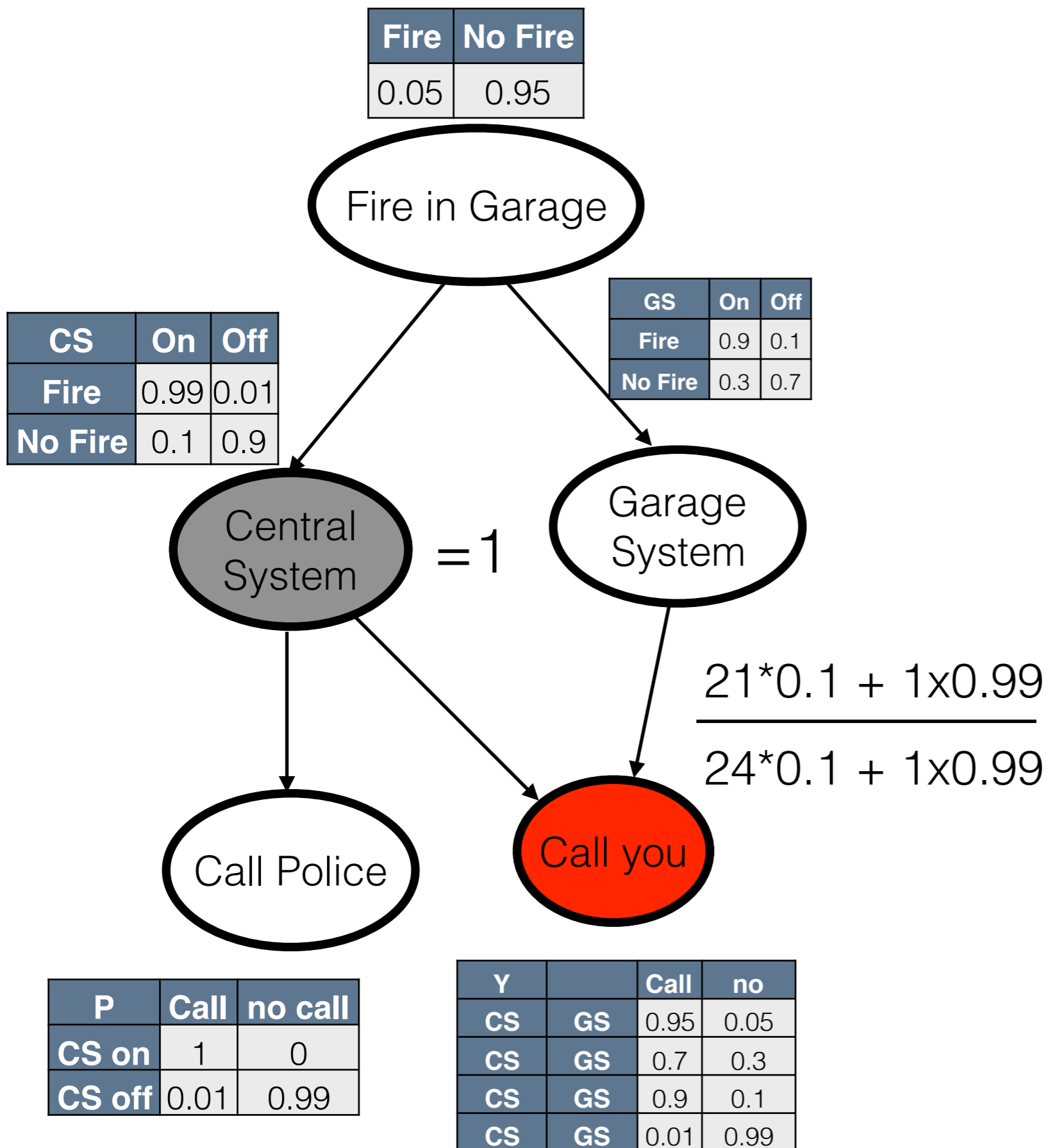
	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	0.1
2	0	1	1	1	1	0.1
3	1	1	1	1	1	0.99
4	0	1	0	1	1	0.1
5	0	1	0	1	1	0.1
6	0	1	0	1	1	0.1
7	0	1	1	1	1	0.1
8	0	1	0	1	0	0.1
9	0	1	0	1	1	0.1
10	0	1	1	1	1	0.1
11	0	1	0	1	1	0.1
12	0	1	1	1	0	0.1
13	0	1	0	1	1	0.1
14	0	1	1	1	1	0.1
15	0	1	0	1	1	0.1
16	0	1	0	1	1	0.1
17	0	1	0	1	1	0.1
18	0	1	0	1	1	0.1
19	0	1	1	1	1	0.1
20	0	1	0	1	1	0.1
21	0	1	0	1	1	0.1
22	0	1	0	1	1	0.1
23	0	1	1	1	1	0.1
24	0	1	0	1	1	0.1
25	0	1	1	1	1	0.1

