Linear Classifiers and Perceptrons

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Reading: Mitchell Chapter 4.4-4.4.2

Example: Spam Filtering

	viagra	learning	the	dating	nigeria		*spam?*
$\vec{x}_1 = ($	1	0	1	0	0	$)$	$y_1 = -1$
$\vec{x}_2 = ($	0	1	1	0	0	$)$	$y_2 = +1$
$\vec{x}_3 = ($	0	0	0	0	1	$)$	$y_3 = -1$

- Instance Space X:
  - Feature vector of word occurrences => binary features
  - N features (N typically > 50000)
- Target Concept c:
  - Spam (-1) / Ham (+1)

Linear Classification Rules

- Hypotheses of the form
  - unbiased: $h_{\vec{w}}(\vec{x}) = \begin{cases} +1 & w_1 x_1 + \ \ldots + w_N x_N > 0 \\ -1 & else \end{cases}$
  - biased: $h_{\vec{w},b}(\vec{x}) = \begin{cases} +1 & w_1 x_1 + \ \ldots + w_N x_N + b > 0 \\ -1 & else \end{cases}$
  - Parameter vector $\vec{w}$, scalar b
- Hypothesis space H
  - $H_{unbiased} = \{\, h_{\vec{w}} \colon \vec{w} \in \Re^N \,\}$
  - $H_{biased} = \left\{\, h_{\vec{w},b} \colon \vec{w} \in \Re^N, b \in \Re \,\right\}$
- Notation
  - $w_1 x_1 + \ \ldots + w_N x_N = \vec{w} \cdot \vec{x}$ and $sign(a) = \begin{cases} +1 & a > 0 \\ -1 & else \end{cases}$
  - $h_{\vec{w}}(\vec{x}) = sign(\vec{w} \cdot \vec{x})$
  - $h_{\vec{w},b}(\vec{x}) = sign(\vec{w} \cdot \vec{x} + \mathrm{b})$

(Batch) Perceptron Algorithm

Input: $S = ((\vec{x}_1, y_1), ..., (\vec{x}_n, y_n))$, $\vec{x}_i \in \Re^N$, $y_i \in \{-1, 1\}$,
$I \in [1, 2, ..]$

Algorithm:

- $\vec{w}_0 = \vec{0}$, $k = 0$
- repeat
  - FOR i=1 TO n
    - IF $y_i(\vec{w}_k \cdot \vec{x}_i) \leq 0$ ### makes mistake
      - $\vec{w}_{k+1} = \vec{w}_k + y_i\vec{x}_i$
      - $k = k + 1$
    - ENDIF
  - ENDFOR
- until I iterations reached

Training Data:

	x_1	x_2		y
$\vec{x}_1 = ($	1	2	$)$	$y_1 = 1$
$\vec{x}_2 = ($	2	1	$)$	$y_2 = 1$
$\vec{x}_3 = ($	-1	-1	$)$	$y_3 = -1$
$\vec{x}_4 = ($	-1	1	$)$	$y_3 = -1$

Example: Reuters Text Classification