

Discriminative vs. Generative Learning

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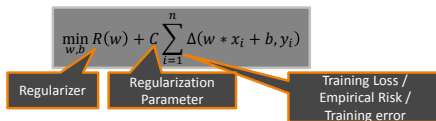
Thorsten Joachims
Cornell University

Reading:
Mitchell, Chapter 6.9 - 6.10
Duda, Hart & Stork, Pages 20-39

Discriminative Learning

- Modeling Step:
 - Select classification rules H to consider (hypothesis space)
- Training Principle:
 - Given training sample $(\vec{x}_1, y_1), \dots, (\vec{x}_n, y_n)$
 - Find h from H with lowest training error
→ Empirical Risk Minimization
 - Argument: low training error leads to low prediction error, if overfitting is controlled.
- Examples: SVM, decision trees, Perceptron

Discriminative Training of Linear Rules



- Soft-Margin SVM
 - $R(w) = \frac{1}{2} w * w$
 - $\Delta(\bar{y}, y_i) = \max(0, 1 - y_i \bar{y})$
- Perceptron
 - $R(w) = 0$
 - $\Delta(\bar{y}, y_i) = \max(0, -y_i \bar{y})$
- Linear Regression
 - $R(w) = 0$
 - $\Delta(\bar{y}, y_i) = (y_i - \bar{y})^2$
- Ridge Regression
 - $R(w) = \frac{1}{2} w * w$
 - $\Delta(\bar{y}, y_i) = (y_i - \bar{y})^2$
- Lasso
 - $R(w) = \frac{1}{2} \sum |w_i|$
 - $\Delta(\bar{y}, y_i) = (y_i - \bar{y})^2$
- Regularized Logistic Regression / Conditional Random Field
 - $R(w) = \frac{1}{2} w * w$
 - $\Delta(\bar{y}, y_i) = \log(1 + e^{-y_i \bar{y}})$

Bayes Decision Rule

- Assumption:
 - learning task $P(X,Y)=P(Y|X) P(X)$ is known
- Question:
 - Given instance x , how should it be classified to minimize prediction error?
- Bayes Decision Rule:

$$h_{\text{bayes}}(\vec{x}) = \operatorname{argmax}_{y \in Y} [P(Y = y | X = \vec{x})]$$

Generative vs. Discriminative Models

Process:

- Generator: Generate descriptions according to distribution $P(X)$.
- Teacher: Assigns a value to each description based on $P(Y|X)$.

Training Examples $(\vec{x}_1, y_1), \dots, (\vec{x}_n, y_n) \sim P(X, Y)$

Discriminative Model

- Select classification rules H to consider (hypothesis space)
- Find h from H with lowest training error
- Argument: low training error leads to low prediction error
- Examples: SVM, decision trees, Perceptron

Generative Model

- Select set of distributions to consider for modeling $P(X,Y)$.
- Find distribution that matches $P(X,Y)$ on training data
- Argument: if match close enough, we can use Bayes' Decision rule
- Examples: naive Bayes, HMM

Bayes Theorem

- It is possible to “switch” conditioning according to the following rule
- Given any two random variables X and Y , it holds that

$$P(Y = y | X = x) = \frac{P(X = x | Y = y) P(Y = y)}{P(X = x)}$$

- Note that

$$P(X = x) = \sum_{y \in Y} P(X = x | Y = y) P(Y = y)$$

Naïve Bayes' Classifier (Multivariate)

- Model for each class

$$P(X = \vec{x} | Y = +1) = \prod_{i=1}^N P(X_i = x_i | Y = +1)$$

$$P(X = \vec{x} | Y = -1) = \prod_{i=1}^N P(X_i = x_i | Y = -1)$$

- Prior probabilities

$$P(Y = +1), P(Y = -1)$$

- Classification rule:

$$h_{naive}(\vec{x}) = \operatorname{argmax}_{y \in \{+1, -1\}} \left\{ P(Y = y) \prod_{i=1}^N P(X_i = x_i | Y = y) \right\}$$

fever (h, l, n)	cough (y, n)	pukes (y, n)	flu?
high	yes	no	1
high	no	yes	1
low	yes	no	-1
low	yes	yes	1
high	no	yes	???

Estimating the Parameters of NB

- Count frequencies in training data
 - n: number of training examples
 - n_+ / n_- : number of pos/neg examples
 - $\#(X_i = x_i, y)$: number of times feature X_i takes value x_i for examples in class y
 - $|X_i|$: number of values attribute X_i can take

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low	yes	yes	1
high	no	yes	???

- Estimating $P(Y)$

– Fraction of positive / negative examples in training data

$$\hat{P}(Y = +1) = \frac{n_+}{n} \quad \hat{P}(Y = -1) = \frac{n_-}{n}$$

- Estimating $P(X|Y)$

– Maximum Likelihood Estimate

$$\hat{P}(X_i = x_i | Y = y) = \frac{\#(X_i = x_i, y)}{n_y}$$

– Smoothing with Laplace estimate

$$\hat{P}(X_i = x_i | Y = y) = \frac{\#(X_i = x_i, y) + 1}{n_y + |X_i|}$$