CS4780 Machine Learning

Prerequisite Test

Instructions

* This quiz must be attempted individually; no collaboration is permitted. However, you may choose to review old material
* The maximum points possible is 20. Each question is annotated with its points available. There are 22 questions in total
* Submission is through CMS
* Weightage for this quiz in the course grade has been announced in class
* Unless noted otherwise, each question has four choices; exactly one choice is correct
* In some questions, the choices are rendered as both math equations and ASCII text. Use the math options to understand the choice, use the text to match the option shown on CMS
* Unattempted questions and wrong choices will receive 0 points

1 Points: 1

A fair coin is tossed repeatedly. What is the probability that Heads comes up for the first time on the \( n \)th toss?

| A. \( \frac{1}{2^n} \) | B. \( \frac{1}{2} \) | C. \( \frac{1}{2^{n+1}} \) | D. \( \frac{1}{2} \) |

2 Points: 1

A biased coin (\( Pr(Heads) = 0.75 \)) is tossed 10 times. What is the probability that Heads comes up exactly 4 times?

| A. \( \frac{3^4}{4^{10}} \) | B. \( \frac{10^4}{2^{10}} \) | C. \( \frac{3^4}{4^4} \) | D. \( \frac{10^4}{10^4} \) |

3 Points: 1

In a class of 150 students, 30 students received A in Math and 50 students received A in Programming Languages. A student is chosen at random from this class; What is the best bound for the probability, \( p \), of that student having received A in either Math or Programming?

| A. \( 0 \leq p < 1 \) \( 0 < p < 1 \) | B. \( \frac{8/15}{1/3} \leq p < 1 \) \( \frac{8/15}{(8/15)} < p < 1 \) | C. \( 1/3 \leq p \leq 8/15 \) \( 1/3 < p < 8/15 \) | D. \( 0 \leq p \leq \frac{8/15}{0 < p < \frac{8/15}} \) |
4 Points: 1

A pair of unbiased 6-sided dice are rolled. Let \( A \) be the event “Sum of rolled faces is 10”, and \( B \) be “Two 5’s were rolled”. What is \( \Pr(B|A) \)?

| A | 1/36 | B | 1/3 | C | 1/6 | D | 1/12 |

5 Points: 1

An urn contains 51 Black and 51 White balls. There is a heap of black balls (more than fifty) nearby. Each turn, we pick 2 balls randomly from the urn, and,
- if both are black, return one ball to the urn and throw the other away
- if both are white, throw both away and enter a black ball from the heap into the urn
- if one black and one white are drawn, return the white ball to the urn, throw the black one away

Each turn, 2 balls are drawn and only one returned. After 101 turns, only one ball will remain. What is the probability that the remaining ball is white?

| A | 0 | B | 1 | C | 1/2 | D | None of these |

6 Points: 1

In an earthquake-prone area (\( \Pr(\text{earthquake}) = 0.4 \)), the probability that a tremor is felt when an earthquake occurs is 5/8. However, tremors can be felt for other reasons too (say, a heavy truck passed by, pneumatic drills etc). On any given day, \( \Pr(\text{tremor}) = 0.75 \). What is the probability that an earthquake occurred, given we felt a tremor?

| A | 1/3 | B | 1/4 | C | 4/10 | D | 3/16 |

7 Points: 1

Consider the roll of a unbiased 6-sided die. \( X \) is a boolean random variable denoting whether the number rolled is even. \( \Pr(X = 1) = ? \)

| A | 1/6 | B | 1/3 | C | 1/2 | D | 0 |

8 Points: 1

Hollywood movies are created on different budgets, and have a wide variety of box office earnings. Let \( X \) be the random variable denoting the IMDb-rating of a released movie (so, \( X \) can take real values in \([0, 10]\)). Let the probability density function for this random variable, \( f \) be given by

\[
f(x) = \begin{cases} 
  x/35 & : x \leq 7 \\
  (10-x)/15 & : \text{otherwise}
\end{cases}
\]

What is the expected value of \( X \)?

| A | 12/5 | B | 5 | C | 7 | D | 17/3 |
9 Points: 1

A biased coin with \( \Pr(Heads) = p \) is tossed \( n \) times. What is the expected number of heads?

\[
\text{A. } np \quad \text{B. } n/2 \quad \text{C. } p/2 \quad \text{D. } p
\]

10 Points: 1

Two cards are drawn from a deck of 32 cards without replacement.

