1 PCA Handout

Let $\mathbf{x}_1, \ldots, \mathbf{x}_n$ be d dimensional vectors. Denote the mean of these vectors by $\mu = \frac{1}{n}\sum_{t=1}^{n} \mathbf{x}_t$. Denote the empirical covariance matrix Σ by

$$\Sigma = \sum_{t=1}^{n} (x_t - \mu)(x_t - \mu)^\top$$

Now let $\mathbf{w}$ be a d dimensional projection vector and the 1 dimensional projection of points $\mathbf{x}_1, \ldots, \mathbf{x}_n$ is obtained by setting

$$y_t = \mathbf{w}^\top \mathbf{x}_t$$

If our goal is to find a w such that w is unit length (i.e. $\|\mathbf{w}\|_2 = 1$) and spread or variance of the y's is maximized then show that the optimization problem we need to solve is:

$$\text{Maximize} \quad \mathbf{w}^\top \Sigma \mathbf{w} \qquad \text{subject to } \|\mathbf{w}\|_2 = 1$$

Start here: We need to find $\mathbf{w}$ s.t. $\|\mathbf{w}\|_2 = 1$ and it maximizes the spread/varinace of y's given by:

$$\text{Variance}(y_1, \ldots, y_n) = \frac{1}{n} \sum_{t=1}^{n} \left(y_t - \frac{1}{n} \sum_{t=1}^{n} y_t \right)^2$$