# CS472-Foundations of Artificial Intelligence
# Assignment 3 – Solutions

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## Problem 1(5pt)

I will use the abreviations: Smoke=S, Fire=F, Heat=H, Big=B, Dumb=D

a) VALID: $S \Rightarrow S \Leftrightarrow S \vee \neg S \Leftrightarrow True$

b) SATISFIABLE: $S = True, F = True$ is solution, $S = True, F = False$ is not solution

c) SATISFIABLE: $S = True, F = True$ is solution, $S = False, F = True$ is not solution

d) VALID: $S \vee F \vee \neg F \Leftrightarrow S \vee True \Leftrightarrow True$

e) VALID: $(S \wedge H) \Rightarrow F \Leftrightarrow \neg S \vee \neg H \vee F \Leftrightarrow (\neg S \vee F) \vee (\neg H \vee F) \Leftrightarrow (S \Rightarrow F) \vee (H \Rightarrow F)$

f) VALID: $(S \Rightarrow F) \Rightarrow ((S \wedge H) \Rightarrow F) \Leftrightarrow (S \wedge \neg F) \vee (\neg S \vee \neg H \vee F) \Leftrightarrow (S \vee \neg S \vee \neg H \vee F) \wedge (\neg F \vee \neg S \vee \neg H \vee F) \Leftrightarrow True \wedge True \Leftrightarrow True$

g) VALID: $B \vee D \vee (B \Rightarrow D) \Leftrightarrow B \vee D \vee \neg B \vee D \Leftrightarrow True \vee D \Leftrightarrow True$

h) SATISFIABLE: $B = True, D = True$ is solution, $B = False, D = True$ is not a solution

One can pretty much gues the correct answers by just looking at the expresions (following the intuition). The troublesome are only e) and g) since usually peope think of implication as being causation.

## Problem 2(5pt)

We have: $Mythical \Rightarrow Immortal(1)$, $\neg Mythical \Rightarrow \neg Immortal \wedge Mammal(2)$, $Immortal \vee Mammal \Rightarrow Horned(3)$, $Horned \Rightarrow Magical(4)$.

$\neg Mythical$ is consistent with the 4 expressions (adding it to the 4 one cannot prove $False$), so $Mythical$ cannot be proved. To be even more specific, There are models in witch $Mythical = True$ and models in witch $Mythical = False$.

If $Mythical = True$ we have $Immortal = True$ (from 1), $Horned = True$(from 3) and $Magical = True$(from 4). If $Mythical = False$ we have $Mammal = True$(from 2), $Horned = True$(from 3) and $Magical = True$(from 4). So $Horned = True$ and $Magical = True$ hold in all the models, wich is equivalent with $Horned$ and $Magical$ are provable.

## Problem 3(5pt)

a) $A \wedge B$: A,B need to be true, C and D can have any value. We have $1 * 2^2 = 4$ models.

b) $A \vee B$: A,B can have any value except false, false and C,D can have any value. We have $3 * 2^2 = 12$ models

c) $A \wedge B \wedge C$: A, B, C need to be true, D can have any value. We have $1 * 2 = 2$ models

## Problem 4(5pt)

Representing the fact, "don't go forward if the wumpus is in front of you" as $WumpusAhead \Rightarrow \neg Forward$ just postpones the problem since now we have to express $WumpusAhead$ in using 64 propositional logic sentences. So it is feasible to use this representation, the 64 runes for don't go forward are replaced with 64 rules with conclusion $WumpusAhead$.

## Problem 5(25pt)

a) 4. $\neg\neg\alpha$ (Hypothesis)

5. $\neg\neg\alpha \Rightarrow (\neg\alpha \Rightarrow \neg\neg\alpha)$ (Ax. I with $\alpha = \neg\neg\alpha$ and $\beta = \neg\alpha$)
6. $\neg\alpha \Rightarrow \neg\neg\alpha$ (Modus Ponens on 4,5)
7. $\alpha$ (Modus Ponens on 6,3)

b) **Soundness**

$$AX \vdash \alpha \Rightarrow P.L. \vDash \alpha$$

which means that formulas provable in the axiomatic system (sintactic proof) are valid in propositional logic.

**Completeness**

$$P.L \vDash \alpha \Rightarrow AX \vdash \alpha$$

which means that for every valid formula in propositional logic, there is a sintactic proof in the axiomatic system.

c) Resolution is refutation complete means that adding the negation of the goal to the axiom system (formulation of the problem) and deriving the empty clause is sound and complete with respect to First Order Logic. This is equivalent with a proof by contradiction, as opposed to the proofstyle form the problem where the problem is added to the set of axioms and a direct proof is made (the sintactic proof tries to derive the goal).

# Problem 6(10pt)

The idea is to express the predicates in tems of gender($Female(x)$, $Male(x)$), $Child(x, y)$ and $Spouse(x, y)$. The definitions are:

$Sibling(x, y) \Leftrightarrow x \neq y \exists z\ Child(x, z) \wedge Child(y, z)$
$GrandChild(c, a) \Leftrightarrow \exists b\ Child(c, b) \wedge Child(b, a)$
$GreatGrandParent(a, d) \Leftrightarrow \exists b, c\ Child(d, c) \wedge Child(c, b) \wedge Child(b, a)$
$Brother(x, y) \Leftrightarrow Male(x) \wedge Sibling(x, y)$
$Sister(x, y) \Leftrightarrow Female(x) \wedge Sibling(x, y)$

