

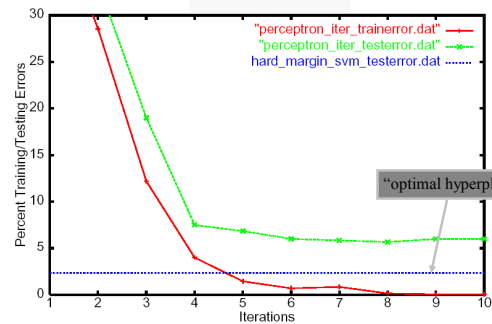
Support Vector Machines: Optimal Hyperplanes

CS4780/5780 – Machine Learning
Fall 2013

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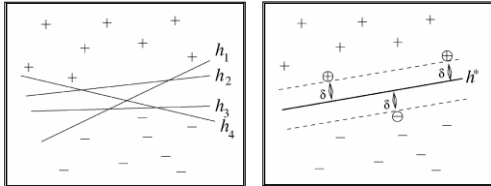
Reading: Schoelkopf/Smola Chapter 7.1-7.3, 7.5

Example: Reuters Text Classification



Optimal Hyperplanes

- Assumption:
 - Training examples are linearly separable.

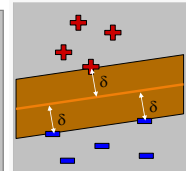


Hard-Margin Separation

- Goal:
 - Find hyperplane with the largest distance to the closest training examples.

Optimization Problem (Primal):

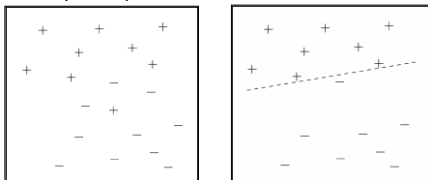
$$\begin{aligned} \min_{\vec{w}, b} \quad & \frac{1}{2} \vec{w} \cdot \vec{w} \\ \text{s.t.} \quad & y_1(\vec{w} \cdot \vec{x}_1 + b) \geq 1 \\ & \dots \\ & y_n(\vec{w} \cdot \vec{x}_n + b) \geq 1 \end{aligned}$$



- Support Vectors:
 - Examples with minimal distance (i.e. margin).

Non-Separable Training Data

- Limitations of hard-margin formulation
 - For some training data, there is no separating hyperplane.
 - Complete separation (i.e. zero training error) can lead to suboptimal prediction error.



Soft-Margin Separation

Idea: Maximize margin and minimize training

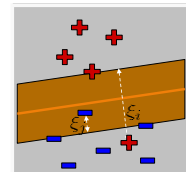
Hard-Margin OP (Primal):

$$\begin{aligned} \min_{\vec{w}, b} \quad & \frac{1}{2} \vec{w} \cdot \vec{w} \\ \text{s.t.} \quad & y_1(\vec{w} \cdot \vec{x}_1 + b) \geq 1 \\ & \dots \\ & y_n(\vec{w} \cdot \vec{x}_n + b) \geq 1 \end{aligned}$$

Soft-Margin OP (Primal):

$$\begin{aligned} \min_{\vec{w}, \xi, b} \quad & \frac{1}{2} \vec{w} \cdot \vec{w} + C \sum_{i=1} \xi_i \\ \text{s.t.} \quad & y_1(\vec{w} \cdot \vec{x}_1 + b) \geq 1 - \xi_1 \wedge \xi_1 \geq 0 \\ & \dots \\ & y_n(\vec{w} \cdot \vec{x}_n + b) \geq 1 - \xi_n \wedge \xi_n \geq 0 \end{aligned}$$

- Slack variable ξ_i measures by how much (x_i, y_i) fails to achieve margin δ
- $\sum \xi_i$ is upper bound on number of training errors
- C is a parameter that controls trade-off between margin and training error.



Controlling Soft-Margin Separation

- $\sum \xi_i$ is upper bound on number of training errors
- C is a parameter that controls trade-off between margin and training error.

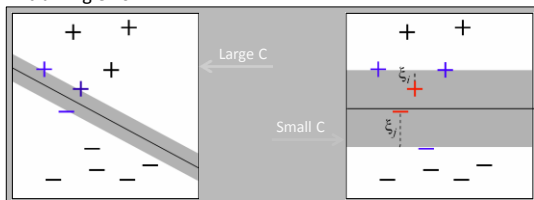
Soft-Margin OP (Primal):

$$\min_{\vec{w}, \xi, b} \frac{1}{2} \vec{w} \cdot \vec{w} + C \sum_{i=1}^n \xi_i$$

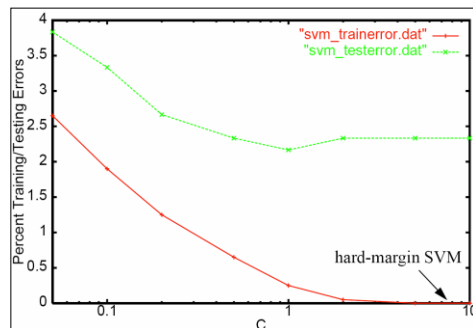
$$s.t. \quad y_1(\vec{w} \cdot \vec{x}_1 + b) \geq 1 - \xi_1 \wedge \xi_1 \geq 0$$

...

$$y_n(\vec{w} \cdot \vec{x}_n + b) \geq 1 - \xi_n \wedge \xi_n \geq 0$$



Example Reuters "acq": Varying C



Example: Margin in High-Dimension

Training Sample S_{train}	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y
	1	0	0	1	0	0	0	1
	1	0	0	0	1	0	0	1
	0	1	0	0	0	1	0	-1
	0	1	0	0	0	0	1	-1
	w_1	w_2	w_3	w_4	w_5	w_6	w_7	b
Hyperplane 1	1	1	0	0	0	0	0	2
Hyperplane 2	0	0	0	1	1	-1	-1	0
Hyperplane 3	1	-1	1	0	0	0	0	0
Hyperplane 4	0.5	-0.5	0	0	0	0	0	0
Hyperplane 5	1	-1	0	0	0	0	0	0
Hyperplane 6	0.95	-0.95	0	0.05	0.05	-0.05	-0.05	0