IMPORTANCE SAMPLING



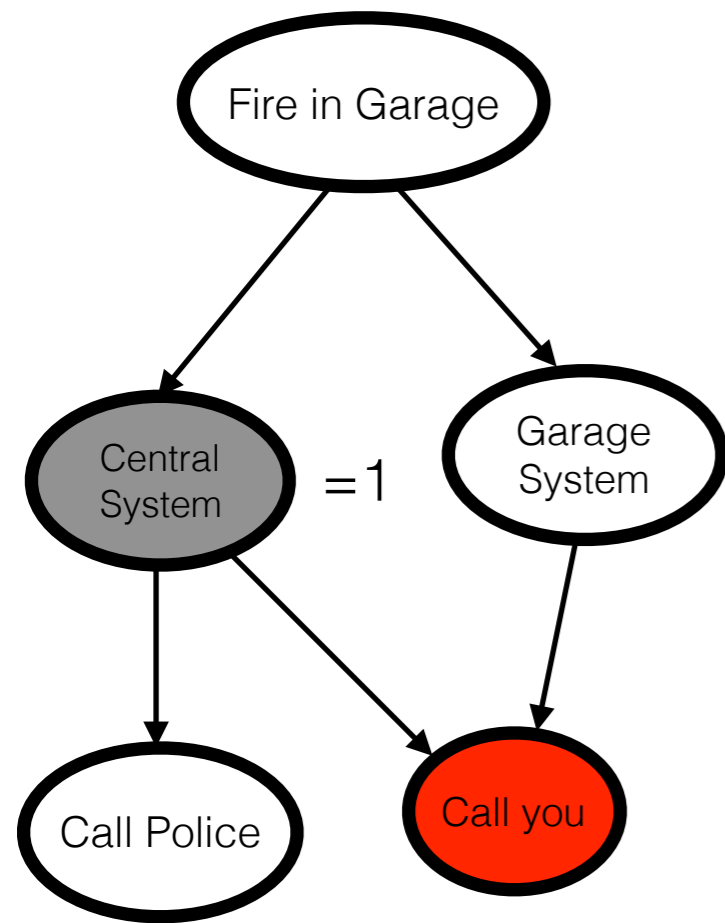
	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	0.1
2	0	1	1	1	1	0.1
3	1	1	1	1	1	0.99
4	0	1	0	1	1	0.1
5	0	1	0	1	1	0.1
6	0	1	0	1	1	0.1
7	0	1	1	1	1	0.1
8	0	1	0	1	0	0.1
9	0	1	0	1	1	0.1
10	0	1	1	1	1	0.1
11	0	1	0	1	1	0.1
12	0	1	1	1	0	0.1
13	0	1	0	1	1	0.1
14	0	1	1	1	1	0.1
15	0	1	0	1	1	0.1
16	0	1	0	1	1	0.1
17	0	1	0	1	1	0.1
18	0	1	0	1	1	0.1
19	0	1	1	1	1	0.1
20	0	1	0	1	1	0.1
21	0	1	0	1	1	0.1
22	0	1	0	1	1	0.1
23	0	1	1	1	1	0.1
24	0	1	0	1	1	0.1
25	0	1	1	1	1	0.1

IMPORTANCE SAMPLING

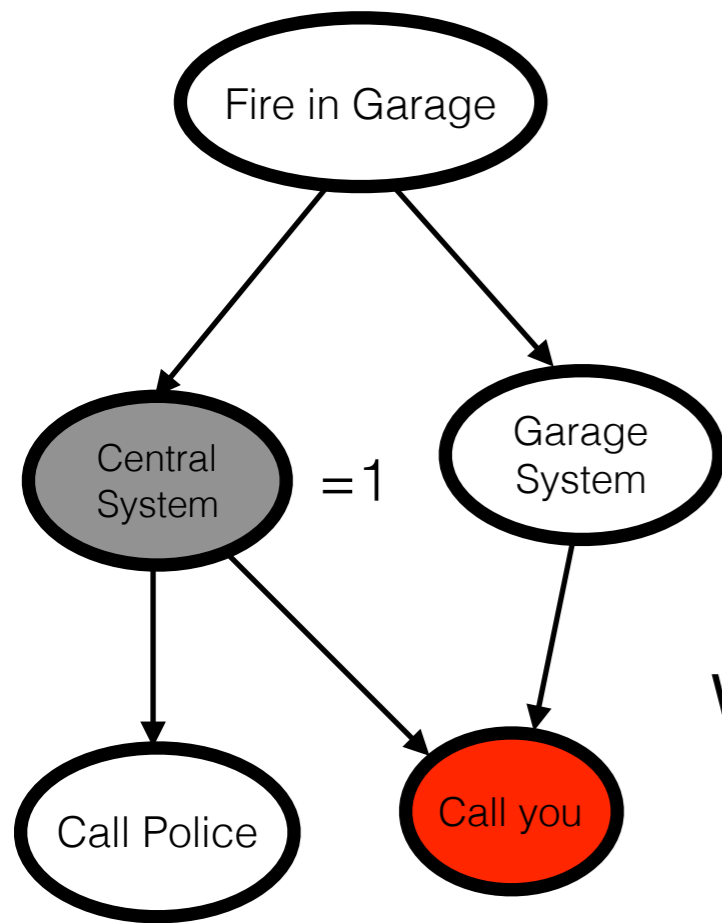


	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	0.1
2	0	1	1	1	1	0.1
3	1	1	1	1	1	0.99
4	0	1	0	1	1	0.1
5	0	1	0	1	1	0.1
6	0	1	0	1	1	0.1
7	0	1	1	1	1	0.1
8	0	1	0	1	0	0.1
9	0	1	0	1	1	0.1
10	0	1	1	1	1	0.1
11	0	1	0	1	1	0.1
12	0	1	1	1	0	0.1
13	0	1	0	1	1	0.1
14	0	1	1	1	1	0.1
15	0	1	0	1	1	0.1
16	0	1	0	1	1	0.1
17	0	1	0	1	1	0.1
18	0	1	0	1	1	0.1
19	0	1	1	1	1	0.1
20	0	1	0	1	1	0.1
21	0	1	0	1	1	0.1
22	0	1	0	1	1	0.1
23	0	1	1	1	1	0.1
24	0	1	0	1	1	0.1
25	0	1	1	1	1	0.1

