Policy Gradients

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Switch from costs to rewards

All optimal control / planning literature written as costs

All RL literature written as rewards

Cost = -Reward

All min() become max()
The Likelihood Ratio Trick!
Algorithm 20: The REINFORCE algorithm.

Start with an arbitrary initial policy $\pi_\theta$

while not converged do

 Run simulator with $\pi_\theta$ to collect $\{\xi^{(i)}\}_{i=1}^N$
 Compute estimated gradient

$$\tilde{\nabla}_\theta J = \frac{1}{N} \sum_{i=1}^N \left[ \left( \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta \left( a_t^{(i)} | s_t^{(i)} \right) \right) R(\xi^{(i)}) \right]$$

 Update parameters $\theta \leftarrow \theta + \alpha \tilde{\nabla}_\theta J$

return $\pi_\theta$
Causality: Can actions affect the past?

How can we

\[
\nabla_\theta J = \left[ \left( \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta (a_t^{(i)} | s_t^{(i)}) \right) \sum_{t=0}^{T-1} r(s_t, a_t) \right].
\]

\[
\frac{\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta (a_t^{(i)} | s_t^{(i)})}{r(s_t, a_t)}.
\]
The Policy Gradient Theorem

\[ \nabla_\theta J = E_{p(\tilde{z}|\theta)} \left[ \sum_{t=0}^{T-1} \left( \nabla_\theta \log \pi_\theta(a_t|s_t) \left( \sum_{t'=0}^{t-1} r(s_{t'}, a_{t'}) + \sum_{t'=t}^{T-1} r(s_{t'}, a_{t'}) \right) \right) \right] \\
= E_{p(\tilde{z}|\theta)} \left[ \sum_{t=0}^{T-1} \left( \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=t}^{T-1} r(s_{t'}, a_{t'}) \right) \right], \]
The Policy Gradient Theorem

$$\nabla_{\theta} J = E_{p(\xi|\theta)} \left[ \sum_{t=0}^{T-1} \left( \nabla_{\theta} \log \pi_{\theta}(a_t|s_t) \left( \sum_{t'=0}^{t-1} r(s_{t'}, a_{t'}) + \sum_{t'=t}^{T-1} r(s_{t'}, a_{t'}) \right) \right) \right]$$

$$= E_{p(\xi|\theta)} \left[ \sum_{t=0}^{T-1} \left( \nabla_{\theta} \log \pi_{\theta}(a_t|s_t) \sum_{t'=t}^{T-1} r(s_{t'}, a_{t'}) \right) \right],$$

$$Q^{\pi_{\theta}}(s_t, a_t)$$

(The reward to go)
The Policy Gradient Theorem

(Finite Horizon Version)

\[ \nabla_\theta J = E_{p(\xi|\theta)} \left[ \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) Q^{\pi_\theta}(s_t, a_t) \right] \]
The Policy Gradient Theorem

(Finite Horizon Version)

$$\nabla_\theta J = E_{p(\xi|\theta)} \left[ \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t) Q^{\pi_\theta}(s_t, a_t) \right]$$

(Infinite Horizon Version)

$$\nabla_\theta J = E_{s \sim d^{\pi_\theta}(s), a \sim \pi_\theta(a|s)} \left[ \nabla_\theta \log \pi_\theta(a|s) Q^{\pi_\theta}(s, a) \right]$$
c) Door opening: This task involves both the arm and the hand working in tandem to open a door. The robot must learn to approach the door, grip the handle, and then pull backwards. This task has more degrees of freedom given the additional arm, and involves the sequence of actions: going to the door, gripping the door, and then pulling away.

Fig. 5: Opening door with flexible handle
The state space is all the joint angles of the hand, the Cartesian position of the arm, the current angle of the door, and last action taken. The action space is the position space of the hand and horizontal position of the wrist of the arm. The reward function is provided as

\[ r = -(d\theta)^2 - (x_{\text{arm}} - x_{\text{door}}) \]

\[ d\theta := \theta_{\text{door}} - \theta_{\text{closed}} \]

We define a trajectory as a success if at any point \( d\theta > 30^\circ \).
a) Valve Rotation: This task involves turning a valve or faucet to a target position. The fingers must cooperatively push and move out of the way, posing an exploration challenge. Furthermore, the contact forces with the valve complicate the dynamics. For our task, the valve must be rotated from 0° to 180°.