$Daughter(d,p) \Leftrightarrow Female(d) \wedge Child(d,p)$
$Son(s,p) \Leftrightarrow Male(s) \wedge Child(s,p)$
$AuntOrUncle(a,c) \Leftrightarrow \exists p\ Child(c,p) \wedge Sibling(a,p)$
$Aunt(a,c) \Leftrightarrow Female(a) \wedge AuntOrUncle(a,c)$
$Uncle(a,c) \Leftrightarrow Male(a) \wedge AuntOrUncle(a,c)$
$BrotherInLaw(b,x) \Leftrightarrow \exists m\ Spouse(x,m) \wedge Brother(b,m)$
$SisterInLaw(s,x) \Leftrightarrow \exists m\ Spouse(x,m) \wedge Sister(s,m)$
$FirstCousin(c,k) \Leftrightarrow \exists p\ AuntOrUncle(p,c) \wedge Child(k,p)$
$NthCousin(1,c,k) \Leftrightarrow FirstClousin(c,k)$
$NthCousin(n,c,k) \Leftrightarrow n > 1 \wedge \exists p,f\ Parent(p,c) \wedge NthCousin(n-1,f,p) \wedge$
$Child(k,f)$
$MthCousinNtimesRemoved(0,n,c,k) \Leftrightarrow NthCousin(n,c,k)$
$MthClusinNtimesRemoved(m,n,c,k) \Leftrightarrow m > 0 \wedge \exists p\ MthClosinNtimesRemoved(m-$
$1,n,c,p) \wedge Child(k,p)$

## Problem 7(10pt)

The problem referes to the situation calculus from chapter 7.

Shooting the wumpus makes it dead, but there is no action to make it come alive (use equivalence not implication):

$$\forall a,s\ Alive(Wumpus, Result(a,s)) \Leftrightarrow [Alive(x,y,s) \wedge \\ \neg(a = Shoot \wedge Has(Agent, Arrow, s) \wedge \\ Facing(Agent, Wumpus, s))]$$

Where $x$ and $y$ is the current location of the Agent, and Facing predicate is defined appropriately in terms of locations of the Agent and Wumpus and the orientation of the Agent.

Possesion of the arrow is lost by shooting and there is no way to get it back (equivalence again):

$$\forall a,s\ Has(Agent, Arrow, Result(a,s)) \Leftrightarrow [Has(Agent, Arrow, s) \wedge (a \neq Shoot)]$$

Note: Solutions that express resonable facts about the shoot action in first order logic will be considered correct.

## Problem 8(10pt)

1. {x/A,y/B,z/B}
2. No unifier since $x$ cannot be linked to both $A$ and $B$
3. {y/John,x/John}
4. No unifier since $y$ doesn't unify with $Father(y)$.

## Problem 9(25pt)

**a) Translation in first order logic** The translation in first order logic is direct and most of you got it right. You got 2 pt for each one of the four translations if correct, 1.5 pt if you missed a small thing, 1 pt if you got serious mistakes, 0.5 pt if you tried to answer and 0 pt otherwise.

1. $\forall X\ R(X) \Rightarrow P(X) \lor S(X)$
2. $\forall X\ R(X) \land S(X) \Rightarrow \exists Y\ R(Y) \land A(Y, X)$
3. $\forall X, Y\ S(X) \land A(Y, X) \Rightarrow P(Y)$
4. $\forall X\ R(X)$

**b) Proving "Some professor is in room B11"** The correct solution to the problem involves the following steps:

**Express the goal in first order logic:**

$$\exists X\ R(X) \land P(X)$$

Translate the negation of the goal in CNF (negate and drop the universal quantifier):

$$\neg R(X_1) \lor \neg P(X_1) \tag{1}$$

**Normal Form:** Translate the first order formulas at point a) in CNF.

To translate i) we just drop the universal quantifier and transform implication in disjunction $((p \Rightarrow q) \Leftrightarrow (\neg p \vee q))$:

$$\neg R(X_2) \vee P(X_2) \vee S(X_2) \tag{2}$$

To translate ii) we first use skolemnization to eliminate the existential quantifier on $Y$ by replacing $Y$ with $\mathcal{Y}(X)$, a function that for every $X$ returns an object. After that we drop universal quantifiers and transform implication. We get two clauses in CNF for this first order formula

$$\neg R(X_3) \vee \neg S(X_3) \vee R(\mathcal{Y}(X_3)) \tag{3}$$

$$\neg R(X_4) \vee \neg S(X_4) \vee A(\mathcal{Y}(X_4), X_4) \tag{4}$$

To translate iii) we eliminate implication and drop the universal quantifiers:

$$\neg S(X_5) \vee \neg A(Y_1, X_5) \vee P(Y_1) \tag{5}$$

To translate iv) we skolemnize $X$ by replacing it with an object $\mathcal{X}$:

$$R(\mathcal{X}) \tag{6}$$

**Resolution:** From now on we will use only resolution and unification on two of the clauses to produce a new one until we produce $\Lambda$ (the empty clause).

$$\{X_1/\mathcal{X}\} \quad (6,1) \quad \neg P(\mathcal{X}) \tag{7}$$

$$\{X_5/X_4, Y_1/\mathcal{Y}(X_4)\} \quad (4,5) \quad \neg R(X_4) \vee \neg S(X_4) \vee P(\mathcal{Y}(X_4)) \tag{8}$$

$$\{X_1/\mathcal{Y}(X_3)\} \quad (3,1) \quad \neg R(X_3) \vee \neg S(X_3) \vee \neg P(\mathcal{Y}(X_3)) \tag{9}$$

$$\{X_4/X_3\} \quad (8,9) \quad \neg R(X_3) \vee \neg S(X_3) \tag{10}$$

$$\{X_3/X_2\} \quad (10,2) \quad \neg R(X_2) \vee P(X_2) \tag{11}$$

$$\{X_2/\mathcal{X}\} \quad (6,11) \quad P(\mathcal{X}) \tag{12}$$

$$(7,12) \quad \Lambda \tag{13}$$