IMPORTANCE SAMPLING

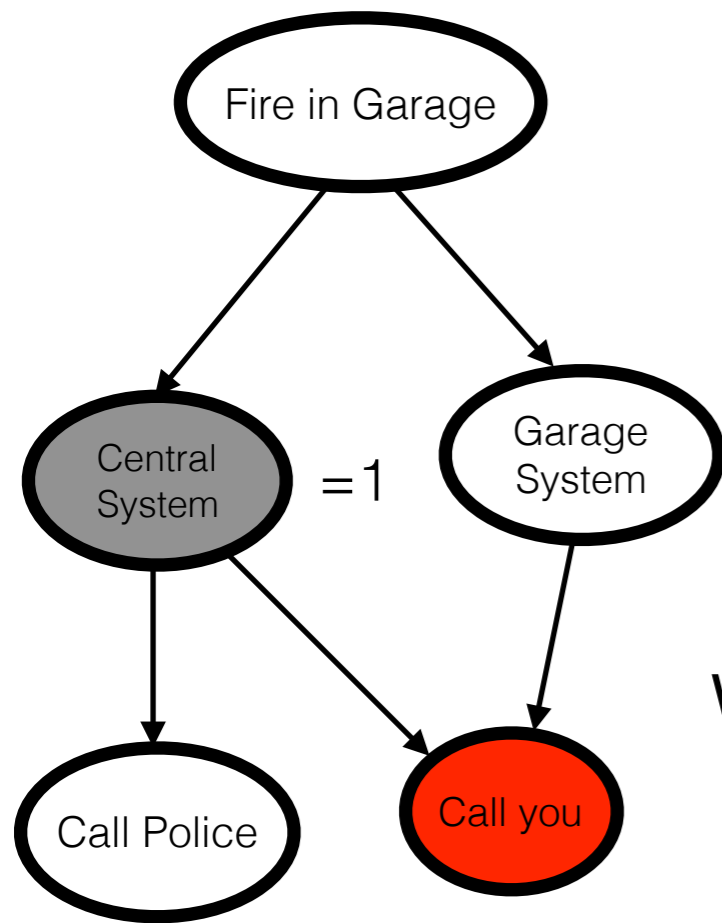


IMPORTANCE SAMPLING



What we want: Draw from $P(F,GS,P,Y | CS=1)$

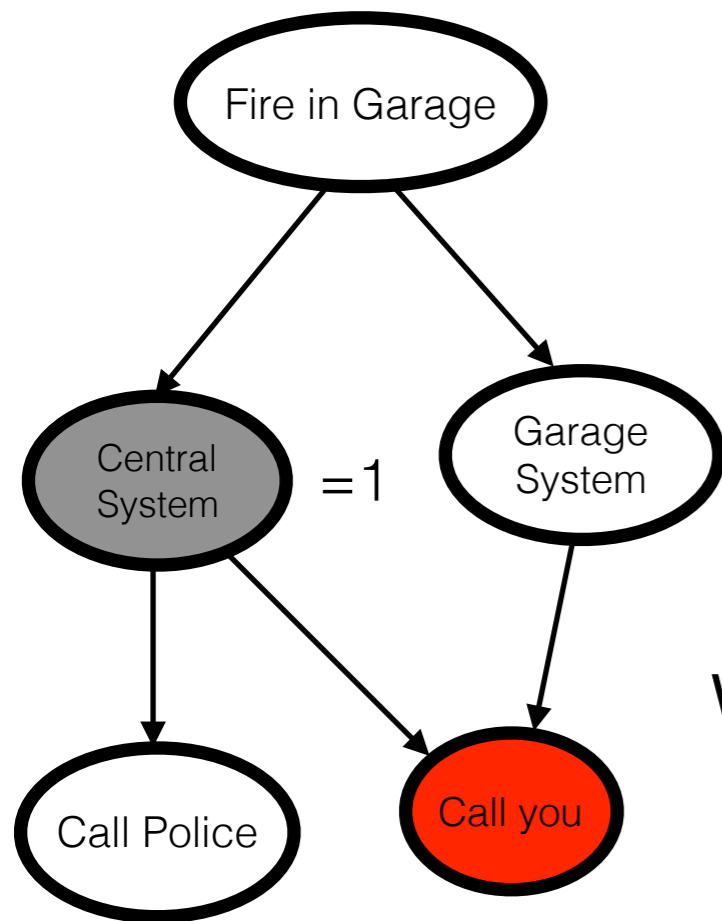
IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

