1 Particle Filters (70 pts.)

In this question, you will implement a particle filter for the non-linear system defined over three state variables, and given by a deterministic state transition:

\[
\begin{pmatrix}
    x' \\
    y' \\
    \theta'
\end{pmatrix} = \begin{pmatrix}
    x + \cos \theta \\
    y + \sin \theta \\
    \theta
\end{pmatrix}
\]

The initial state estimate has:

\[
\mu = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}
\]
\[
\Sigma = \begin{bmatrix}
    0.01 & 0 & 0 \\
    0 & 0.01 & 0 \\
    0 & 0 & 10000
\end{bmatrix}
\]

For all parts, you should sample on the order of 100 or more particles. When plotting, you can just plot the x-y coordinates of the particles, or use MATLAB’s quiver function or similar to include orientation. When showing propagation, plot particles for a few timesteps, on different axes but with the same limits. You may need to choose non-consecutive timesteps (e.g. 1, 5, 10) to properly illustrate that you have each part working.

A. Give a suitable initial estimate for the particle prior, which reflects the state of knowledge in the gaussian prior.

B. Implement a particle filter and run its prediction step. Compare the resulting prior with the one from your intuitive analysis. What can be said about the resolution of the \(x - y\) co-ordinates and the orientation \(\theta\) in your particle filter?

C. Now let us add a measurement to our estimate. The measurement is a noisy projection of the \(x\)-coordinate of the robot, with covariance \(Q=0.01\). Measure the ground-truth position of the robot using an additional “ground-truth” particle drawn and propagated in the same way as your others.

   Implement the step, compute the result and plot it. Compare this result with your intuition on particle filters.

2 Beyond Probability (10 pts.)

The key idea of probabilistic robotics is to maintain probability distributions over unknown quantities such as robot poses and maps. Can you imagine situations where a probability distribution might be insufficient to accurately characterize the state of knowledge? If yes, describe one. If not, argue why no such situation might exist.
3 Readings (10 pts.)

From the Stanley autonomous vehicle paper: Please describe the motion model, \( P(x'|x) \), used in the Bayes Filter (UKF).

4 POMDP (10 pts.)

Please comment on why POMDPs are hard to apply to real-world problems (in their current form, as discussed in the class).