Previous Lecture

Probability distributions for modeling the sensor data.

Maximum Likelihood approach for estimation.

(First homework problem in HW2 explores this.)
Robot Localization

- Data from sensors is affected by measurement errors.

- We can only compute the probability that the robot is in a given configuration.
Localization

- Robot is placed along a line, but it does not know where.

- What is $p(l)$?
Multimodal Distribution

Robot near ‘a’ or ‘b’

Which distribution to use to model this?
Dirac Distribution

Robot is at ‘a’ with probability 1.0

$$\delta(l - a)$$

$$\delta(l) = \begin{cases} \infty & l = 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{with} \quad \int_{-\infty}^{+\infty} \delta(l) dl = 1$$
Gaussian Distribution

- Used in Kalman filters.
Probabilistic approach for Robot Localization

- Initial probability distribution $p(l)$
- Statistical model characterizing the error of each sensor $p(s|l)$
- Data from the sensor $s=s_i$
- Map of the environment.
Two phases in Robot Operation

- **Action**
  - Take a known action
  - E.g., move the robot by $\Delta S$
  - Often movement is not perfect.
Two phases in Robot Operation

- **Perception**
  - Use sensors to figure out where it is.
  - Sensors are not perfect.
  - Use probability distributions.
Example

- Initial estimate $p(l)$
Example

- Robot uses sonar.
- Sonar statistical error: $p(s \mid l)$

What is $p(l \mid s)$?
Bayes Rule

\[ P(l \mid s) = \frac{p(s \mid l) p(l)}{p(s)} \]
Example 2

• Prob. Distribution of robot position = belief

Initial belief:

\[ p(l) \]

Ashutosh Saxena
Example 2

- The robot moves $\Delta S$, and assume the action is “perfect”.

\[ p_{Enc}(l) = \delta(l - \Delta S) \]

- What is the new robot belief?
Involves a convolution operation

\[ P_{\text{new}}(l) = p(l) \ast P_{\text{enc}}(l) \]

\[ = \sum_{l'} p(l') P_{\text{enc}}(l-l') \]
New Robot Belief

• Before:

\[ p(l) \]

\[ a \quad b \quad c \quad d \]

• After taking the action:

\[ p(l) \]

\[ a + \Delta s \quad b + \Delta s \quad c + \Delta s \quad d + \Delta s \]
Error in Action

- Suppose the action is not perfect, but rather approximately moves the robot.

- What is the new robot belief?
New Robot Belief

- **Before**

\[ p(l) \]

- **After taking the action:**

\[ p(l) \]
Initially, the robot is put somewhere, but not told its location. $P(l)$ is uniform.

**Perception**, robot queries its sensor, but does not know which one.

$$p(l|s_1) = p(s_1|l) \frac{p(l)}{p(s_1)}$$

**Action**, robot moves one meter forward. Error in motion makes $p(l|s)$ smoother.

$$p(l) = p(l) * p_{enc}(l)$$

**Perception**, robot queries again

$$p(l|s_2) = p(s_2|l) \frac{p(l)}{p(s_2)}$$
Where is the robot?

\[ l^* = \arg \max_l p(l | s_1, s_2) \]

Maximum likelihood estimate.

Can also do:
\[ l^* = \arg \max_l \log p(l | s_1, s_2) \]
Summary

- Decide state space, i.e., how to represent ‘l’.

- Choose a statistical model for sensors $P(s \mid l)$

- Choose a statistical model for action $P_{enc}(l)$
  - Only needed when the robot is moving.

- Use *Bayes rule / conditional independence* properties to compute the probability $p(l \mid s_1, s_2, \ldots)$
  - Compute $\arg \max p(l \mid s_1, s_2, \ldots)$
Two approaches

Grid-based localization
- Discretize the state into many cells.
  - E.g., for 1D problem:
    \[ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \]
  - For 3D problem: need \(100 \times 100 \times 100 = 1000000\) cells.

Kalman filter based localization
- Only use Gaussian distribution to model robot motion and sensors.
- Benefit: Only need \(\mu\) and \(\Sigma\) (fewer numbers)
- Cons: ?
Grid-Based (Markov localization)

For a 2D robot (e.g., a car): \((x, y, \theta)\)
- We need 3D grid.

For a 3D robot (e.g., a helicopter)
- We need 6D grid
Example

Let us assume that the robot probability distribution at t=0 is the one below:

9.1 Action Phase:
Let us assume that the robot moves forward with the following statistical model:
After Action phase

- How would $p(l)$ look?

$$p(l) \times p_{Enc}(l) = \sum_{m=0}^{m=9} p[m] \cdot p_{Enc}[l - m]$$

That is:

![Histogram of $p(l)$](image)
Perception phase

\[ p(s \mid l) \]

0.5

0 1 2 3 4 5 6 7 8 9

Ashutosh Saxena
Robot's final belief

Before perception:

After perception
2D localization

- Sensors (laser)
- 2D grid
Sensor Modeling

How to model $p(s|l)$?

Ultrasonic (left)  Laser (right)

Ashutosh Saxena
2D grid: beliefs at different times

- Example 1: Office Building

Courtesy of W. Burgard
Real Example 2