What we want: Draw from $P(F,GS,P,Y | CS=1)$

IMPORTANCE SAMPLING

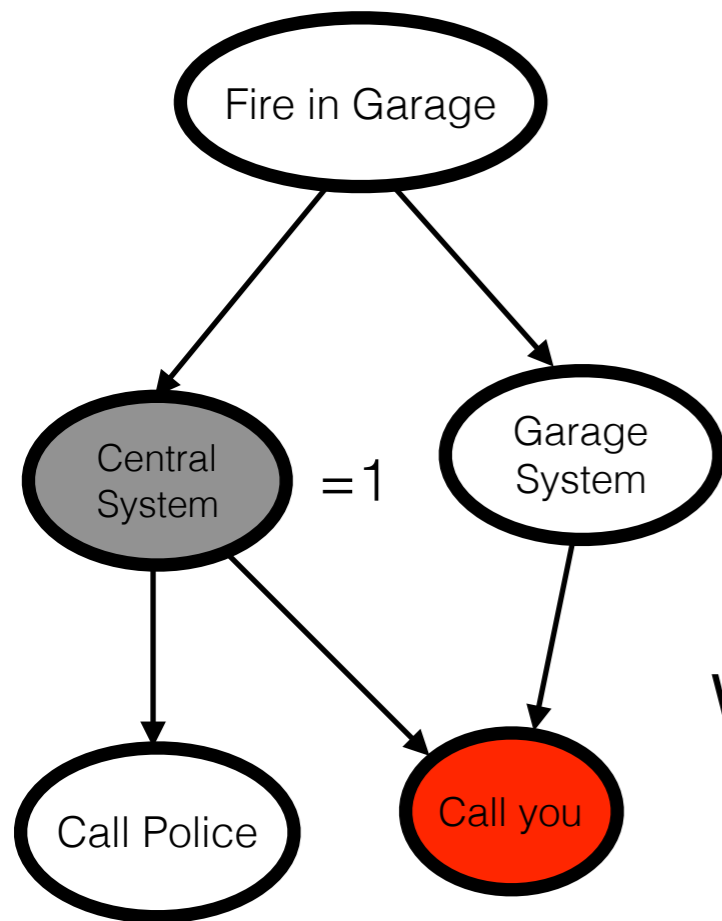


	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

What we want: Draw from $P(F, GS, P, Y | CS=1)$

$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) :$$

IMPORTANCE SAMPLING



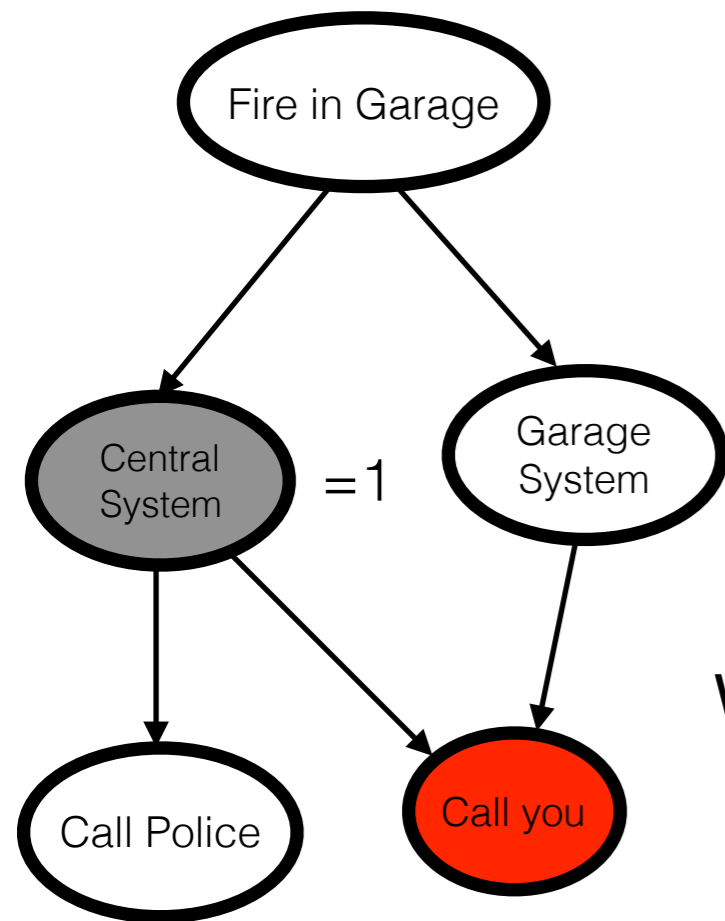
	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

What we want: Draw from $P(F, GS, P, Y | CS=1)$

$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) :$$

Instead we draw from ?

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

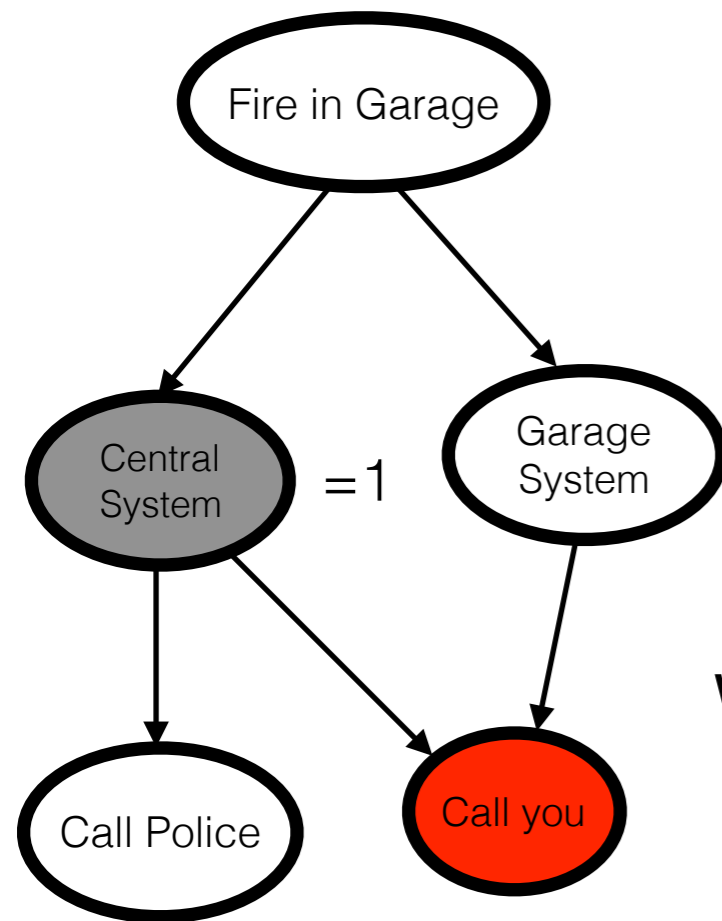
What we want: Draw from $P(F, GS, P, Y | CS=1)$

$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) :$$

Instead we draw from ?

$$P(F = 0)$$

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

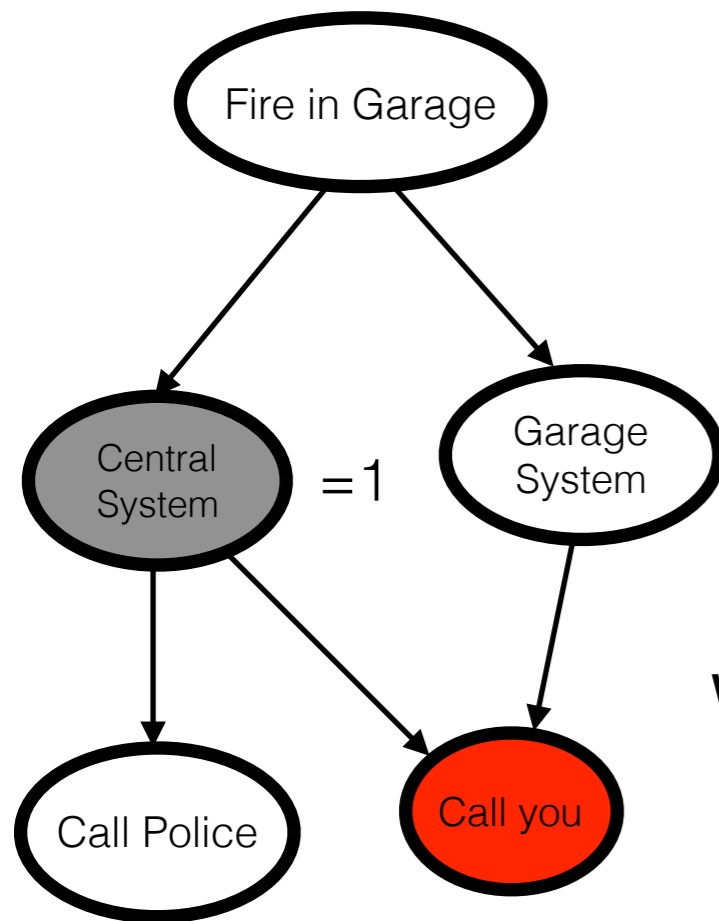
What we want: Draw from $P(F, GS, P, Y | CS=1)$

$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) :$$

Instead we draw from ?