- A be the event “First card drawn is Queen of Hearts”
- B be the event “Second card drawn is Queen of Hearts”
- C be the event “Second card drawn is a Queen”

Which of the following is true?

\[
\begin{array}{|c|c|c|c|}
\hline
\text{A. } & \text{A and B are independent} & \text{B. } & \text{B and C are independent} \\
\text{AB} & \text{BC} & \text{C. } & \text{A and B are conditionally independent given C} \\
& & \text{AB|C} & \\
\hline
\end{array}
\]

11 Points: 0.5 Yes/No

When tossing a fair coin, we note that Heads has come up six times consecutively. Hence, Tails should be more likely to come up on the seventh toss.

\[
\text{A. Yes} \quad \text{B. No}
\]

12 Points: 0.5 Yes/No

If we plot the probability distribution function of a random variable \( X \), the highest point in the graph should be at the expected value of \( X \).

\[
\text{A. Yes} \quad \text{B. No}
\]

13 Points: 1

Consider the vector \( \vec{v} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} \) in \( \mathbb{R}^3 \) (cartesian co-ordinates: \( x \) for the first dimension, \( y \) for the second and \( z \) for the third). The equation of the plane passing through \( \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \) and having \( \vec{v} \) as its normal is?

\[
\begin{array}{|c|c|c|c|}
\hline
\text{A. } & 6x + 2y + 3z = 0 & \text{B. } & x/6 + y/2 + z/3 = 0 \\
\text{C. } & 6x + 2y + 3z = 11 & \text{D. } & x/6 + y/2 + z/3 = 1 \\
\hline
\end{array}
\]
14 Points: 1
System of linear equations in \( x, y, z \) \((x, y, z \in \mathbb{R})\)
\[
\begin{align*}
    x + y + z &= 1 \\
    x &= y \\
    y &= 1 + z
\end{align*}
\]
Which of these is true for the solution set?

A. Empty  |  B. Exactly one  |  C. Infinitely many  |  D. None of these

15 Points: 1
Mike has just reached Manhattan. He finds the streets are parallel to the X-axis, and are spaced 40m apart, the avenues are parallel to the Y-axis and spaced 60m apart. He is at the intersection of 42\(^{nd}\) Street and 7\(^{th}\) Avenue, and he needs to get to 34\(^{th}\) Street and 5\(^{th}\) Avenue. If he can walk only along streets and avenues (no shortcuts), what is the least distance he must travel?

A. 440m  |  B. 340m  |  C. 560m  |  D. \(40\sqrt{73}\)m

16 Points: 1
A line segment is drawn in \( \mathbb{R}^3 \), joining \( \begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix} \) to \( \begin{pmatrix} -4 \\ 4 \\ 8 \end{pmatrix} \). What is the length of the segment?

A. 18  |  B. 13  |  C. \(\sqrt{42}\)  |  D. \(\sqrt{170}\)

17 Points: 1
You are given the 8-dimensional vector \( v = \begin{pmatrix} 3 & 4 & -1 & 2 & -2 & 1 & -2 & -3 \end{pmatrix}^T \)
Let its \( L_1 \)-norm be denoted by \( v_1 \) and its \( L_2 \)-norm by \( v_2 \). Which of these is true?

A. \( 2 = v_1 < v_2 = 12 \)  |  B. \( 12 = v_2 < v_1 = 18 \)  |  C. \( 4\sqrt{3} = v_2 < v_1 = 18 \)  |  D. \( 2 = v_1 < v_2 = 4\sqrt{3} \)

18 Points: 1
Consider the hyperplane given by \( x_1 + 2x_2 + \sqrt{2}x_3 - 3x_4 - 7 = 0 \)
The Euclidean distance of \( \begin{pmatrix} 2 \\ -5 \\ \sqrt{2} \\ -1 \end{pmatrix} \) to this hyperplane is?
19 Points: 1

Consider a real-valued function defined on strictly positive real numbers,

\[ f(x) = x^2 \log_2 x^2 : x > 0 \]

The function achieves its minimum at \( x = ? \)

A. \( \frac{1}{\sqrt{e}} \)  
B. \( \sqrt{e} \)  
C. \( \frac{1}{\sqrt{2}} \)  
D. \( \sqrt{2} \)

20 Points: 1

Consider the set \( S \) of points in \( \mathbb{R}^3 \) which satisfy the following inequalities.

\[
\begin{align*}
  x + y + z &\leq 1 \\
  z &\leq 0.5 \\
  2x + y &\geq 2
\end{align*}
\]

The cardinality of set \( S \) is ?

A. 0  
B. 1  
C. Infinite  
D. None of these

21 Points: 0.5 Yes/No

Let \( W = \begin{pmatrix} 1 & -3 \\ 2 & -2 \\ 3 & 1 \end{pmatrix}, X = \begin{pmatrix} 3 & 2 & 1 \\ -1 & 2 & 3 \\ 0 & -1 & 1 \end{pmatrix}, Y = \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix}, Z = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \)

Then, \( YW - Z^2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \).

A. Yes  
B. No

22 Points: 0.5 Yes/No

Let \( W, X, Y, Z \) be defined as in 21

Then, \( WY - X = \begin{pmatrix} -2 & 3 & -7 \\ 3 & 4 & -7 \\ 1 & 6 & 1 \end{pmatrix} \)

A. Yes  
B. No