Modeling Sequence Data: Markov Models

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Reading:
Manning/Schuetze, Sections 9.1-9.3 (except 9.3.1)
Leeds Online HMM Tutorial (except Forward and Forward/Backward Algorithm)
(http://www.comp.leeds.ac.uk/roger/HiddenMarkovModels/html_dev/main.html)

“Less Naïve” Bayes Classifier

- Example: Classify sentences as insulting / not insulting

text	Insult?
$\bar{x}_1 = (Peter, is, nice, and, not, stupid)$	-1
$\bar{x}_2 = (Peter, is, not, nice, and, stupid)$	+1

- Assumption (l words in document)
  - $P(X = x|Y = +1)$

$$= P(W_1 = w_1|Y = +1) \prod_{i=2}^{l} P(W_\mathrm{i} = w_i|W_{\mathrm{i}-1} = w_{i-1}, Y = +1)$$

  - $P(X = x|Y = -1)$

$$= P(W_1 = w_1|Y = -1) \prod_{i=2}^{l} P(W_\mathrm{i} = w_i|\, W_{\mathrm{i}-1} = w_{i-1}, Y = -1)$$

- Decision Rule

$$h_{less}(x) = \underset{y \in \{+1,-1\}}{\operatorname{argmax}} \left\{ P(Y = y) P(W_1 = w_1|Y = y) \prod_{i=2}^{l} P(W_i = w_i|W_{i-1} = w_{i-1}, Y = y) \right\}$$

Markov Model

- Definition
  - Set of States: $s_1,\ldots,s_k$
  - Start probabilities: $P(S_1=s)$
  - Transition probabilities: $P(S_i=s \mid S_{i-1}=s')$
- Random walk on graph
  - Start in state *s* with probability $P(S_1=s)$
  - Move to next state with probability $P(S_i=s \mid S_{i-1}=s')$
- Assumptions
  - Limited dependence: Next state depends only on previous state, but no other state (i.e. first order Markov model)
  - Stationary: $P(S_i=s \mid S_{i-1}=s')$ is the same for all i

Part-of-Speech Tagging Task

- Assign the correct part of speech (word class) to each word in a document

  "The/DT planet/NN Jupiter/NNP and/CC its/PRP moons/NNS are/VBP in/IN effect/NN a/DT mini-solar/JJ system/NN ,/, and/CC Jupiter/NNP itself/PRP is/VBZ often/RB called/VBN a/DT star/NN that/IN never/RB caught/VBN fire/NN ./."

- Needed as an initial processing step for a number of language technology applications
  - Information extraction
  - Answer extraction in QA
  - Base step in identifying syntactic phrases for IR systems
  - Critical for word-sense disambiguation (WordNet apps)
  - …

Why is POS Tagging Hard?

- Ambiguity
  - He will race/VB the car.
  - When will the race/NN end?
  - I bank/VB at CFCU.
  - Go to the bank/NN!
- Average of ~2 parts of speech for each word
  - The number of tags used by different systems varies a lot. Some systems use < 20 tags, while others use > 400.

The POS Learning Problem

- Example

sentence	POS
$\bar{x}_1 = (I, bank, at, CFCU)$	$\bar{y}_1 = (PRP, V, PREP, N)$
$\bar{x}_2 = (Go, to, the, bank)$	$\bar{y}_2 = (V, PREP, DET, N)$

Hidden Markov Model for POS Tagging

- States
  - Think about as nodes of a graph
  - One for each POS tag
  - special start state (and maybe end state)
- Transitions
  - Think about as directed edges in a graph
  - Edges have transition probabilities
- Output
  - Each state also produces a word of the sequence
  - Sentence is generated by a walk through the graph

Hidden Markov Model

- States: $y \in \{s_1,\ldots,s_k\}$
- Outputs symbols: $x \in \{o_1,\ldots,o_m\}$
- Starting probability $P(Y_1 = y_1)$
  - Specifies where the sequence starts
- Transition probability $P(Y_i = y_i \mid Y_{i-1} = y_{i-1})$
  - Probability that one states succeeds another
- Output/Emission probability $P(X_i = x_i \mid Y_i = y_i)$
  - Probability that word is generated in this state

=> Every output+state sequence has a probability

$$P(x, y) = P(x_1, \ldots, x_l, y_1, \ldots, y_l)$$

$$= P(y_1)P(x_1|y_1) \prod_{i=2}^{l} P(x_i|y_i)P(y_i|y_{i-1})$$

Estimating the Probabilities

- Given: Fully observed data
  - Pairs of output sequence with their state sequence
- Estimating transition probabilities $P(Y_i|Y_{i-1})$

$$P(Y_i = a|Y_{i-1} = b) = \frac{\text{\# of times state a follows state b}}{\text{\# of times state b occurs}}$$

- Estimating emission probabilities $P(X_i|Y_i)$

$$P(X_i = a|Y_i = b) = \frac{\text{\# of times output a is observed in state b}}{\text{\# of times state b occurs}}$$

- Smoothing the estimates
  - Laplace smoothing -> uniform prior
  - See naïve Bayes for text classification
- Partially observed data
  - Expectation Maximization (EM)