$$P(F = 0) \times P(GS = 0 | F = 0)$$

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

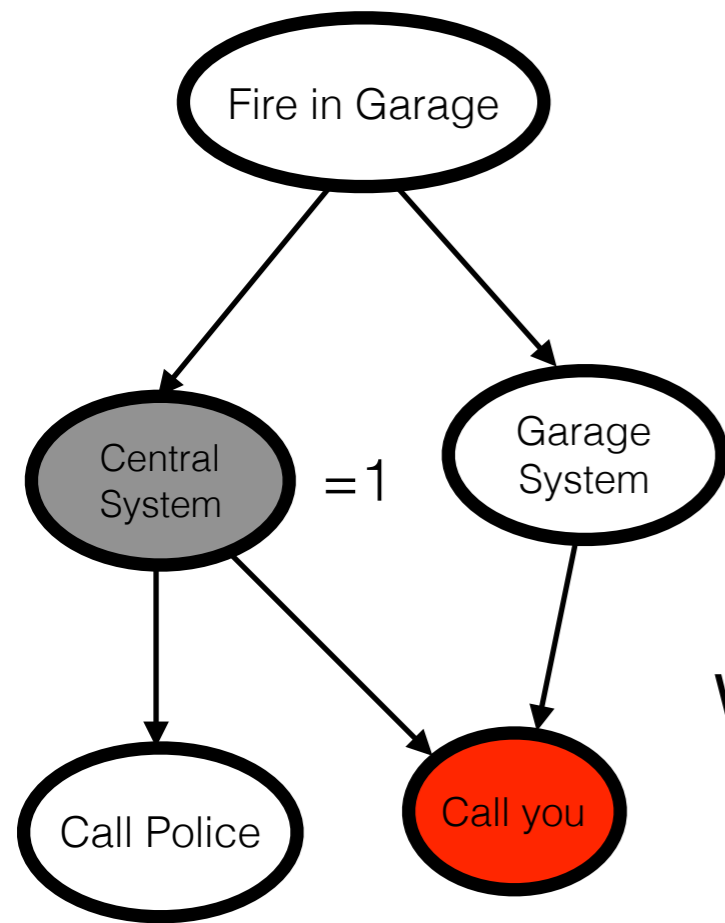
What we want: Draw from $P(F, GS, P, Y | CS=1)$

$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) :$$

Instead we draw from ?

$$P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1)$$

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

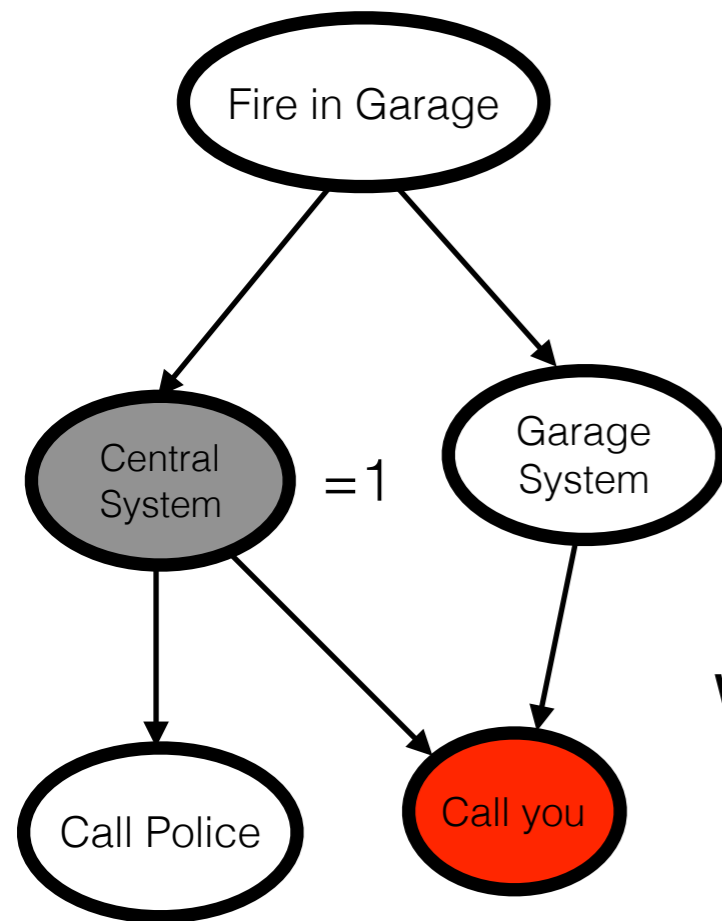
What we want: Draw from $P(F, GS, P, Y | CS=1)$

$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) :$$

Instead we draw from ?

$$P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1) \times P(Y = 0 | CS = 1, GS = 0)$$

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

What we want: Draw from $P(F, GS, P, Y | CS=1)$

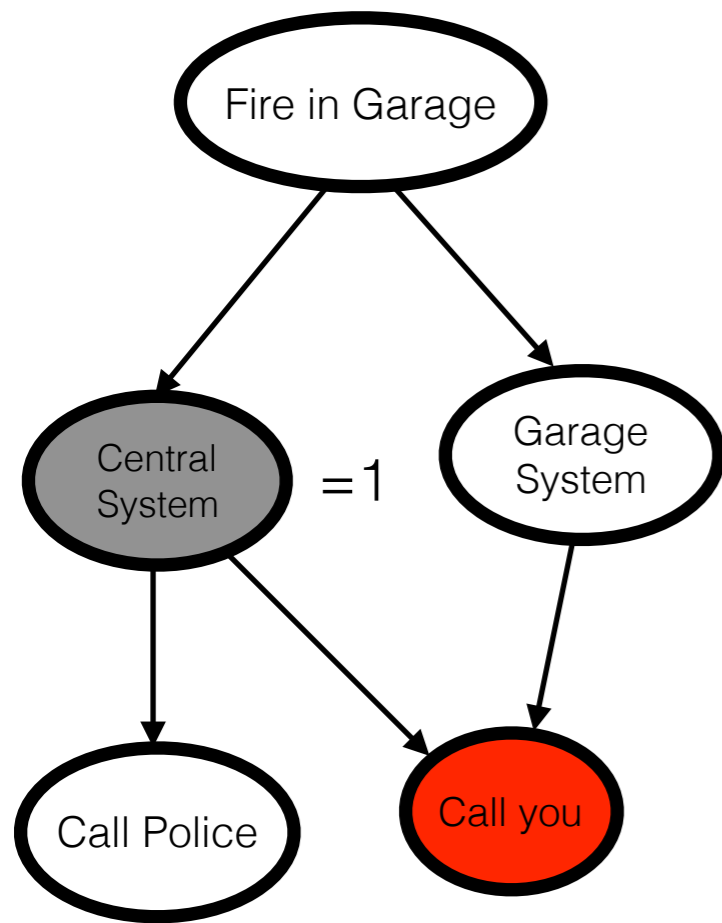
$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) :$$