Fig. 3: Illustration of valve rotation
Fig. 3: Illustration of valve rotation

The state space consists of all the joint angles of the hand, the current angle of rotation of the valve \([\theta_{\text{valve}}]\), the distance to the goal angle \([d\theta]\), and the last action taken. The action space is joint angles of the hand and the reward function is

\[
r = -|d\theta| + 10 * \mathbb{1}_{|d\theta|<0.1} + 50 * \mathbb{1}_{|d\theta|<0.05}
\]

\[
d\theta := \theta_{\text{valve}} - \theta_{\text{goal}}
\]

We define a trajectory as a success if \(|d\theta| < 20^\circ\) for at least 20% of the trajectory.
Activity
On-policy vs Off-policy

On-policy RL algorithms:

You must collect data according to your current policy to update learner parameters

Off-policy RL algorithms:

Your learner can learn from data from any policy
On-policy vs Off-policy

On-policy RL algorithms:
You must collect data according to your current policy to update learner parameters

Off-policy RL algorithms:
Your learner can learn from data from any policy

When poll is active respond at PollEv.com/sc2582
Are we done?
No!

Three major nightmares with policy gradients
Nightmare 1: High Variance
Consider the following MDP

\[
\pi_\theta(a = U \mid s_0) = \theta \\
\pi_\theta(a = D \mid s_0) = 1 - \theta
\]

Suppose we init \(\theta = 0.5\), and draw 4 samples with our policy
And then apply PG
When Q values for all rollouts in a batch are high?

\[ \nabla_\theta J = E_{s \sim d^{\pi_\theta}(s), a \sim \pi_\theta(a|s)} \left[ \nabla_\theta \log \pi_\theta(a|s) Q^{\pi_\theta}(s, a) \right] \]

Recall that one of the reasons for the high variance is that the algorithm does not know how well the trajectories perform compared to other trajectories. Therefore, by introducing a baseline for the total reward (or reward to go), we can update the policy based on how well the policy performs compared to a baseline.
Solution: Subtract a baseline!

\[ \pi_\theta(a = U \mid s_0) = \theta \]

\[ \pi_\theta(a = D \mid s_0) = 1 - \theta \]

Suppose we subtracted of \( V^\pi(s_0) = 10.5 \) from the reward to go

\[ \nabla_\theta J = E_{d^\pi_\theta(s)} E_{\pi_\theta(a \mid s)} \left[ \nabla_\theta \log(\pi_\theta(a \mid s)) \left(Q^{\pi_\theta}(s, a) - V^{\pi_\theta}(s)\right) \right] \]
Solution: Subtract a baseline!

$$\nabla_\theta J = E_{d^{\pi_\theta}(s)} E_{\pi_\theta(a|s)} \left[ \nabla_\theta \log(\pi_\theta(a|s)) \left( Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s) \right) \right].$$

We can prove that this does not change the gradient.
Solution: Subtract a baseline!

\[ \nabla_\theta J = E_{d^{\pi_\theta}(s)} E_{\pi_\theta(a|s)} \left[ \nabla_\theta \log(\pi_\theta(a|s)) \left( Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s) \right) \right] \]

We can prove that this does not change the gradient

\[ \nabla_\theta J = E_{d^{\pi_\theta}(s)} E_{\pi_\theta(a|s)} \left[ \nabla_\theta \log(\pi_\theta(a|s)) A^{\pi_\theta}(s,a) \right] \]

But turns Q values into advantage (which is lower variance)
Solution: Subtract a baseline!

$$\nabla_\theta J = E_d^{\pi_\theta(s)} E_{\pi_\theta(a|s)} \left[ \nabla_\theta \log(\pi_\theta(a|s)) \left( Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s) \right) \right]$$

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$$\nabla_\theta J = E_d^{\pi_\theta(s)} E_{\pi_\theta(a|s)} \left[ \nabla_\theta \log(\pi_\theta(a|s)) A^{\pi_\theta}(s,a) \right]$$

But turns Q values into advantage (which is lower variance)

Can we justify this move using the PDL?
Nightmare 2: Distribution Shift
What happens if your step-size is large?

\[ \nabla_\theta J = E_d^{\pi_\theta(s)} E_{\pi_\theta(a|s)} \left[ \nabla_\theta \log(\pi_\theta(a|s) \Delta^{\pi_\theta}(s, a)) \right] \]
What happens if your step-size is large?

