CS 4758/6758: Robot Learning: Homework 1 (Basics)

Due: Feb 10, 2011, 5pm

1 Probability review: Multivariate Gaussians

Let $X = (x_1, x_2, x_3) \in \mathbb{R}^3$ be a Gaussian random vector with mean μ and variance Σ.

$$\mu = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Are the following also normally distributed? If yes, give the mean and variance of each. (5 points)

A) $x_1 x_2$

B) $x_1 + x_2$

C) Ax, where $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$

2 Transformations and Sensors (20 pts)

We want to measure the acceleration $\alpha \in \mathbb{R}^3$ of a quadrotor. The accelration has three components in x, y, and z directions. We have three sensors that measure scalar acceleration along *their* z-axis.

Because of the mechanical design constraints, we placed the three sensors at different positions and orientations. The orientations of the sensors (as a set of Euler angles[1]) and their output at a single instant of time are given below. Assuming no measurement error, what is the acceleration $\alpha \in \mathbb{R}^3$ of the quadrotor?

Hint: You might have to solve a set of linear equations. (20 points)

α	β	γ	acceleration measured
$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	3
$-\frac{\pi}{3}$	$\frac{\pi}{3}$	$-\frac{\pi}{3}$	4
$\frac{2\pi}{3}$	$-\frac{\pi}{2}$	0	-1

3 Forward and Inverse Kinematics (50 pts)

Figure 1 shows the schematic of a robot arm lying in the $X-Y$ plane. The robot has three links each of length $l_1 = 20$ cm, $l_2 = 10$ cm, $l_3 = 10cm$. The first link of the robot is located at point (0,0) on the $X - Y$ plane. The angles at each of these joints are $\theta_1, \theta_2, \theta_3$.

1. **Forward Kinematics**: Given $\theta_1, \theta_2, \theta_3$, compute the position of robot's hand (X_{hand}, Y_{hand}). (a) Make rotation matrix[2] for each of these links. (b) Write the homogenous transformation matrices T_1^0, T_2^1 and T_3^2. (3x3 matrix: rotation matrix appended with translation) (c) Write the expression for obtaining

[1] To know about the convention for Euler angles, visit: http://en.wikipedia.org/wiki/Euler_angles

[2] Hint: Rotation matrix would be 2x2.

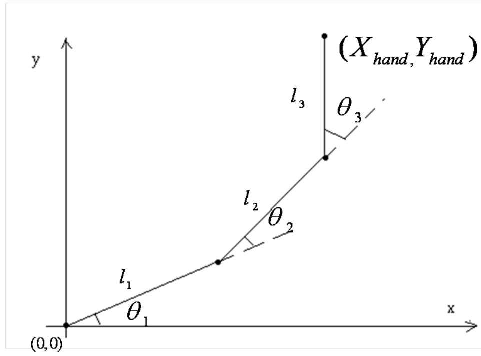

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Figure 1: A planar robot arm with three links.

(X_{hand}, Y_{hand}) as a function of homogenous tranformation matrices. (d) Evaluate the expression for $\theta_1 = 45^o, \theta_2 = 30^o, \theta_3 = 30^o$, and report the (X_{hand}, Y_{hand}).
(15 points)

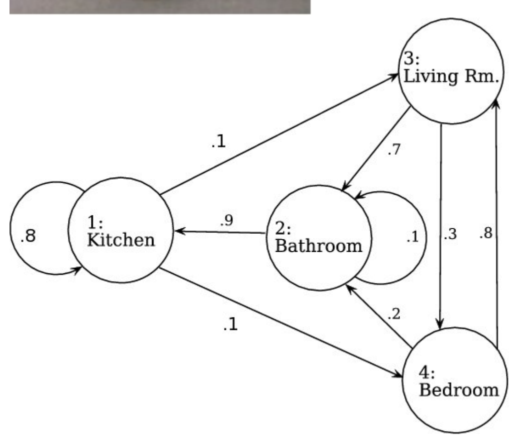
2. **Inverse Kinematics (Programming assignment)**: Given: (X_{hand}, Y_{hand}), find $\theta_1, \theta_2, \theta_3$ (there could be multiple solutions) The angles are constrained to lie in the following ranges: $\theta_1 \in [-30, 30], \theta_2 \in [-30, 30], \theta_3 \in [-45, 45]$.

   Implement a simple inverse kinematics solver to compute poses for our system. You are not allowed to use an existing kinematics solver. You can use any algorithm, but it should be extensible to a different arm. For example, you can use nearest neighbor (with linear interpolation or gradient descent).

   Use your program to compute the angles for the (X_{hand}, Y_{hand}) given in the following file:
   http://www.cs.cornell.edu/courses/CS4758/2011sp/materials/hw1ik.txt
   Submit your code, together with the printout of the computed angles. Make sure that in the text, you also briefly explain your approach. (35 points)

4 Markov Chains (25 pts)

A ground robot is let loose in an apartment with four rooms and begins to randomly wander. After each minute passes, there is a chance it moves to a different room (assume the robot doesn't change rooms more than once a minute). So, for instance, if the robot starts in the kitchen, one minute later there is a .1 probability it moves to the living room, a .1 probability it goes to the bedroom, and .8 chance it remains in the kitchen. Using the information given in Figure on the right, answer the following:



A) If the robot starts at $t = 0$ in the kitchen, what is the probability it is in the living room at $t = 3$?

B) Suppose instead the robot begins at time $t = 0$ in a random room given by the probability distribution p_0. (p_0 is a four-dimensional vector; the first element of p_0 is the probability of starting in the first room, etc.). Then similar distributions $p_1, p_2, ...$ are defined for all subsequent t. Give a general expression for p_t. (Hint: Use 4x4 transition matrices at each time.)

C) Evaluate $\lim_{t\to\infty} p_t$. Does it depend on p_0?

D) Let $p_0 = (.25, .25, .25, .25)$. At $t = 100$, what is the expected probability of time spent in each room?