Instead we draw from ?

$$P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1) \times P(Y = 0 | CS = 1, GS = 0)$$

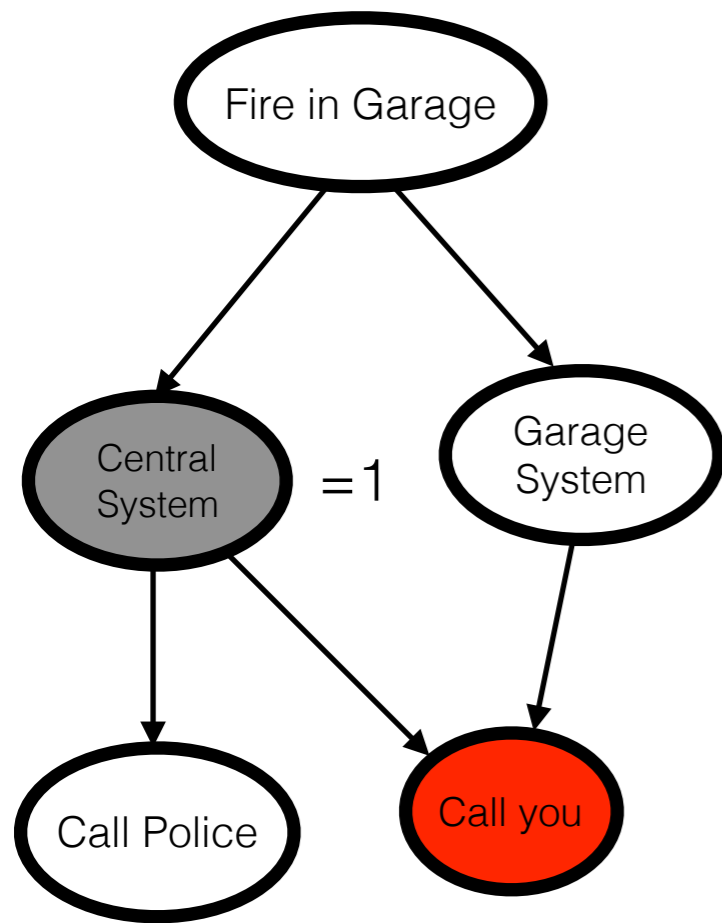
Weigh each sample by ratio of Prob we want / Prob of draw

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

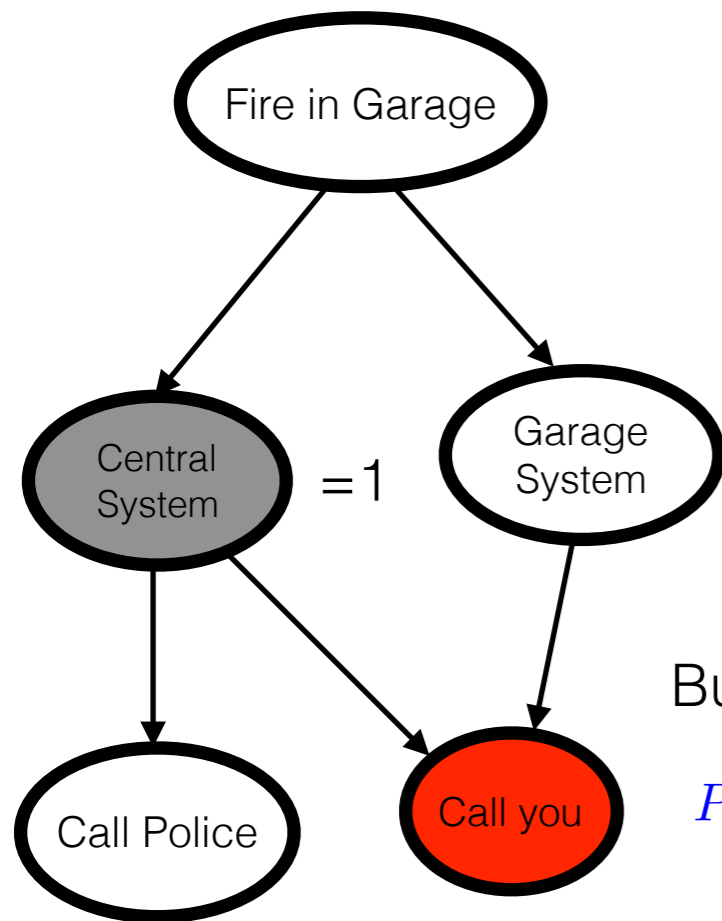
IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

$$\text{weight} \propto \frac{P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1)}{P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1) \times P(Y = 1 | GS = 0, CS = 1)}$$

IMPORTANCE SAMPLING



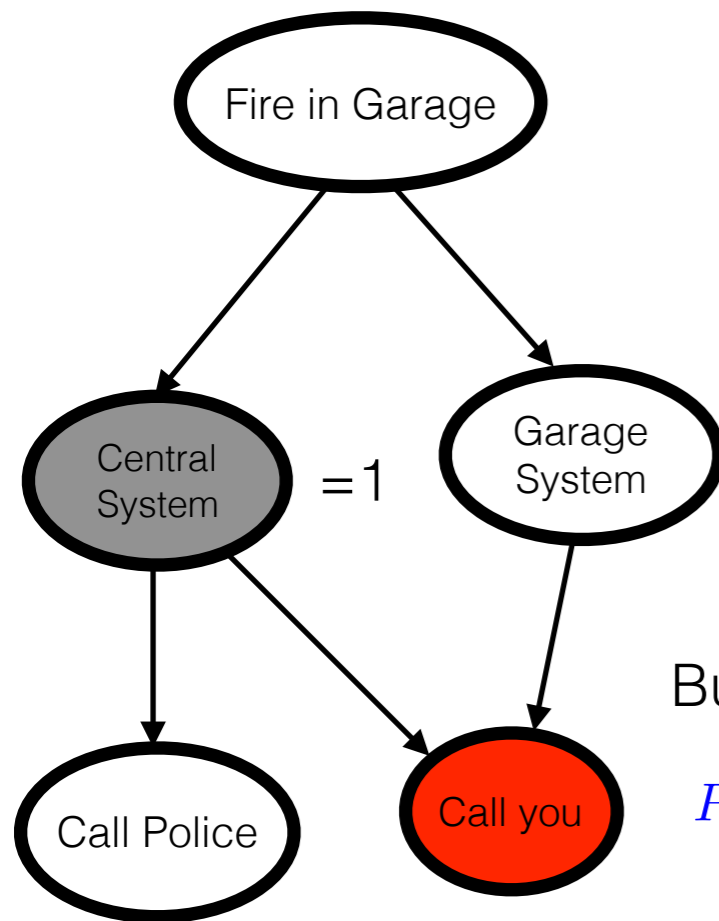
	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