\[ \nabla_\theta J = E_d^{\pi_\theta(s)} E_{\pi_\theta(a|s)} \left[ \nabla_\theta \log(\pi_\theta(a|s)) A_{\theta_0}(s, a) \right] \]

\[ \hat{A}^{\pi_\theta}(s, a) \]

We are estimating the advantage from roll-outs.
The problem of distribution shift

True Advantage
The problem of distribution shift

Estimated Advantage

True Advantage

Our new policy wants to go all the way to the RIGHT
The problem of distribution shift

True advantage of new policy
The problem of distribution shift

Estimated Advantage

Our new policy wants to go all the way to the LEFT
Recap: Problem with Approximate Policy Iteration

\[ V^{\pi^+}(s_0) - V^\pi(s_0) = \sum_{t=0}^{T-1} \mathbb{E}_{s_t \sim d_t^{\pi^+}} A^\pi(s_t, \pi^+) \]

PDL requires accurate \( Q^\pi_\theta \) on states that \( \pi^+ \) will visit! \((d_t^{\pi^+})\)

But we only have states that \( \pi \) visits \((d_t^\pi)\)

If \( \pi^+ \) changes drastically from \( \pi \), then \(|d_t^{\pi^+} - d_t^\pi|\) is big!
Be stable

Slowly change policies

Keep $d_t^{\pi^+}$ close to $d_t^{\pi}$
Goal: Change distributions slowly

$$\max_{\Delta \theta} J(\theta + \Delta \theta)$$

s.t. $d^{\pi_{\theta+\Delta \theta}}$ is close to $d^{\pi_{\theta}}$

How do we measure distance between distributions?
Goal: Change distributions slowly

\[
\max_{\Delta \theta} J(\theta + \Delta \theta)
\]

s.t. \( KL(d^{\pi_{\theta + \Delta \theta}} || d^{\pi_{\theta}}) \leq \epsilon \)
This gives us a new type of gradient descent

$$\max_{\Delta \theta} J(\theta + \Delta \theta)$$

s.t. $KL(d^{\pi_{\theta+\Delta \theta}} || d^{\pi_{\theta}}) \leq \epsilon$

$$\theta \leftarrow \theta + \eta G^{-1}(\theta) \nabla_{\theta} J(\theta)$$

Where $G(\theta)$ is the Fischer Information Matrix

$$G(\theta) = \mathbb{E}_{s, a \sim d^\pi_\theta} \left[ \nabla_{\theta} \log \pi_{\theta}(a \mid s) \nabla_{\theta} \log \pi_{\theta}(a \mid s)^T \right]$$
This is also called a “natural” gradient

\[
\theta \leftarrow \theta + \eta G^{-1}(\theta) \nabla_\theta J(\theta)
\]

Where \( G(\theta) \) is the Fischer Information Matrix

\[
G(\theta) = \mathbb{E}_{s,a \sim d_\theta^\pi} \left[ \nabla_\theta \log \pi_\theta(a \mid s) \nabla_\theta \log \pi_\theta(a \mid s)^T \right]
\]
“Natural” Gradient Descent

Start with an arbitrary initial policy $\pi_\theta$

while not converged do
  Run simulator with $\pi_\theta$ to collect $\{\xi^{(i)}\}_{i=1}^N$
  Compute estimated gradient

\[
\tilde{\nabla}_\theta I = \frac{1}{N} \sum_{i=1}^N \left[ \left( \sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta \left( a^{(i)}_t | s^{(i)}_t \right) \right) R(\xi^{(i)}) \right]
\]

\[
\tilde{G}(\theta) = \frac{1}{N} \sum_{i=1}^N \left[ \nabla_\theta \log \pi_\theta(a_i|s_i) \nabla_\theta \log \pi_\theta(a_i|s_i)^T \right]
\]

Update parameters $\theta \leftarrow \theta + \alpha \tilde{G}^{-1}(\theta) \tilde{\nabla}_\theta I$.

return $\pi_\theta$

Modern variants are TRPO, PPO, etc
Nightmare 3: Local Optima
The Ring of Fire
The Ring of Fire
The Ring of Fire

Get's sucked into a local optima!!
Idea: What if we had a “good reset distribution?”
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Run REINFORCE from different start states
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