But:

$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) = \frac{P(F = 0, GS = 0, P = 1, Y = 0, CS = 1)}{P(CS = 1)}$$

$$\text{weight} \propto \frac{P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1)}{P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1) \times P(Y = 1 | GS = 0, CS = 1)}$$

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

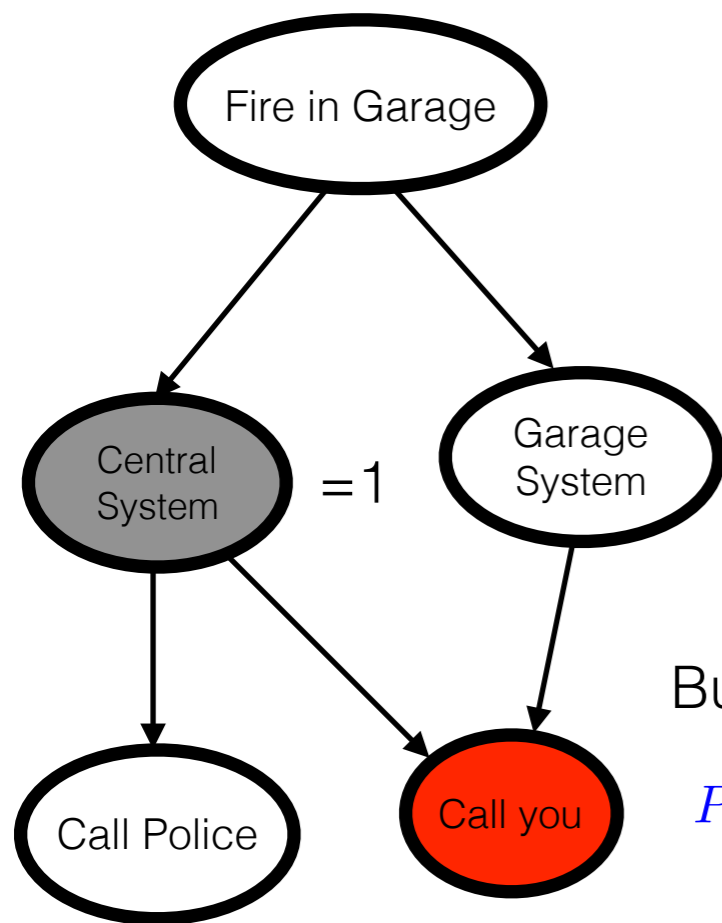
But:

$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) = \frac{P(F = 0, GS = 0, P = 1, Y = 0, CS = 1)}{P(CS = 1)}$$

$$\text{weight} \propto \frac{P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1)}{P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1) \times P(Y = 1 | GS = 0, CS = 1)}$$

$$\propto \frac{1}{P(CS = 1)} \cdot \frac{P(F = 0, GS = 0, P = 1, Y = 0, CS = 1)}{P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1), P(Y = 0 | CS = 1, GS = 0)}$$

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

But:

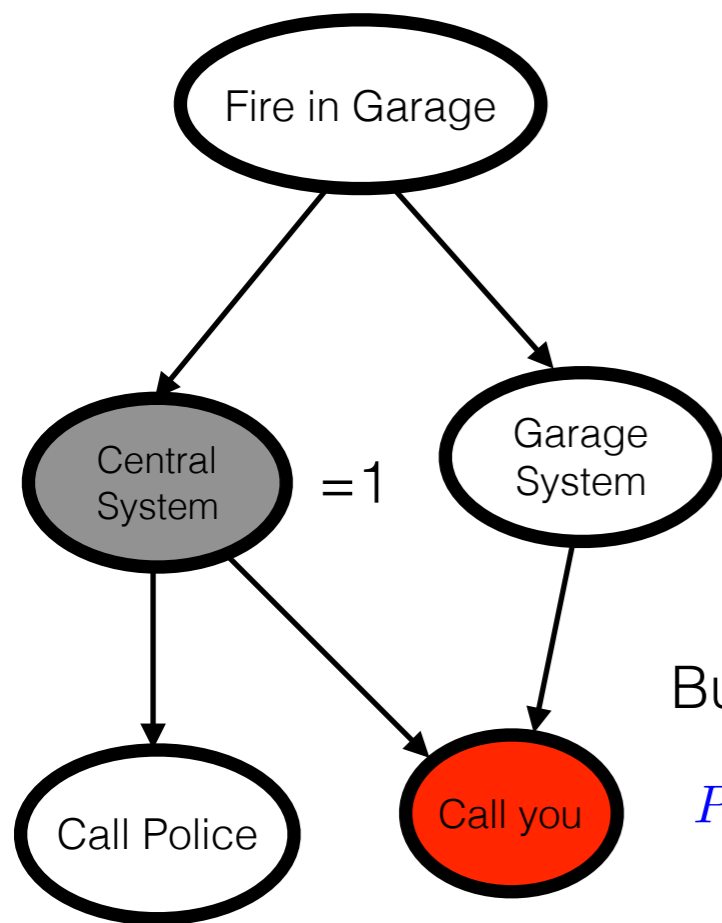
$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) = \frac{P(F = 0, GS = 0, P = 1, Y = 0, CS = 1)}{P(CS = 1)}$$

$$\text{weight} \propto \frac{P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1)}{P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1) \times P(Y = 1 | GS = 0, CS = 1)}$$

$$\propto \frac{1}{P(CS = 1)} \cdot \frac{P(F = 0, GS = 0, P = 1, Y = 0, CS = 1)}{P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1), P(Y = 0 | CS = 1, GS = 0)}$$

$$\propto \frac{P(CS = 1 | F = 0)}{P(CS = 1)}$$

IMPORTANCE SAMPLING



	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	

But:

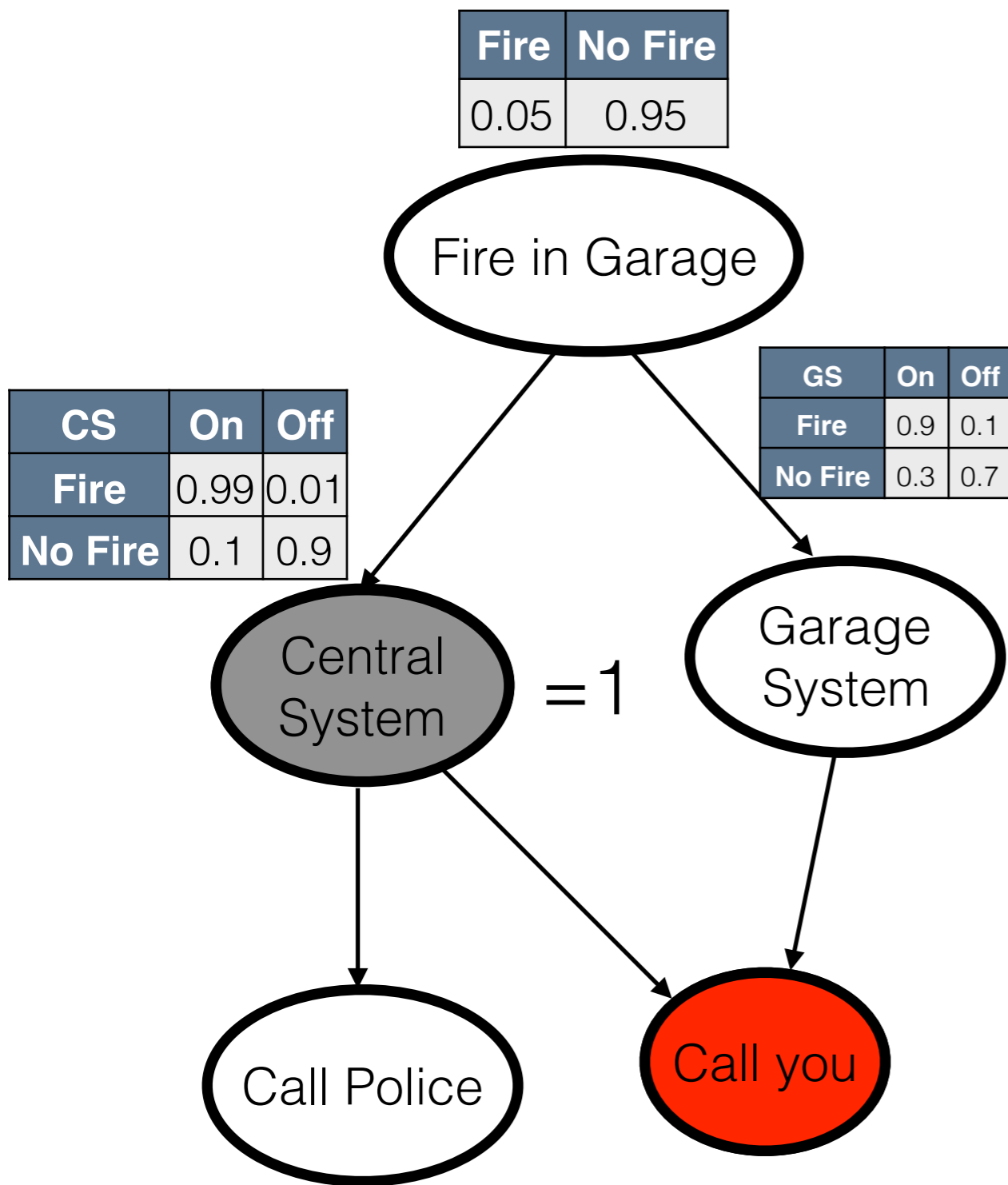
$$P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1) = \frac{P(F = 0, GS = 0, P = 1, Y = 0, CS = 1)}{P(CS = 1)}$$

$$\text{weight} \propto \frac{P(F = 0, GS = 0, P = 1, Y = 0 | CS = 1)}{P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1) \times P(Y = 1 | GS = 0, CS = 1)}$$

$$\propto \frac{1}{P(CS = 1)} \cdot \frac{P(F = 0, GS = 0, P = 1, Y = 0, CS = 1)}{P(F = 0) \times P(GS = 0 | F = 0) \times P(P = 1 | CS = 1), P(Y = 0 | CS = 1, GS = 0)}$$

$$\propto \frac{P(CS = 1 | F = 0)}{P(CS = 1)} \propto P(CS = 1 | F = 0)$$

IMPORTANCE SAMPLING



P	Call	no call
CS on	1	0
CS off	0.01	0.99

Y		Call	no
CS	GS	0.95	0.05
CS	GS	0.7	0.3
CS	GS	0.9	0.1
CS	GS	0.01	0.99

	F	CS	GS	P	Y	Weight
1	0	1	0	1	0	0.1
2	0	1	1	1	1	0.1
3	1	1	1	1	1	0.99
4	0	1	0	1	1	0.1
5	0	1	0	1	1	0.1
6	0	1	0	1	1	0.1
7	0	1	1	1	1	0.1
8	0	1	0	1	0	0.1
9	0	1	0	1	1	0.1
10	0	1	1	1	1	0.1
11	0	1	0	1	1	0.1
12	0	1	1	1	0	0.1
13	0	1	0	1	1	0.1
14	0	1	1	1	1	0.1
15	0	1	0	1	1	0.1
16	0	1	0	1	1	0.1
17	0	1	0	1	1	0.1
18	0	1	0	1	1	0.1
19	0	1	1	1	1	0.1
20	0	1	0	1	1	0.1
21	0	1	0	1	1	0.1
22	0	1	0	1	1	0.1
23	0	1	1	1	1	0.1
24	0	1	0	1	1	0.1
25	0	1	1	1	1	0.1